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Evocation and Enactment: Understanding Trajectories of Conceptual Development in Artifact-Mediated Situations

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Abstract: When instructional designers develop content-targeted pedagogical situations, their practice can be theorized as engineering students' development of conceptual schemes. To account for the contributions of students' prior schemes and situated experiences towards the development of conceptual schemes, we suggest a distinction between the evocation of schemes and enactment of situations. The former suggests that the design of an instructional situation can activate students' prior schemes. The latter suggests that the structure of students' activity in an instructional situation determines the nature of newly constructed schemes. We contextualize our views using empirical data from case studies involving students' interactions with educational technology designed for embodying and grounding the concept of proportionality.

Keywords: Educational technology, learning theory, embodied interaction, embodied cognition, mathematics education, design-based research

Introduction

For better or worse, the measure of a successful instructional activity is frequently defined by whether or not the interaction leads students to master the targeted learning objective. While this statement is self-evident, various national performance indicators (e.g., [17]), guidelines ([e.g., 16]), as well as the research literature cautioning on the (mis)interpretation of students' 'misconceptions' ([e.g., 20]) can be interpreted as indirect evidence that, as a field, our collective understanding of instructional design practice may be underdeveloped.

Consider the challenges that students have with learning rational-number concepts such as fractions, ratio, and proportion (for an overview, see [14]). A major concern in early rational-number instruction is that many students cope with the topic's inherent challenge by progressively learning to rely primarily on their ability to perform procedural algorithms and symbol manipulation, which they apply as a means of demonstrating their mathematical competence [9]. The pitfalls and limitations of relying on purely procedural fraction knowledge are poignantly demonstrated in Liping Ma's [15] research on the pedagogical practices of elementary school teachers: While perfectly capable of performing the appropriate arithmetic procedures for solving fraction-division problems, these teachers were unable to articulate their solution procedures in the form of mathematically accurate scenarios.

Consequentially, more work is needed to explicate the rationales underlying the otherwise tacit design choices that inform instructional design practice, as well as to understand the effects that these choices can have upon students' cognition and learning outcomes. A primary objective of this paper will be to explore how the design of an instructional situation can influence students' mathematical cognition, so as to inform a general framework for both fostering and analyzing students' interactions with instructional artifacts (i.e., technologies) and activities.

The claim to be advanced is that an instructional activity may be designed so as to purposefully and productively *evoke* students' existing funds of knowledge, and/or to *enact* situations that support students' development of targeted concepts/schemes ([5], see also [6]).

This thesis emerged from, and will be supported by empirical data from case studies involving educational technology (see Figures 1–3 below) designed to provide students with embodied experiences of mathematical concepts such as proportional equivalence [10]. Our analysis will explore how the material embodiment of a mathematical task (in the form of an instructional activity) contributes to students' cognitive scheme. The functional unit of analysis guiding this examination is the interplay between students' *schemes*, and the instructional *situations* that both influence and give rise to them [26].

1. Background and Theoretical Framework

1.1 Expanding our Notions of Inter-Subjectivity

A socio-cultural perspective of learning is that teachers use artifacts, broadly construed, to mediate the speech, actions, and perceptions of learners; and that learners appropriate and internalize the socially established meanings of cultural forms such as mathematical signs and concepts ([27], see also [19]).

Although the inter-subjective dialectic between instructor and learner is clearly a vital component of teaching and learning, this framing essentially constrains any analyses along two dimensions—the knowledge, goals, and beliefs of the instructor [15,23]; and the prior knowledge and cognitive capacity of the learner. Given that not all instructional interactions are successful in supporting students' development of a targeted concept/scheme, a purely socio-cultural analyses would necessarily imply some failing on either the part of the instructor or learner.

Rather than continue to problematize unsuccessful teaching and learning interactions in terms of the participants, we believe it may be more productive to problematize our existing theoretical frameworks. We suggest that a potential gap in socio-cultural accounts of learning is that they do not adequately address the contributions of learners' tacit, multimodal activity to cognition, or their prior knowledge to learning (cf. [1]). A potentially more productive approach towards designing and understanding learning interactions may be to incorporate a more nuanced examination of how the *intra*-subjective contributions of students' multi-modal perceptions and prior funds of knowledge interact within a designed, instructional situation.

