Chen, W. et al. (Eds.) (2016). Proceedings of the 24th International Conference on Computers in Education. India: Asia-Pacific Society for Computers in Education

Practical Use of Triangle Block Model for Bridging between Problem and Solution in Arithmetic Word Problems

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Abstract: This paper reports a practical use of triangle block model for learning of arithmetic word problems. The triangle block model has been proposed as a bridging model between conceptual representation of a word problem and quantitative representation of its solution. Based on this model, we have developed an interactive environment where a pupil is able to manipulate an integrated representation of the conceptual and quantitative ones as learning of arithmetic word problems. We also designed a series of lessons to use the environment. The lessons were practically conducted for 75 4th grade pupils (in two classes) in an elementary school for 7 class times. As the results, it was confirmed that (1) the pupils and their responsible teacher accepted the lessons as useful ones, and (2) the lessons had learning effect as improvement of structural understanding for the problems.

Keywords: Problem Comprehension, Bridging Model, Word Problem

1. Introduction

A problem represented by natural language and solved by calculation with quantitative relationships is often called a "word/story problem". Problem-solving exercise of word problems is an important and indispensable step in learning of arithmetic/mathematics, physics and so on. Especially in arithmetic/mathematics leaning, many researchers have already investigated solving process of the word problems and they have agreed that the process is divided into two sub-processes: (1) comprehension phase and (2) solution phase (Polya 1945; Riley et al. 1983; Cummins et al. 1988; Hegarty et al. 1995; Heffernan et al. 1998). They have also agreed that comprehension phase is the main origin of the difficulty of the word problems.

In order to derive an answer of a word problem by quantitative calculation, it is necessary to derive quantitative relationships from the problem representation. Therefore, in the comprehension phase, a learner is required to interpret the representation written by words and to create quantitative relationships. Here, several researches assumed that the product of the comprehension phase is a representation that connecting "problem (conceptual) representation" and "quantitative representation" (Nathan et al. 1992; Reusser 1996; Koedinger and Nathan 2004). We call this connection "bridging" and the product of comprehension is called "Concept-Quantity Representation" (CQ representation for short) in this paper. This framework is illustrated in Figure 1.

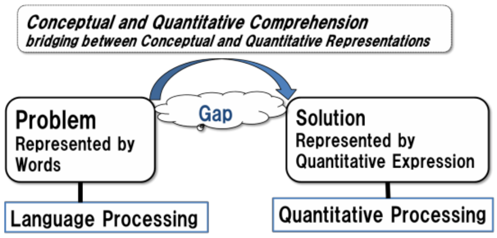


Figure 1. A Framework of Solving Process of Word Problem.

The framework shown in Figure 1 suggests that a gap in the representations between a problem and its solution is the main origin of the difficulty of the word problems. For the representation of the problem, there are a formal expression and its processing method as a natural language and its grammar. For the representation of the solution, there are also a formal expression and its processing method as mathematics. However, regarding the connection process between the two representations (in this figure, the process is conceptual and quantitative comprehension), there is no formal one. There are several researches that have attempted to build a model composed of a CQ representation and its treatment (Nathan 1992; Hirashima et al. 1992; Reusser 1996; Chang et al. 2006; Arnau et al. 2013). Although teaching and learning of arithmetic word problems start at elementary school, there is no report about CQ representation that is manipulatable for a pupil at elementary school and diagnosable for a system.

Based on these considerations, "Triangle Block Model" has been proposed as a CQ representation that satisfies following requirements: (1) a pupil is allowed to build the CQ representation, (2) concepts constituting both the CQ representation and the problem representation are the same ones, and (3) the CQ representation is able to be diagnosed (Hirashima et al. 2015). This paper reports the design and development of a series of lessons of arithmetical word problems based on the model. Target pupils are 4th grade pupils in an elementary school who start to learn arithmetic word problems that are solved by using multiple arithmetic operations. One lesson is composed of (I) teacher's teaching of arithmetical word problems with the model and (II) exercises with an interactive learning environment developed based on the model. In the environment, a pupil is able to manipulate the CQ representation. In order to carry out the both steps in the same usual classroom, the learning environment has been implemented on a tablet PC. The series of lessons was conducted for 75 4th grade pupils in two classes. Through this practical use, we have confirmed that the teaching and environment designed based on the model were accepted as a useful method and tool for learning word problems by the pupils and their responsible teacher. We have also found the learning effect of the activities.

