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Inculcating Mathematical thinking through Epistemic Agency

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Abstract. The ability to formulate and apply principles are crucial 21st Century skills. These skills are inherent in Mathematical thinking processes, which require learners to search for abstract problem-solving methods that would serve as analogy-enhancing bridges enabling transfer between different task situations. Consequently, two component skills that need to be mastered are pattern recognition and reasoning. In this exploratory study, we adopted an inquiry-based approach to design technological scaffolds. Wre aimed to investigate the relation between the teacher's beliefs, his design of instructional practices, the design of technology and students' learning outcomes. We discovered that epistemic agency can be used as the core design factor, redefining earlier definitions of context. Furthermore, teachers' beliefs clearly influenced how he sequenced the classroom curriculum, how he relates Mathematical problems with real life, how he identifies, interprets and addresses students' misconceptions (especially with regards to the lower-performing students) and how students should be motivated to learn. From the students' perspective, based on the Technology Acceptance Model, the highest score was for satisfaction when using the system, followed by usefulness (i.e., the system helped them to reason and think), ease of use and opportunities to practice. We inferred from this high score that students liked being challenged by diverse problems of increasing difficulty. comparisons between their pattern recognition and fill in the blanks answers eshowed that our system was able to identify implicitly how students actually think, areas students need further help with and most importantly, may be able to utilize students's thinking strategies to implicitly predict student performance in Mathematics. Based on these findings, we suggest implications to teacher professional development/TPACK.

Keywords: Inculcating Mathematical thinking, epistemic agency, design of technology-mediated environments, teacher professional development

1. Introduction

The 21st century and globalization have challenged teaching-learning beliefs and practices in every classroom. Hence, there is a need to regard classrooms as emergent systems, dynamically adaptive to real-world demands, constantly revising and refining or even designing and developing new teaching-learning models.

Schoenfeld (1992) and Balacheff and Kaput (1996) argue that learning Mathematics is not only about content and algorithms but about Mathematical thinking processes, i.e., helping learners to search for abstract problem-solving methods that would serve as analogy-enhancing bridges enabling transfer between different task situations. In addition, English (2002) highlights that all students should have democratic access to powerful Mathematics ideas. Democratic access involves not only access to content and technology but also cognitive access and provision of more engaging/quality Mathematical learning experiences where understanding and analysis are of a higher educational priority than Mathematical computations. Consequently, one of the priority themes and issues for Mathematics education research in the 21st century is how to prepare Mathematics education researchers (Sowder, 2000) with regards to theories and methodologies related to teacher professional development, students' knowledge building and the design of learning contexts.

Some evidence of technology's crucial role in scaffolding learning in emergent classrooms are Jones, Langrall, Thornton, and Nisbet (2002)'s findings at the elementary school level. These findings are for specific content domains such as Schwartz & Yerushalmy's (1984) Geometric Supposer, Laborde's (1985) Carbi-geometry, Battista (1988) Shape Maker and Yim (2010).

The Humanity-based approach (Chan, 2007) attempts to develop an individual's capacity both from the individual and the social perspectives, drawing from a direct link between research outcomes and practical applications and refinement of theories/research. We are interested in one of the key factors, which determines the design of learning environments and learning activities and thus determines how successfully theory and practice can be bridged, i.e. teacher beliefs. Prior research by Song and Looi (2011) investigates how specific teacher beliefs impact specific teacher practices and how these practices influence the design of student inquiry learning in specific domains in a CSCL environment. Elaboration on these aspects are presented in Table 1.

Table 1. Aspects that influence the design of student inquiry learning

Teacher's beliefs	Teaching philosophy will affect teachers in curriculum planning and teaching methods (Crawford, 2007).
Teacher's practice	Analysis of teachers' teaching experience, understanding the site conditions, the problems encountered in the practice teaching will help the design of more effective technology-assisted learning environment (Song and Looi, 2011).
Students' learning	Conduct a situation analysis to explore the difficulties encountered by the students in learning Mathematics, in order to improve the learning process and outcomes of all students, with special emphasis on lower-achieving students.

Findings confirm that the two teachers' enactment of different beliefs led to different practices. Consequently, these shape the students' opportunities for progressive inquiry and learning. They discover that the teacher who has "innovation-oriented" beliefs is inclined to enact the lesson in patterns of inquiry-principle-based practices as well as enhance learning experiences with the use of technology. These patterns in turn refine student inquiry processes and the effective use of technology affordances.

2. Objectives

Based on the above studies, we aimed to investigate how the teacher's beliefs influence his design of instructional practices and consequently, students' learning outcomes. Next, we hypothesized that it is possible to inculcate Mathematical thinking by regarding its learning as an inquiry process and by regarding epistemic agency as the core design factor for the design of practices in the classroom. We chose to focus on epistemic agency because students' ability to self-direct their own learning (epistemic agency) will determine how meaningful and effective learning will be. Furthermore, to contribute to real community needs, students need to identify and apply principles by themselves.