1.2 Evoking Schemes and Enacting Situations

Elaborating on the work of Piaget, Vergnaud [26] highlights the relationship between an individual's mathematical schemes and the particular situations that give rise to and transform said schemes. In brief, the schemes an individual possesses determine the forms and organization of activity the individual applies towards novel situations. Reciprocally, the situations an individual encounters determine the schemes that are elicited and/or constructed. This subjective coupling of situations and schemes occurs as an accommodation of existing schemes—a person will assimilate a novel situation if it is amenable for the application of the scheme within the context of some task demand.

Notably, whereas some learners may experience some activity situations as conceptually "meaningless" and others as "meaningful," what can be common to these activities are the material objects students encounter, the operations they conduct upon and with these objects, and the discernable results of these actions [cf. 22].

Here, we propose utilizing the verb "to *evoke*" to refer to the activation of students' prior schemes. It is widely accepted that familiar artifacts, symbols, and signs can evoke an individual's pre-existing schemes. Similarly, students' schemes may be evoked upon their discovery of familiar features present in an instructional situation. Understanding and anticipating the schemes that are likely to be evoked by particular design features would position designers and educators to better interpret and address students' responses as they guide them towards the desired learning objective.

In tandem, we propose utilizing the verb "to *enact*" to describe how students' experience of a situation can be purposefully designed so as to support a particular learning objective. Bruner [4] had originally used the term "enactive representation" to elucidate how children's actions and activities in the world contribute to their development of iconic and/or symbolic forms of representation. Bruner had suggested that only after "something" is first acted upon and experienced in the world, can it be referenced; first as an object of thought, and later as an icon and/or symbol (see also [12]).

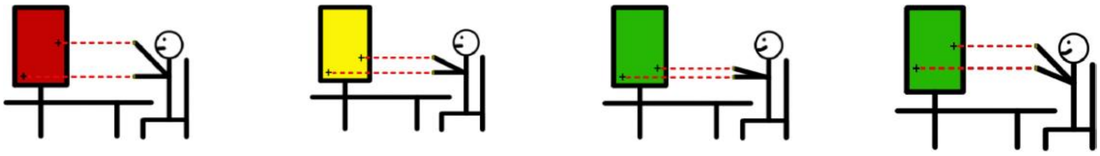
Following this reasoning, we argue that the educator/designer can guide students to dynamically enact situations—to act upon, observe, and/or produce pedagogically-meaningful situated phenomena. From an embodied cognition perspective [13,18,20], when students are guided to generate, alter, and/or attend to specific phenomena, they are fostering multi-modal image schemas, the experience of which can be assimilated into, and lead to the accommodation of students' cognitive schemes.

In summary, given the prominent and established role that designed artifacts play in supporting instruction, it is important to understand how the design and utilization of these objects can influence students' mathematical cognition. We have proposed the constructs of evocation and enactment in order to further elucidate the relationship between the schemes that are activated when students' notice particular features of a design, the forms of interactions that the design supports, and ultimately, the contributions of situated activity towards mathematical learning.

2. Design and Modes of Inquiry

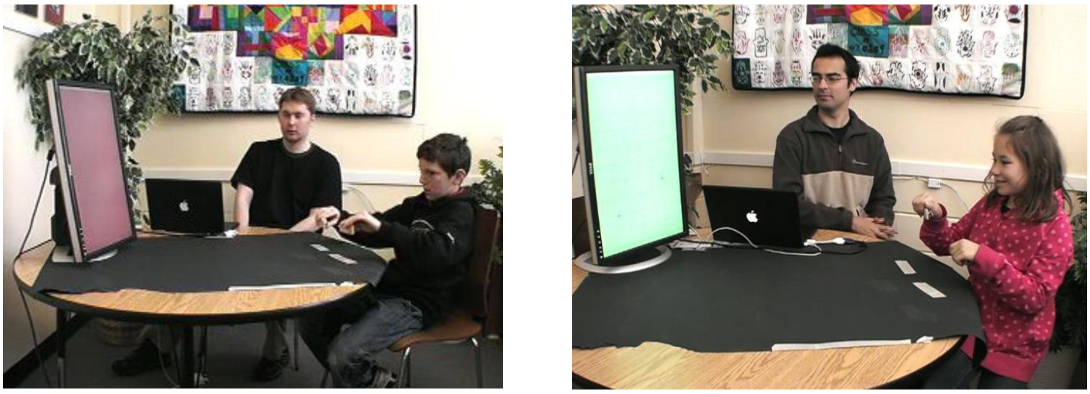
The theoretical perspectives presented in this paper drew on data gathered in an ongoing design-based research study [3,8] investigating the emergence of mathematical concepts from guided embodied interactional (EI) activities.