In the next section, the framework of the triangle block model is described. Then, design of lessons composed of teacher's teaching and exercises with the learning environment are described. The results and analysis of the lessons are also reported.

2. Triangle Block Model

2.1 Unit Problem and Triangle Block

An arithmetic word problem that can be solved by one of four arithmetic operations is a minimum size problem. We call this problem a unit problem. Because the four arithmetic operations are binary operations, one calculation is composed of three numerical values (that is, two operands and one result) and one arithmetic operation. In an arithmetic word problem, then, each numerical value has its own meaning. In this research, a conceptual expression of a numerical value in an arithmetic word problem is called a quantity concept. Therefore, it is possible to characterize a unit problem by using three quantity concepts and one arithmetic operation. Therefore, in comprehension phase of a unit problem, it is necessary for a learner to recognize three quantity concepts, and to find an arithmetic relation (arithmetic operation) among them. The triangle block model is a representation that visualizes a unit problem. The three quantity concepts are expressed at three apexes and one arithmetic operation is expressed at the base edge as shown in Figure 2.

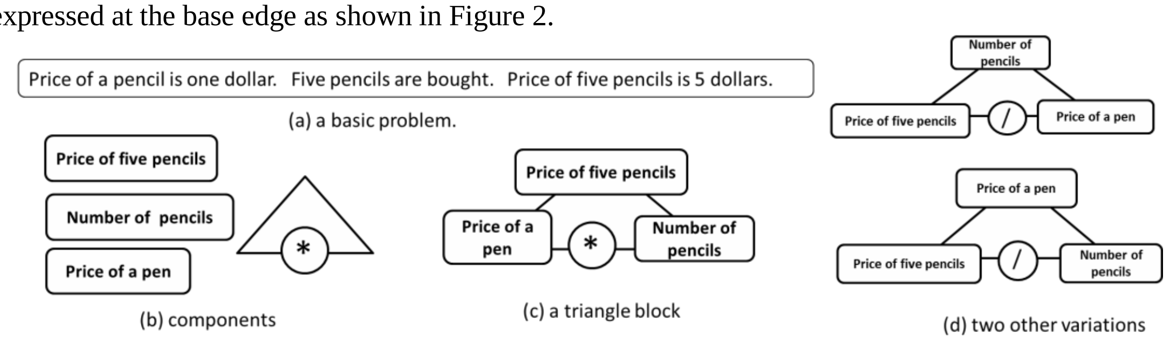


Figure 2. Example of Triangle Block Representation.

Figure 2(a) is a unit problem (story) and Figure 2(b) shows the components characterizing the unit problem. The unit problem includes three quantity concepts, that is, “price of a pen”, “number of pencils”, and “price of five pencils”. Then, following the story of the problem, it is possible to derive “multiplication” as an arithmetic operation. The three quantity concepts are arranged in three apexes of a triangle. In the triangle expression, multiplication as the operation of the story is placed in the base edge of the triangle. Then, two quantity concepts (operands of the operation) arranged in both ends of the base edge. A quantity concept (result of the operation) is placed at the remaining apex opposite to the base edge. By using the components shown in Figure 2(b), the triangle block expression shown in Figure 2(c) is built.

In a unit problem, the quantitative relation can be expressed by three ways in logically. If a problem has a quantitative relation expressed “X*Y=Z”, it includes two more quantitative relations expressed as “Z/Y=X” and “Z/X=Y”. Addition and subtraction have the same relation. These relationships are called “one-addition and two-subtraction” and “one multiplication and two divisions” and have been investigated as an important learning target in arithmetic word problems (Hirashima et al. 2014). A triangle block is an expression that explicitly expresses one operational relation with the base edge. The triangle block also implicitly suggests two remaining operational relations with the two oblique edges of the triangle. The two other triangle blocks are created by changing an oblique edge to the base edge and by making the implicit operation on the oblique edge explicit one. One of the most important characteristics of the triangle block is that the two oblique edges visualize the existence of two other relations between the other two pairs of quantity concepts.

