

# Developing Digital Technologies for Undergraduate University Mathematics: Challenges, Issues and Perspectives

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**Abstract:** Our research effort presented in this paper relates with developing digital tools for mathematics education at undergraduate university level. It focuses specifically on studies where mathematics is not a core subject but it is very important in order to cope with core subjects. For our design, we adopt a participatory design method, involving collaboration with students and teachers. As a first step in our design, we developed in collaboration with teachers a set of visualization applets using GeoGebra for the “Mathematics for Multimedia Applications” course taught for Media Technology students at Aalborg University Copenhagen. Then we conducted focus groups with students where they reflected on the introduction of these applets and proposed ways to improve them or alternative ways to present the specific part of the curriculum. At the same time, we conducted observations of teachers and students during lectures and exercise time. During these observations we were able to investigate how the applets were used in practice but also to get insight in the challenges that the students face during mathematics learning. These findings together with student feedback inspire the next round of design requirements for the development of digital tools that support mathematics teaching and learning at university level.

**Keywords:** Mathematics teaching and learning, participatory design, technology-enhanced learning, GeoGebra

## 1. Introduction

The importance of mathematics education is widely acknowledged. Its (inter)disciplinary, practical, and cultural value has been highlighted since the first schools made their appearance. Back in the 17<sup>th</sup> century, Locke argued that mastery in mathematics allows children to develop into autonomous, useful individuals who will understand “the natural rights of man,” “will seek the true measures of right and wrong,” and will apply themselves “to that wherein [they] may be serviceable to [their] country” (Locke, 1880). Moreover, the social and political dimension of mathematics education has been extensively studied (Valero & Zevenbergen, 2004).

Despite its importance, performance of many undergraduate university students in mathematics education is poor (Bialek & Botstein, 2004; Timcenko, 2009). Various causes of poor performance in undergraduate mathematics are identified in literature (Peelo & Whitehead, 2006). One of them is the lack of necessary background in the subject, which is deteriorated by the fact that over the last years an increased number of students are taking time out from studies after completing high school - often referred to as a “gap year” (Martin, 2010). A gap year makes the transition from high school to higher education harder and aggregates the so-called “transition problems” (Gueudet, 2008). Moreover, wider participation in higher education has brought students with very different backgrounds to university classes, transforming them to a really heterogeneous group of people. Finally, changes in the specification of qualifications for high school students have also made traditional assumptions about mathematics knowledge of entrants to university courses obsolete (Greene & Forster, 2003).

In the last decades, the rapid development in ICT has provided new possibilities for education to integrate digital technologies into schooling and thus enhance teaching and learning. Over the past decades, research has extensively addressed various aspects of the introduction and use of digital

technologies in primary and secondary mathematics education. However, little is known about the current extent of technology use and mathematicians' practices in university teaching (Lavicza, 2007). This is because research in university level mostly concentrated on issues related to students' learning in undergraduate mathematics and described or evaluated theoretical frames that are used in technology-related research in the domain of mathematics education (Drijvers et al., 2010). Our research explores the potential of use of digital technologies in mathematics education at university level, taking the theoretical frame (i.e. Problem Based Learning - PBL) as granted. Therefore, a brief description of research efforts focusing mainly on introducing technology in mathematics education at university level is provided below.

The use of technology for mathematics teaching and learning can be classified in two dimensions: the use of domain-specific mathematical software and general use of learning technologies and e-learning tools. In approaches where domain-specific mathematical software is used, Matlab, GeoGebra and Computer Algebra Systems (CAS) are the most popular choices.

Matlab, which is an environment for numerical computation, visualization, and programming, has been particularly popular in mathematics courses intended for engineering students. (Behrens et al., 2010; Chang, 2011). GeoGebra, which is open source dynamic mathematics software that combines the ease-of-use of dynamic geometry software with some basic features of computer algebra systems, has been used mainly for undergraduate mathematics (Diković, 2009; Jaworski, 2010). Finally, CAS are software programs that support manipulation of mathematical expressions in symbolic form and their educational uses have been examined by a number of studies (Brito et al., 2009; Thomas & Holton, 2003).