Our design conjecture is that some mathematical concepts are difficult to learn because mundane life does not occasion opportunities to embody and rehearse particular schemes that constitute the requisite cognitive substrate for meaningfully appropriating these concepts' numerical procedures. Specifically, we conjectured that students' canonically incorrect solutions for rational-number problems—"fixed difference" solutions, e.g., "2/3 = 4/5"—indicate students' lack of multimodal action images to ground proportion-related concepts [21]. Accordingly, we engineered an EI inquiry activity for students to enact, discover, rehearse, and embody pre-symbolic notions pertaining to the mathematics of proportional transformation. At the center of our instructional design is the Mathematical Imagery Trainer (MIT; see Figures 1-3, below; for detailed descriptions of the study's rationale and technical properties as well as initial human-computer interaction findings, see [11,25]).



a. b. c. d.

Figure 1. MIT interaction schematics, with the device set at a 1:2 ratio, so that the right hand needs to be twice as high than the left hand: (a) incorrect performance (red feedback on exploratory gestures); (b) almost correct performance (yellow feedback); (c) correct performance (green feedback); and (d) another correct performance.

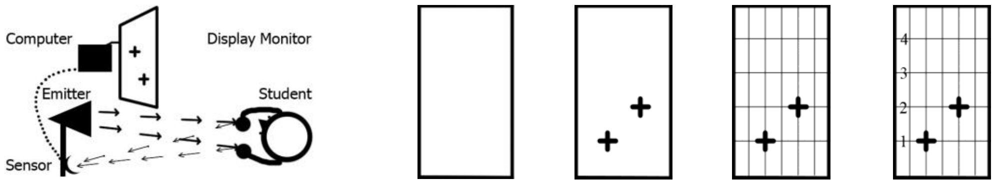


a. b.

Figure 2. MIT in action: (a) "incorrect" enactment turns the screen red; and (b) "correct" enactment turns the screen green. See http://www.youtube.com/watch?v=n9xVC76PlWc for a CyberLearning 2011 conference demonstration and http://tinyurl.com/edrl-mit2 showing the MIT in use by students.

The MIT measures the height of the users' hands above the desk. When these heights (e.g., 10" & 20") match the unknown ratio set on the interviewer's console (e.g., 1:2), the screen is green. So if the user then raises her hands in front of this pre-symbolic "what's-my-rule" artifact proportionately increasing distances (e.g., 15" & 30"), the screen will remain green but will otherwise turn red (e.g., 15" & 25", fixed distance). Study participants were tasked first to find green then to maintain it while moving their hands. The protocol included layering a set of mathematical artifacts onto the display, such as an adaptable Cartesian grid (see Figure 3, next page), to shepherd progressive mathematization of emergent strategies.

Participants included 22 students from a private K–8 suburban school in the greater San Francisco Bay Area (33% on financial aid; 10% minority students). Care was taken to balance students of both genders from low-, middle-, and high-achieving groups as ranked by their teachers. Students participated either individually or paired in a semi-structured clinical interview (duration of mean 70 min.; SD 20 min.). Interviews consisted primarily of working with the MIT. At first, the condition for green was set at a 1:2 ratio, and no feedback other than background color was given (see Figure 3b; we used this challenging condition only in the last six interviews). Then, crosshairs were introduced (see Figure 3c): these virtual objects mirrored the location of participants' hands in space yet, so doing, became the objects users acted *on*, then *through*. Next, a grid was overlain on the display (see Figure 3d) to help students plan, execute, and interpret their manipulations and, so doing, begin to articulate quantitative verbal assertions. In time, numerical labels "1, 2, 3,…" were overlain along the grid's *y*-axis (see Figure 3e): these enabled students to construct further meanings by evoking prior arithmetic knowledge and skills germane to the problem-solving task.



a. b. c. d. e.