In the case of Figure 2(c), the left oblique edge suggests an implicit arithmetic operation “division” with “price of five pencils” and “price of a pen” as operands and “number of pencils” as a result. Then, the right oblique edge suggests an implicit arithmetic operation “division” with “price of five pencils” and “number of pencils” as operands and “price of a pen” as a result. These two implicit numerical relations in the triangle block shown in Figure 2(c) are explicitly expressed as the triangle blocks shown in Figure 2(d). This rotation of the base edge in a triangle block is a unique and important difference from other CQ representations

2.2 Combination of Triangle Blocks

In the triangle block model, a word problem that is solved by using more than one operation is characterized by using more than one triangle block. For the problem shown in Figure 3, it is possible to find four quantity concepts that correspond to the four sentences in the problem as shown in the right-side of Figure 3. A value of a quantity concept expressed with bold-line rectangle is explicitly given in the problem and a value of a quantity concept expressed with a gray rectangle is an answer of the problem. These four quantity concepts cannot be connected to one directly. So, it is necessary to find a quantity concept that intermediates to connect the quantity concepts given in the problem. In this problem, “price of five pencils” is able to be derived from the pair of “current Tom’s money” and “previous Tom’s money”, or the pair of “price of a pen” and “number of pencils”. By using the derived quantity concept, it is possible to connect the four explicitly given quantity concepts to one. We call this derived quantity concept an intermediate concept.

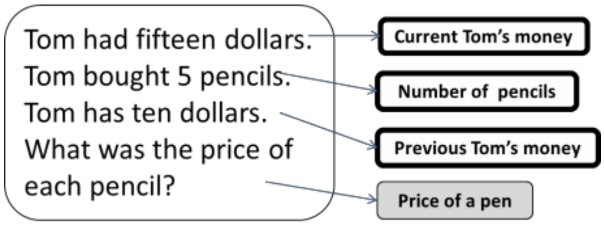


Figure 3. A Word Problem and Quantity concepts.

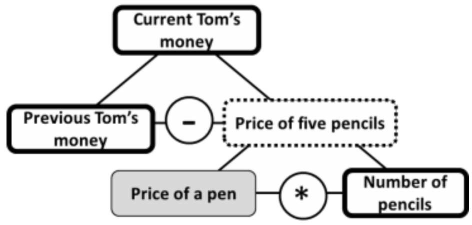


Figure 4. Combined Triangle Block.

By combining the intermediate concept with the two pairs, it is possible to make two triangle blocks. Then, the two triangle blocks can be combined as shown in Figure 4 as an example. The rectangle expressed with dashed line is the intermediate concept. This combined one is called a combined triangle block and the problem corresponding to the combined triangle block is called a combined word problem.

The combined triangle block shown in Figure 4 expresses a series of numerical relation: Previous Tom's money - (Price of a pen * Number of pencils) = Current Tom's money. In this numerical relation, there is only one unknown value: "price of a pen". So, it is possible to calculate the value arithmetically. In the combined triangle block, this means that there is only one unknown node of the nodes that are not connected to another triangle block. In order to form an arithmetic calculatable numerical relation, in the triangle block model, it is allowed only to connect an operand node of a triangle block to a result node of another triangle block.

3. Design of Tasks with Triangle Block Model

In this section, first of all, as a preparation of design of tasks with the triangle block model, two equations for the same problem, one is a problem equation and the other is a calculation equation, are introduced. Then, several tasks designed based on the triangle block model are introduced. These tasks have been implemented in an interactive learning environment called MONSAKUN-TB and used it in practical lessons explained in the next section.

3.1 Problem Equation and Calculation Equation

In the triangle block model, a unit problem is composed of three quantity concepts. Then, a combination of the three concepts decides a quantitative relation among them. The quantitative relation of a unit problem is expressed by three concrete equations that are corresponding to variations of a triangle block explained in Section 2.1. Then, one or two equations have following two roles, one role is expression of natural numerical relation in the story/problem, and the other is expression of calculation procedure to derive its answer. We call an equation with the former role "problem equation" and an equation with the latter role "calculation equation". For example, assume that "a price of a pen" is unknown in the problem shown in Figure 2(a). Because the cover story has a form of "bought several something", an associated operation to the cover story is multiplication. So, the story equation is expressed as "price of a pen" * "number of pencils" = "price of five pencils" that corresponds to the triangle block shown in Figure 2(c). In order to derive the unknown value, it is necessary to calculate "price of five pencils" / "number of pencils" = "price of a pen". This is the calculation equation of the problem. In this case, the problem equation is different from the calculation equation. This type of problem is often called reverse-thinking problem. Assuming that "price of five pencils" is unknown, the calculation equation is the same one with the problem equation. This type of problem is often called forward-thinking problem. As for combined word problems, both problem equation (and corresponding triangle block) and calculation equation (and corresponding triangle block) are important in variations of equations and combined triangle blocks. A triangle block corresponding to problem equation (PE) is called a PE triangle block and a triangle block corresponding to calculation equation (CE) a CE triangle block (in practical classes and implementation, the unknown value is expressed with "?", and "equation" is called "numerical expression with unknown value").