Advances in e-learning tools can be used in an innovative manner for enhancing students' experience of mathematics teaching and learning and for enabling students' autonomy in the learning process. Specific e-learning services provide support for mathematics instruction in higher education, such as ALEKS (Assessment and Learning Knowledge Spaces, [www.aleks.com](http://www.aleks.com)) and WebCT (Course Tools, [www.blackboard.com](http://www.blackboard.com)) (Brinkman, Rae, & Dwivedi, 2007; Tempelaar et al., 2011).

With reference to the above-mentioned work, we have identified a couple of deficiencies in the existing approaches that require further research attention. The first aspect that we consider worth researching is involving the users (i.e. teachers and students) in the design of such approaches. Therefore, we are adopting participatory design as a method for developing our approach. Another interesting research issue is the development of technologies that are not connected to a specific course or study but focus on mathematics at university level in general. The only limitation of our research is that it focuses on undergraduate university studies where mathematics is not a core subject, but still is important for coping with core subjects. We focus on such studies because students lack not only background but also motivation in studying mathematics. On comparing our research efforts with the aforementioned projects and the identified deficiencies, our approach tries to first investigate teachers and students needs and difficulties in mathematics education, and then design learning material that takes into account these findings. These efforts take place into a PBL environment that provides a theoretical framework for our design.

The paper is organized as follows: section two discusses Problem Based Learning (PBL) that constitutes the theoretical framework of our approach while section three presents our methodological approach, namely participatory design, which aims at involving teachers and students in this research effort. Afterwards, we introduce a set of design implications for digital tools in mathematics education that was compiled during our collaboration with teachers and students. The paper ends with a discussion and an outline of our future work.

## 2. Theoretical Perspective

PBL is a student-centered pedagogy in which students learn through the experience of problem solving (Kolmos, Krogh, & Fink, 2004). The goals of PBL are to help the students develop flexible knowledge, effective problem solving skills, self-directed learning, effective collaboration skills and intrinsic motivation. In PBL learning begins with a problem to be solved, posed in such a way that students need to gain new knowledge before they can solve the problem, and thereby learning both thinking strategies and domain knowledge. PBL supports also group work. Working in groups, students identify what they already know, what they need to know, and how and where to access new

information that may lead to resolution of the problem. Moreover, PBL represents a paradigm shift from traditional classroom/lecture teaching. The role of the instructor in PBL (known as the tutor) is to facilitate learning by supporting, guiding, and monitoring the learning process.

Mathematics courses in our department are conducted in the form of lectures, which are followed by exercise sessions in groups. The lectures are given by a professor, who follows a transmission teaching model and uses slides projected on the wall for communicating the content of the curriculum. After these lectures, the students have to work in groups in hand-in assignments, based on the notion of PBL group work. The assignments are typical math exercises that are solved by hand. In some cases (e.g. numerical integration, linear algebra), students are asked to hand in a solution in a computer program (i.e. Matlab), along with the by-hand solution. During the exercise time, there are two teaching assistants, who support the students during this process but only when the students ask for it. In order to complete successfully the course, the students have to pass an individual written examination.

Our current research effort focuses on building educational material for transforming assignments for mathematics education according to PBL stance. We intent to introduce new learning activities, where students can experiment with pre-made digital mathematical visualizations but where also are asked to create their own in order to solve course assignments. An important aspect of our research is therefore the combination of PBL group work, where students are considered to be active agents who engage in social knowledge construction, with activities, where students are building mathematical concepts through development of computer applications.

### **3. Methodological Approach**

Participatory design – also called cooperative design – is the inclusion of all stakeholders (e.g. employees, partners, customers, citizens, end users) within a development team, such that they actively help in setting design goals and planning prototypes (Schuler & Namioka, 1993). It is an approach that attempts to ensure that the product designed meets the stakeholders' needs.

In the field of technology-enhanced learning, participatory design has been used in order to enable teachers and/or students to participate effectively in the design of digital educational tools (Winters & Mor, 2008). It is expected that active participation of the target group will result in tools that will correspond to their needs and interests, and thus be engaging and better accepted (Magnussen, Buch, & Misfeldt, 2003; Siozos, Palaigeorgiou, Triantafyllakos, & Despotakis, 2009).