Figure 3. The Mathematical Imagery Trainer: (a) overview of the system featuring an earlier MIT version, in which students held tennis balls with reflective tape. Figures 3b – 3e are schematic representations of different display configurations, beginning with (b) a blank screen, and then featuring a set of symbolical objects that are incrementally overlain onto the display: (c) crosshairs; (d) a grid; and (e) numerals along the *y*-axis of the grid (in the actual design, the default range of the grid and corresponding numerals was 1 to 10).

For the purposes of this paper, we focus on two brief episodes as context for elaborating on our theoretical constructs.

3. Results and Discussion

The MIT design essentially allows students to enact two situations: the generation of either a red or green screen. The choice of colors is intended to mediate students' activity by evoking socio-normative associations (i.e. green = go/good, red = stop/bad, yellow = almost green). In order to consistently generate the green screen, students must learn to position the pointers at proportional distances. All of the students we worked with ultimately succeeded in devising and articulating strategies for making the screen green, and these strategies were aligned with the mathematical content of proportionality. We observed some minor variation in individual participants' initial interpretation of the task as well as consequent variation in their subsequent trajectory through the protocol. However, by and large the students progressed through similar problem-solving stages, with the more mathematically competent students generating more strategies and coordinating more among quantitative properties, relations, and patterns they noticed.

Students were expected to discover the rules governing the interaction. In attempting to discover the workings of the system, students exhibited the following strategies in their attempts to enact "green": working with only one hand at a time; waving both hands up and down in opposite directions; or lifting both hands up at the same pace, possibly in abrupt gestures. They soon realized that a coordinated movement of both hands was necessary to make the screen green, and that the vertical distance between their hands was a critical factor. This conscious embodiment of the rules for enacting a green screen would ultimately serve as the experiential bases for their development of concepts such as ratio and proportionality.

The following data excerpts will sketch the relationship between the EI design and the schemes it evoked for students, as well as how the perceptual-motor enactment of specific green situations is ultimately used to mediate students' mathematically instrumented re-descriptions.

3.1 Leara

While working without the Cartesian grid, Leara, a 5th-grade female student, consistently moved her hands up in a fixed-distance motion, received red feedback, and adjusted the left hand down for green. Asked to explain her strategy, she responds as follows:

Leara: I think if I keep them apart and keep going up, it stays the same...

Int: If you keep them apart and you keep going up it stays the same?
Leara: It's not becoming red, but... it's like a ladder.
Int: So... how are you thinking about keeping them apart?
Leara: Oh maybe it's more. If it's farther up, then it has to be…they have to be more apart.

The mismatch between her gestured action and verbal explanation is a telltale indication of conceptual transition [9], as seen here by her evoking the embodied experience of using a ladder to describe a fixed-difference hands movement. Yet this scheme is quickly discarded when she realizes its enactment is at odds with the situation, that is, that the "further up" her hands move, "they have to be more apart."

Later, upon the introduction of the grid and numerals, Leara is asked to predict green locations without moving her hands. Her attention immediately shifts to the grid lines. She notes that there is "1 row" in between the crosshairs when the left hand is at 1 and the right hand is at 2, making green. She extends the thought:

Leara: And if you go… 10… if you go up to 10, there's gonna be like 4 or 5 rows. [*i.e., if the right hand is at 10, 4-5 rows are needed between the hands for "green."*]

Seemingly, the grid and numbers evoke productive problem-solving heuristics Leara may have learned from similar instructional situations. Leara proceeds to instrumentalize the grid to enhance her previous qualitative strategy for green, namely that the "farther up" her hands are, the "more apart" they ought to be. However, the 1:2 ratio has yet to become articulated, as her guess indicates she is still thinking in terms of approximate magnitude, "4 or 5," rather than relying upon more powerful mathematics (e.g., half of ten is five). Yet here precisely comes the moment of transition to the more powerful mode of reasoning, multiplicative relations, as seen from the following exchange where the interviewer prompts her to decide between 4 and 5:

Leara: No… five!
Int: Five? How did you do that so quickly? How did you know it was five?
Leara: Half of ten is five.