3.2 Design of Tasks

The tasks are categorized into two: one is tasks with a unit problem (Task-I) and the other is tasks with a combined problem (Task-II). In the following this subsection, these tasks are explained. In the tasks, a unit problem is expressed with three sentences. On the right side of Figure 6, an arithmetic word problem is provided. The problem is composed of three sentences. Each sentence expresses a quantity concept, and then, a unit arithmetic word problem is expressed with three sentences. We call this framework to express a unit problem "triplet structure model" (Hirashima et al. 2014) and have already implemented several learning environments for learning by problem-posing for the unit problems (Hirashima et al. 2000, Hirashima et al. 2007, Hirashima and Kurayama 2011). Practical uses of them in classroom with tablet have been also reported (Yamamoto et al. 2012, Yamamoto et al. 2013). We call the series of learning environment called MONSAKUN. Triangle block model is an extension of the triple structure model and a learning environment extended based on the triangle block model is MONSAKUN-TB (MONSAKUN Triangle Block). Assuming pupils have already experienced to deal with unit arithmetic problems expressed with three sentences, the tasks of MONSAKUN-TB are designed.

3.2.1 *Task-I: Tasks with a unit problem*

Tasks in Task-I request a pupil to deal with a unit problem, its equations (PE and CE) and corresponding triangle blocks. Four tasks are designed as follows: (Task-I-1) make a PE triangle block from a unit problem, (Task-I-2) select a correct pair of problem equation and calculation equation for a problem, and then make triangle blocks corresponding to the equations, (Task-I-3) select a correct problem equation for a problem, make a PE triangle block corresponding to the problem equation, and then, select a correct CE triangle block for the PE triangle block, (Task-I-4) pose a problem by combining three sentences and complete a triangle block to the posed problem.

Figure 5 shows interface of Task-I-1 (words in figures of system interface are translated into English from Japanese). On the left side of the interface, a unit problem compose of three sentences is provided. A pupil is requested to complete a triangle block corresponding to the problem on a field of the right side. The field is called “triangle block field”. In the triangle block field, three concept nodes corresponding to sentences and a triangle block with one operation (operation in PE) are provided. With drag and drop manipulation, the three nodes are put on the three apexes in the triangle block. When the pupil pushes “answer” button, the system diagnoses the triangle block and gives the pupil feedback based on the diagnosis.

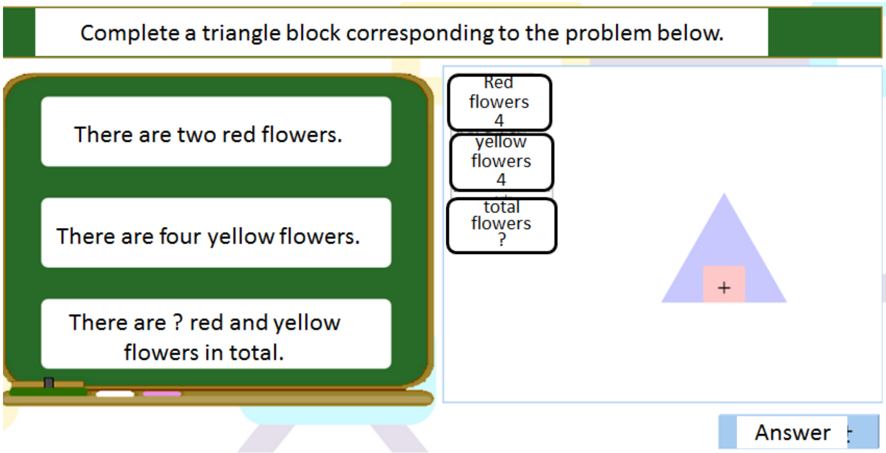


Figure 5. Task-I-1.

Figure 6 shows the interface of Task-I-2. In this task, a unit problem and four pairs of PE and CE are provided, and then, a pupil is requested to select an adequate pair to the unit problem from the four choices. After selecting a pair by clicking an area of a pair, the pupil is requested to push “answer” button. Then, the system diagnoses the selection and gives feedback. Here, when the pupil thinks that there is no adequate choice, the pupil is allowed to push “no correct choice” button. There are a few assignments where “no correct choice” is a right answer. After selecting correct PE and CE, the pupil is requested to complete two triangle blocks corresponding to PE and CE respectively as shown in Figure 7.