Consequently, we adopted Song and Looi's (2011) constructs as criteria for evaluating the teacher's beliefs and practices in the classroom and subsequently, attempt to associate and derive implications to students' learning. We expect that if our hypothesis is found to be sound, then our contribution is to confirm the crucial role of with epistemic agency as a key design factor in designing learning context.

3. Related work

The theory of "identical elements", currently the most prevalent notion about transfer, first came into conception from Thorndike's (1906) study. He notes that transfer of learning occurs when both learning source and learning target share common stimulus-response elements. His findings concur with Gage and Berliner's (1983) findings that learning and achievement levels are dependent mainly on learning and achievement prerequisites. The notion that transfer of learning depends on learning and achievement prerequisites eventually led to the development of a hierarchical curricular structure in education, which is often referred to as the spiral learning approach. Hence, designing learning content and activities to help students recognize patterns and incrementally increase difficulty is likely to facilitate transfer and consolidation of learning.

Studies such as by Sandoval and Daniszewski (2004), Tillema and Orland-Barak (2006), Weinberger and Fischer (2006), Crawford (2007), Jacobson, So, Teo, Lee, Pathak, and Lossman (2010) and Song (2011) indicate that teacher beliefs and teaching practices are intrinsically related.

These studies highlight that changes in teachers' beliefs influence their planning decisions and subsequently, the design of curriculum. Hence, by understanding the teacher's beliefs, we can better understand how the curriculum and learning practices are designed (and how to design technological aids) to improve the learning process and outcomes. Furthermore, since learning involves inquiry, teacher practices need to be evaluated based on inquiry instructional principles. Consequently, mapping between student and teacher inquiry processes may help capture specific key elements in inquiry practices.

4. Methodology

This study is exploratory in nature. Hence, we used the mixed method approach. We interviewed an elementary school teacher who has seven years of teaching experience to better understand the socio-cultural contexts within which teacher teach and students learn Mathematical thinking. Subsequently, we demonstrated the system to the teacher and the students used the system for two class periods. We video-captured students' learning processes and evaluated their perception towards our system based on the Technology Acceptance Model (Venkatesh, Morris, Davis & Davis, 2003).

5. System design

Our main objective was to scaffold students to discover the patterns/principles underlying the concept addition (non-carry forward) through implicit inquiry via repeated practice. Each round of practice increases in difficulty. We hoped that once the students were able to recognize these patterns/principles, they would be able to transfer these patterns/principles to answer all questions regardless of the level of complexity. We also hoped that through this system, we could cultivate curiosity in Mathematics and provide the playground for learning Mathematical reasoning after school hours.

We designed and developed two main types of problems, i.e., pattern recognition (Figure 1) and fill in the blanks (Figure 2). For each type of problem, there are different levels. These levels are designed to chunk learning and subsequently, help reduce cognitive load.

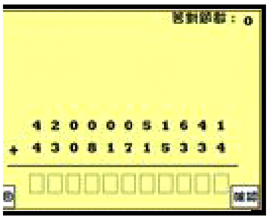

Figure 1. Example of pattern recognition exercise

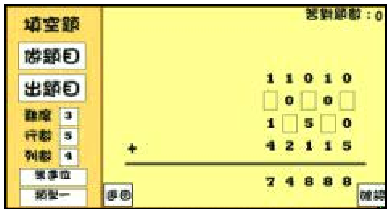

Figure 2. Example of fill-in-the-blanks exercise

5. Findings

The teacher demonstrated "innovation-oriented" beliefs which inclined him to enact the lesson in patterns of inquiry-principle-based practices. He was positive about enriching learning experiences with the use of technology. These beliefs resulted in the students' good scores in the hands-on session. The following subsections elaborate further on these points.

5.1 How do the teacher's beliefs influence his design of teaching strategies?

The teacher whom we interviewed was an elementary school teacher with seven years of teaching experience. We present below our interview findings.

5.1.1 Sequencing

Reference to prerequisite concepts and comparison between examples are two of the more common methods that he used to teach concepts. However, he believed that making comparisons is difficult.

He would begin with the Big Ideas and then zoom in into the examples and calculations. For example, for the problem 1 + 8 + 2, we know that if we calculate 8 + 2 first, it will equal 10. Add on 1 and it will equal 11. The Big Idea here is that numbers can be represented differently using different combinations and grouping of numbers.

5.1.2. Relating Mathematical problems with real life.

He made the lesson interesting by linking learning Mathematics with real life situations. He found that linking problems with real life applications are especially helpful to lower-achieving students.

5.1.3 Identifying students' misconceptions

He identified students' misconceptions of the concepts as a matter of whether they start calculating from the left or from the right. If they started from the left, then they would face difficulty when it comes to problems involving carry forward of place values. He believed that both visualization and linking learning mathematics with real life situations were necessary to create deep processing and understanding.