Later, during the post-interview debriefing, Leara reflects on the activity of finding green.

Leara: It's not just moving hands… it's… [*Leara moves her hands up and down*]… it's… you're trying to do *something* and get the number.

For Leara, the embodied enactment now functions as the basis for mathematical reflection.

3.2 Benjamin

Next we present the case of Benjamin, another Grade 5 student. We shall witness that Benjamin brings to the activity a sequence of mathematical meanings that evolve from the qualitative to the quantitative and from additive to multiplicative. Having explored the problem space, Benjamin makes initial observations about requisite hand positions for accomplishing the objective of rendering the screen green.

Ben: You have to make the right hand go higher than the, uhm, left hand.

As he interacts with the device, Benjamin refines his previous statement by bringing in the notion of speed.

Ben: I start with my right hand going a little faster than the left hand.

The interviewer encourages Benjamin's inclusion of speed. This appears to evoke another set of associations, and in response, the student elaborates further by discussing a car race.

Ben: If you keep them in the same…pace…like, for a car, if you wanted to do this with a car, it would sort of be the same speed limit. [He gestures with his two hands moving back and forth horizontally to indicate two racing cars.] Like this—one's going 20, and this one's going 50.

The grid and symbols are layered onto the screen. The mathematical associations that are evoked by the symbols steer Benjamins towards quantitative additive reasoning.

Ben: The right hand always goes up two and the left hand goes up one.

Ultimately, Benjamin notices that the measure of right hand is twice that of left and reconciles this fact with previous observations.

Ben: If it was a car race, then the one on the right would be twice as fast.

Thus Ben, like Leara, *comes to reinterpret his newly developed skill of "green-finding" via mathematics*, concurrently using the situated enactment of green as an objective, feedback, and "conservation" for an ontogenesis of proportion.

4. Summary and Conclusions

We have proposed two mechanisms for characterizing students' interactions with an instructional artifact or activity. First, we highlight the fact that instructional activities can evoke pre-existing schemes in the mind of the learner. As a design heuristic, this notion of evocation leads the designer to consider how students might perceive and experience an instructional situation. Vitally, it prepares designers to better account for the otherwise unexpected ways that a student might make sense of a novel instructional situation.

Knowing a priori that a particular design feature is likely to evoke a particular scheme for students (e.g., how the discrete units of a Cartesian-grid evoke counting schemes) could inform how an educator/designer chooses to mediate a learning activity.

Second, we use the term "enact" to describe how students' dynamic, situated activity can be structured so as to help facilitate their construction of a given scheme. Schemes are not constructed ex nihilo but from students' encounters with, and assimilation of, new situations. Instructional artifacts may be utilized to purposefully enact situations that designers believe will productively support the development of students' conceptual schemes.

It stands to reason then that the manner in which students' situated activity is organized and orchestrated is central to their conceptual development. Instructional designers can selectively determine students' experiences and thereby influence the trajectory of their cognitive development. Students' enactment of an instructional situation can in turn evoke prior schemes, or furnish students with a set of experiences that may form the cognitive bases for the instructor's targeted learning objective.

Although evidence from only two brief episodes are provided, the full corpus of data supports the argument that the embodied enactment helped students' to ground their emerging mathematical ideas. We observe, however, that students could not fully articulate their experiences *mathematically* until provided with the Cartesian grid and numerical inscriptions. This illustrates the necessity for designers to carefully consider the prior knowledge and conceptual resources that must also be evoked in order to productively complement an enacted instructional situation.

The argument advanced by introducing the constructs of evocation and enactment is this: Educators/designers effectively constrain and even determine the actions and outcomes that arise from students' interactions within a particular situation, hence significantly influencing students' schemes. Conceptualizing instructional practice in terms of designing learning situations for evocation and enactment of schemes may provide educators, designers, and researchers alike with productive insights for anticipating and evaluating the effectiveness of instructional designs.

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