The main task of Task-I-3 is to select a triangle block with the same meaning. In triangle block model, a triangle block can be changed to two other triangle blocks keeping their numerical relation as mentioned in 2.1. Here, it is regarded that these three triangle blocks has the same meaning. Figure 8 is the snapshot of this task. In Task-I-4, at first, a pupil is requested to pose a problem as shown in Figure 9. The pupil selects three cards from a set of cards on the right side and puts them in the three blanks on the left side. A request sentence shown in the upper side in the figure is conditions that the posed problem should satisfy. After the pupil correctly posed a problem, he/she is requested to complete a triangle block as shown in Figure 10. In this task, a student is requested to select the arithmetic operation of a triangle by him/herself. When the student clicks an arithmetic operation at the right bottom, a triangle block with the operation appears.

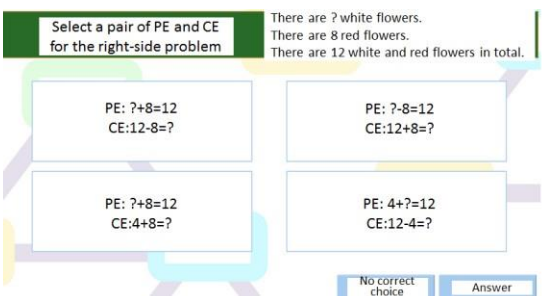


Figure 6. Task-I-2(a).

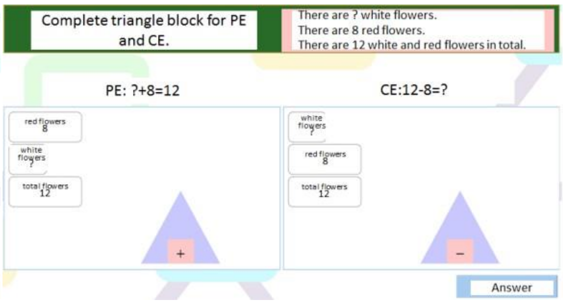


Figure 7. Task-I-2(b).

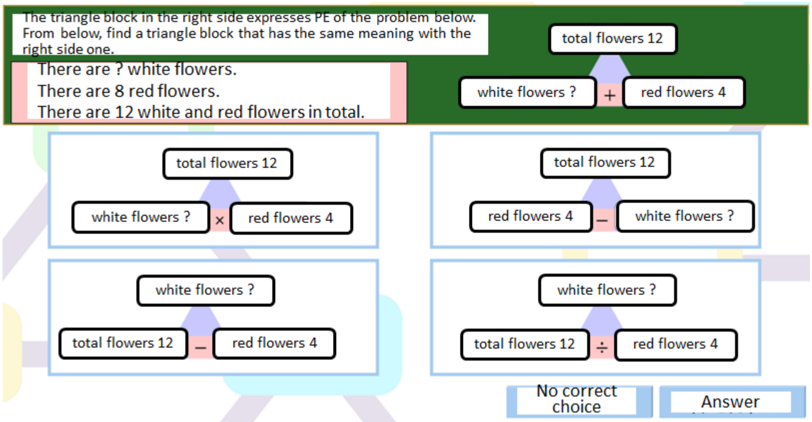


Figure 8. Task-I-3.

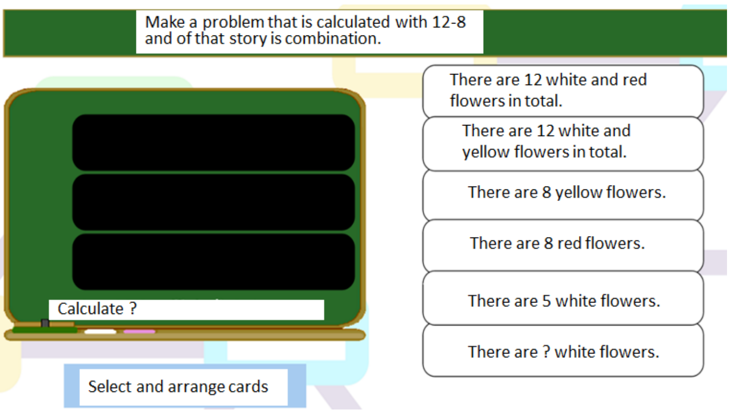


Figure 9. Task-I-4(a).