Since we are focusing on students who lack motivation, we decided to adopt a participatory design method. By involving students in the design, we aim at developing tools that students will find interesting and attractive, and consequently at increasing their motivation to study. Furthermore, we want to investigate how teachers and students can contribute to this design collaboration. We are guided by beliefs that both groups should participate effectively in the design of educational applications, and that their expertise in education (each group is considered to be an expert in different aspects of education) could be especially critical. Each participant should be seen as an equal element contributing to the design, but at the same time none of them should have the full responsibility for the participatory design decisions.

As a first step in our design, we decided to investigate how the “Mathematics for Multimedia Applications” course is taught for Media Technology students and how digital technologies could contribute in improving teaching and learning for this course. This course is given during the second (spring) semester and introduces mathematics needed for media technology applications. The course covers basic elements of Calculus, Trigonometry, Geometry, and Algebra.

Our participatory design involves two professors, two teaching assistants, and seven students all related to the “Mathematics for Multimedia Applications” course. Our hypothesis was that incorporating all participants in one design team would create barriers in expressing criticism for established practices or for other participants' performance. Therefore we approached each group separately and established different ways of collaborating with each group.

In order to establish a common ground, and to avoid obstacles imposed by the culture and professional jargon of software design in our interaction with professors, we created a cooperative relationship between the professors and a software engineer, which spanned the fall semester of 2012. The software engineer worked with them in five sessions to design an intervention based on digital

technologies for the “Mathematics for Multimedia Applications” course. During this phase, we gave full control to the professors to decide what kind of digital technologies should be introduced in the course, and which parts of the curriculum would be suitable for working with such technologies.

The students group composed of three females and four males. The collaboration with this group of students took part in thirteen focus group discussions, because this type of discussion produces data and insights that would be less accessible without interaction found in a group setting. During these sessions one of our research team moderated the focus group discussion on students’ experience with the specific course assignments.

Although teaching assistants play a subsidiary role in the “Mathematics for Multimedia Applications” course, we believe that their contribution to our design is valuable. By having the opportunity of supporting the students while they are working on their assignments, they get insight into which parts of the curriculum are challenging for students. Our collaboration with them was established in the form of group interviews during the semester. During these interviews we aimed at getting feedback on how students worked in their assignments and whether and how they used the developed applets.

In order to get insight in the process of teaching and learning in its natural setting, we have conducted overt, direct observations of seven lectures and of seven exercise sessions (for the group that we are collaborating with). During these observations we gathered data on individual behaviors of students and teachers (the professor and teaching assistants) and interactions between them. Moreover, we were able to evaluate the introduction of the developed applets in the course. These findings will be incorporated in the future steps of our design and are presented in the next section.

#### 4. Results

The preliminary results reported in this paper concern the initial phase of our participatory design, where an intervention conceptualized by teachers was performed in an introductory mathematics course, and then feedback from students and teaching assistants was gathered, together with suggestions for alternative interventions. Table 1 summarizes the findings regarding which parts of the curriculum are considered challenging by our groups, informed also by our own observations of students working on their assignments.