5.1.4. Motivating students

He believed that students should compare their own current learning with prior progress. If there is a sense of accomplishment, they would be motivated to learn further. Furthermore, students should learn by collaborating, not competing. Such practice will require a change in students' attitudes. Hence, teaching methods should be designed to create not only a sense of accomplishment but more importantly involve a change in students' attitudes towards what constitutes learning and success.

5.2 Teacher's perception towards the design of the problems (pattern recognition and fill in the blanks) posed by our system

The teacher believed that as the students have learned the concept of addition, they should be able to cope with the problems posed by the learning system. For the open-ended fill in the blank questions, he perceived these as challenging but the difficulty level was not beyond the students. Their primary motivation were a cumulative sense of accomplishment, and this motivated students to continue learning. Furthermore, he agreed with the design of the pattern recognition and fill in the blanks exercises because he believed that by practicing, students would be able to identify the patterns in reasoning and consequently grasp the underlying principles.

From the above, both the teacher's beliefs and our system's epistemology were similar. As such, he was able to accept our system's design and regarded our system as having positive usefulness in helping students think deeper to identify the principles underlying the various problems posed by our system. He also regarded the system as having potential in helping students learn the concept of carry forward more effectively.

We will next look at students' learning performance to better understand and identify areas that we need to look further into in designing technological scaffolds that would help even the lower-performing students to improve in Mathematical thinking.

5.3 Students' learning performance with regards to our system

Students were all grade 2 students in one of Taiwan's experimental school. As the students were in grade 2, they were used to answering questions which require them to calculate the sum. They were positive about the system. The highest mean was 4.24 indicating high satisfaction when using the system. The second highest mean was 4.12 indicating that most of the students found the system useful. The third highest mean at 3.92 provided support for why they found the system useful, i.e. in helping them reason and think. Another positive aspect highly rated by students was the ease of use and opportunities to practice with the high means of 3.88 and 3.84 respectively. We inferred from this high score that students liked being challenged by diverse problems of increasing difficulty.

For more detailed analyses, we looked at whether students calculated from right to left or from left to right. For the pattern recognition problems, all the students except two managed to score above 90%. For the pattern recognition problems, fourteen out of twenty three students obtained 100% while seven students obtained 92%. As for the fill-in-the-blank exercises, some students did not do well for the first two questions but subsequently they appeared to have made sense of the problems posed and managed to answer correctly although the difficulty level had increased. Based on the teacher's practice in emphasizing principles over mechanical computation, we inferred that a possible reason was because they were able to identify the principles that were being tested and were able to apply the correct principles.

For the fill-in-the-blank exercises, students made more mistakes in the first two questions. However the number of errors reduced towards the later part. It was possible that students were trying to figure out the principles to apply and once they had figured it out, they managed to perform better.

Four students were consistently calculating from left to right. Three scored 100% for the pattern recognition problem and one scored 92%. Out of the three, two managed to answer all fill in the blanks exercises correctly whereas one answered question two wrongly. Based on comparisons between their pattern recognition and fill in the blanks answers, we predicted that student id 179 would likely perform best in class followed by student id 174.

Eleven students exhibited uncertainty when answering the fill in the blanks problems. However, nine of them were very certain when answering the pattern recognition problems. This implied that they may not have fully grasped the principle being tested. This finding was interesting as it showed that our system was able to identify implicitly how students actually think, areas students need further help with and *most importantly, may be able to predict student performance in Mathematics.* We will investigate deeper into the types of questions they had doubts with, in our future work.

Last but not least, considering that the students' performance with regards to the problems posed by our system is good with only two scores below 90% (the lowest score being 75% and 83%), and the highest 100%, and considering that they regard the system as fun, we inferred that this was due to the good match between teacher beliefs and practices, student' understanding and practices and our system design.

6. Implications to teacher professional development

We suggest some implications for teacher professional development based on Technology, Pedagogy, Content and Knowledge or TPACK.

1) Knowledge: In terms of Knowledge, teachers need to be well-versed with principles, Big Ideas and multiple ways to represent numbers and multiple techniques to calculate different representations. Teacher professional development should expose and train teachers with regards to the above and most importantly, highlight the reasons for each representation or technique.
2) Content: The design of the content can be simple but increasing in degrees of difficulty, placing emphasis on pattern recognition and evaluation of these patterns.
3) Pedagogy: The design of curriculum and its planning should consider the diversity of representations and techniques and their suitability to the students' level taking into consideration aspects 1 and 2 above.
4) Technology: Different teachers would bring with them different beliefs from their past experiences and past learning as students themselves. Hence, the design of supporting or mediating technology should take into account these inherent beliefs and practices from the teacher's past and yet incrementally motivate and scaffold them to explore, experiment techniques from their peers through collaborative learning technologies.

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