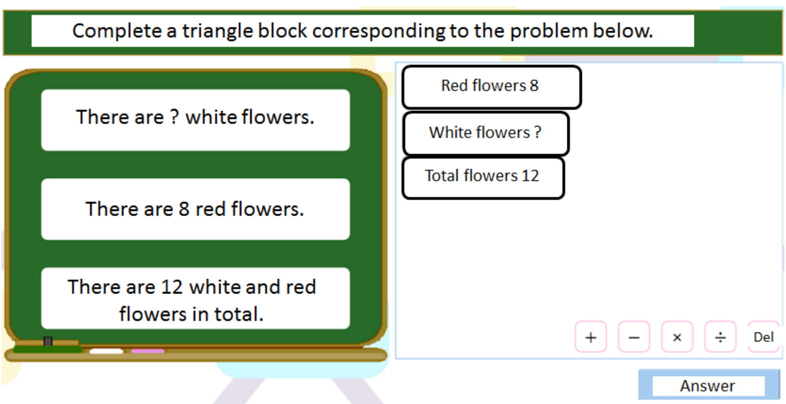


Figure 10. Task-I-4(b).

3.2.2 Task-II: Tasks with a combined problem

Tasks in Task-II request a pupil to deal with a combined problem and its equations and combined triangle blocks. Four tasks are designed as follows: (Task-II-1) build a combined triangle block for a problem by selecting two triangle blocks from provided three and by combining the two, (Task-II-2) selecting a correct problem for a combined triangle block, (Task-II-3) selecting a correct combined triangle block for a problem, and (Task-II-4) selecting a numerical relation for a combined triangle block.

Figure 11 shows Task-II-1. With drag and drop manipulation, by overlapping a common node (in this case, "total price ? yen"), two triangle blocks are connected and a combined triangle block corresponding to the problem is completed. Figure 12 shows Task-II-2. A pupil is requested to interpret the combined triangle block and find a corresponding problem. Figure 13 shows Task-II-3. In Task-II-3, opposite to Task-II-2, a pupil to select a corresponding combined triangle block to a problem. In Task-II-4, as shown in Figure 14, a pupil is requested to select a corresponding numerical relation to a combined triangle block. In these tasks, there are also a few assignments that have no answer. Through conducting these tasks, a pupil is able to experience activities to bridge between a combined problem and a combined triangle block, and between a combined triangle block and equations.

Figure 11. Task-II-1.

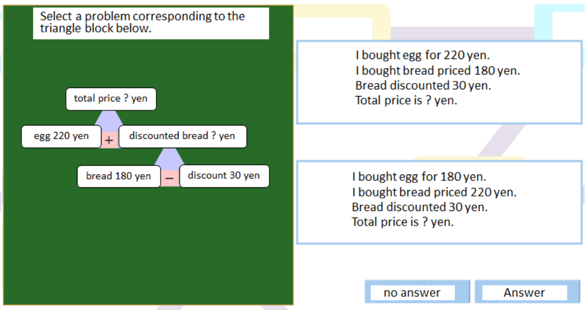


Figure 12. Task-II-2.

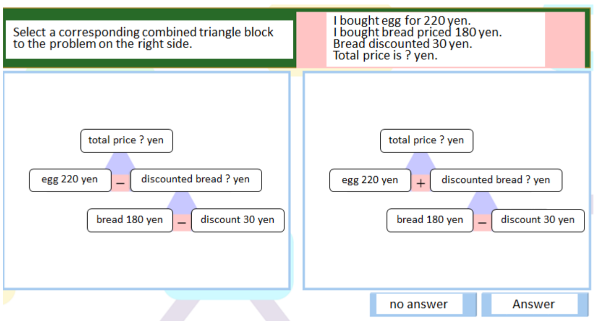


Figure 13. Task-II-3.