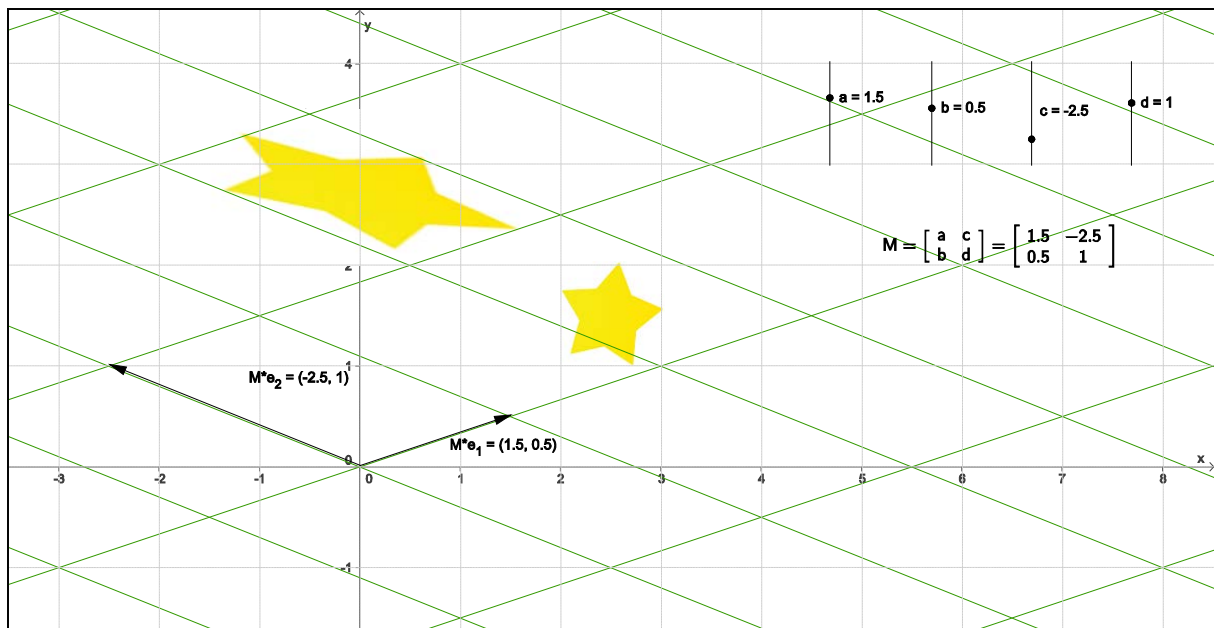
Table 1: Challenges in the curriculum reported by teachers and students.

Topics	Challenges
Basic arithmetic	Negative numbers, absolute value, factors and multiples, adding/multiplying fractions
Algebra	Difference between a variable and a parameter, notation of a function, summation operator ( $\Sigma$ ), properties of systems of linear equations
Trigonometry	Definition of sine and cosine in the unit circle, trigonometric functions and their diagrams related to the unit circle, difference between radians and degrees
Calculus	Definition of derivative, meaning of derivative, applying the chain rule in differentiation, finding the anti-derivative of a function,
Geometry	Vectors: addition, dot product
In various topics	Proof of theorems

Our findings from collaborating with the different groups seem to suggest that professors, teaching assistants and students do not always share same perceptions about how the mathematics curriculum should be taught and which parts of it are challenging. While professors focus on visualizing mathematical concepts (e.g. trigonometric functions, definition of derivative, addition of vectors), students and teaching assistants stress the importance of focusing on basic mathematics first and also presenting applications of mathematics in Media Technology.

At the beginning of our sessions both professors agreed that teaching and learning for the “Mathematics for Multimedia Applications” would be enhanced by incorporating interactive learning material (applets) in teaching in order to visualize core mathematical concepts. An example of an

applet is shown in Figure 1. This material was not connected directly with course assignments, a fact that resulted in students not using it during their exercise time.



**Figure 1.** Visualizing Linear Transformations:

Students can change the matrix of the linear transformation by using the sliders, and observe how the grid and image are transformed.

## 5. Discussion and Future Efforts

In this paper we have illustrated how our initial participatory design efforts resulted in a set of interactive learning activities that were used in teaching a mathematics course in Media Technology students. These activities provide visualization of mathematics concepts that teachers considered important and challenging for students. By conducting focus groups with students following this course, we were able to get feedback on challenging subjects in the course and on improving the learning activities. We also conducted observations of lectures and exercise time of the course, in order to gain insight on problematic areas and adapt our research on dealing with them.

One issue that needs to be considered is the different perceptions of teachers and students about how digital technologies can be introduced to mathematics education and what the challenging topics are. Our participatory design enables us to consult both groups and design interventions that are based on both perceptions. Moreover, an important issue that became evident while observing students in exercise time is that the level of difficulty and interactivity of the developed educational material should be adjusted to students' cognitive ability. If the learning tasks evolve fast and are complicated compared to students' abilities, then they would not have any added value. On the other hand, if learning activities evolve slow or are trivial, students would easily lose interest. Finally, connecting mathematics to its applications is very crucial if one wishes to improve student engagement and performance.

The next step is to use the aforementioned findings to better inform the design of a four-session workshop in mathematics. This workshop will serve as an introduction to the courses "Computer Graphics Rendering" and "Computer Graphics Programming" in Media Technology.

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