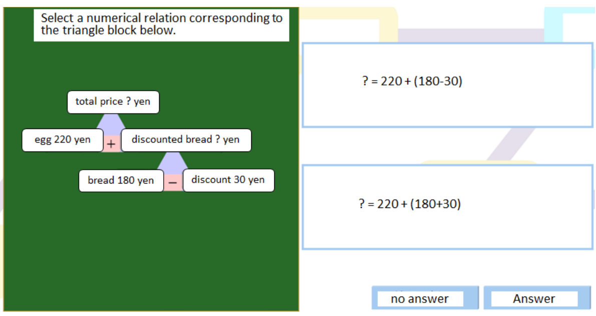
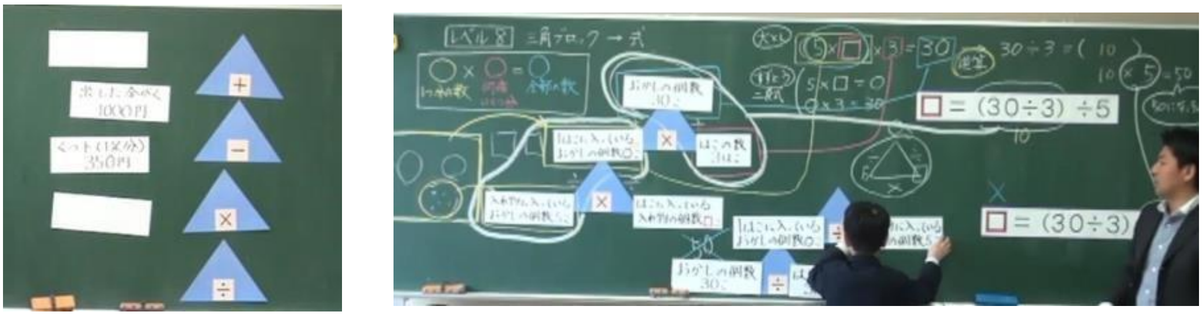


Figure 14. Task-II-4.

3.2.3 Teaching

In the lessons, a responsible teacher introduced the tasks to deal with triangle blocks on a blackboard. In the teacher's introduction, handmade components (a rectangle card with a quantity concept and a triangle with an operation) designed based on the triangle block model were used as shown in Figure 15(a). In this introduction, the teacher usually promoted the pupils to operate and explain the triangle blocks and their components, and to derive numerical expressions by themselves. Figure 15 (b) is a scene a pupil was building a combined triangle block by himself. After the teaching, the pupils used the MONSAKUN-TB.



(a) Components (b) Building of a Combined Triangle Block

Figure 15. Scenes of Teaching with the Triangle Block Model.

4. Practical Use and Results

4.1 Procedure of Practical Use

Seventy-five 4th grade pupils in two classes at an elementary school received 7 lessons (45 minutes per lesson, in 2 weeks) of arithmetic word problems designed based on the triangle block model. They had already learnt combined arithmetic word problems that can be solved by using two operations in a regular curriculum. The responsible teacher of them evaluated that they had no trouble to solve such problems. Therefore, these lessons are regarded as advanced ones. As described in Subsection 3.2, one lesson consists of teacher’s introduction of triangle blocks and exercise with MONSAKUN-TB. Basically, the teaching took from 15 to 25 minutes and the exercise from 20 to 30 minutes.

This use was analyzed by using following three factors: (1) log data of the exercises, (2) questionnaire for the pupils and the teacher, (3) pretest and posttest carried out as a schema priming test (Hirashima et al. 2008; Seta et al. 2014). The pretest was carried out one day before this practice began. The posttest was carried out at the last day of this practice. The schema priming test is a way to evaluate structural understanding for arithmetic word problems. This schema priming test was carried out with software. In this test, sentences constituting a combined problem are provided on the software one by one with two seconds interval. Each sentence includes a quantity concept. After all sentences are provided, the software requests the pupil to find a correct equation for the problem from a list of equations (in case of the problem in Figure 3, “*15- (? * 5) = 10”* is a correct one). If a pupil is able to understand the sentences with structural way, it is possible to build the structure in the priming phase and select a correct equation exactly and quickly. In the test, in addition to 10 usual problems, 10 extraneous problems that include an extraneous sentence (its quantity concept is not used to solve the problem) were used. The extraneous problems are more complex than the usual problems.

4.2 Analysis of the Results

4.2.1 Analysis of Log Data and Questionnaire

For each lesson, 30-40 assignments were prepared in MONSAKUN-TB. A pupil used MONSAKUN-TB for 25.2 minutes and correctly solved 40.9 assignments in average in a lesson (when a pupil solved all assignments, the pupils were allowed to solve the series of the assignments again). If an answer was wrong in an assignment, the pupils were requested to answer it again. They made 0.51 wrong answers to correctly solve an assignment in average.

We conducted a questionnaire for the pupils about the activity using MONSAKUN-TB at the end of the last lesson of this use. Main questions are as follows: Q1: “Did you enjoy the exercise with MONSAKUN-TB?”; Q2: “Do you think that the exercise with MONSAKUN-TB was useful for your learning?". Answers for them were as follows: answers for Q1: {Strong agree: 54, Agree: 21, Disagree: 0, Strong disagree: 0}; Q2: {Strong agree: 35, Agree: 40, Disagree: 0, Strong disagree: 0}. The responsible teacher commented that the pupils were very activated in the lessons than usual. He also commented that the lessons were very useful to let the pupils think arithmetic word problems more deeply. Then, he confirmed that the exercise with MONSAKUN-TB was an indispensable step in the lessons and the pupils engaged in the exercise very eagerly. These results of log data and questionnaire suggest that the lessons were accepted by most of the pupils as meaningful ones. This is important evidence that the triangle block model is a suitable representation for the CQ representation.

4.2.2 Analysis of Results of Pretest and Posttest

In this analysis, accuracy rate (AR (%)) and response time (RT (second)) for the usual problems (P-1) and the extraneous problems (P-2) are used. The posttest was carried out two weeks later of the pretest. In the posttest, the same problems with the posttest were used by permuting the sequence of problems. Two-sided Wilcoxon rank sum test is used for all statistical analyses. As shown in Table 1, in the accuracy rate, there were significant differences between pretest and posttest for both the usual problems and the extraneous problems. In the response time, there were also significant differences between pretest and posttest for the both problems as shown in Table 1. Medium effect sizes were obtained for AR of P-1, AR of P-2 and RT of P-1, and large effect size for RT of P-2. These results suggest that the lessons improve pupil's structural understanding for arithmetic word problems.

As for additional analysis, the pupils were categorized into two groups by using an average of accuracy rate in the pretest of P-1: one is higher group (27 pupils) and the other lower group (48 pupils). In the higher group, although there were no significant differences for the both problems in the accuracy rate, the response time reduced significantly for both problems, as shown in Table 2. In the lower group, the accuracy rate and response time were significantly improved for the both problems, as shown in Table 3. These results suggest that these lessons were more effective for the pupils in the lower group.

Table 1: Results of All Pupils.

All Pupils (75)		Pretest	Posttest	*p*-value	Effect Size (*r*)
AR (%)	P-1	0.70(*SD*=0.14)	0.81(0.12)	2.9e-07	0.42(Medium Size)
	P-2	0.63(0.21)	0.78(0.16)	3.3e-05	0.34(Medium)
RT (second)	P-1	6.1(2.3)	4.1 (1.7)	4.2e-09	0.48(Medium)
	P-2	9.5 (2.7)	7.1(2.5)	7.8e-08	0.55(Large)

Table 2: Results of Higher Group.

Higher Group (27)		Pretest	Posttest	*p*-value	Effect Size (*r*)
AR (%)	P-1	0.84(0.06)	0.84(0.09)	0.856	-
	P-2	0.71(0.23)	0.83(0.16)	0.067	-
RT (second)	P-1	6.0(2.2)	4.2(1.8)	0.0009	0.45(Medium)
	P-2	9.2(2.1)	7.0 (2.6)	0.0013	0.55(Large)

Table 3: Results of Lower Group.

Lower Group (48)		Pretest	Posttest	*p*-value	Effect Size (*r*)
AR (%)	P-1	0.62(0.10)	0.80(0.13)	1.924e-10	0.65(Large)
	P-2	0.59(0.19)	0.75(0.16)	6.843e-05	0.41(Medium)
RT (second)	P-1	6.2(2.3)	4.1(1.6)	2.424e-07	0.53(Large)
	P-2	9.72(2.9)	7.2(2.5)	1.789e-05	0.55(Large)

5. Considerations and Remarks

The results of these lessons suggest that the pupils could use the triangle blocks in the teaching with blackboard and exercise with the learning environment actively, and they thought that the activities were enjoyable and useful for their learning. The responsible teacher also agreed these considerations. Moreover, learning effect was also observed. The teacher commented that the pupils' manipulation and explanation for the components of triangle blocks were really improved through the lessons and the improvement suggested their deep understanding for arithmetic word problems. Based on these results, we have concluded that the triangle block model is a promising as a CQ representation for pupils in elementary school.

This paper is a report of the first trail of practical use of the triangle block model in a specific situation. Based on the results, we will extend this research from the viewpoints of teaching and interactive learning environment, and then will evaluate them through the long-term use by more and various pupils, as our future work.

Acknowledgements

This work was partially supported by JSPS KAKENHI Grant Number 15H02931 and 15K12413.

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