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Study on the Effects of Learning Examples through Production in Problem Posing

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Abstract: Problem posing is identified as an important activity in mathematics education. Although it is difficult for novice learners to pose diverse problems, they must do so. Such novices particularly have difficulty in appropriately composing mathematical structures of solutions when posing new problems. This paper discussed and experimentally studied an approach to support learning of problem posing as a production task. We designed support for learning by imitation of examples, in which learners reproduce problems presented as examples. That is expected to facilitate learners' understanding ideas used in generation of the examples, and to stimulate new idea generation in learners' own problem posing. We then verified the effects of learning an example through imitation, with the results indicating that learning by imitation can facilitate adaptation of ideas used in an example into learners' problem posing, whereas it cannot necessarily facilitate appropriate problem posing when they try to diversify.

Keywords: Mathematical learning, problem posing, learning from examples, idea generation, solution strictures

Introduction

Students generally learn through solving problems provided by a teacher or textbooks in mathematical learning. Besides problem solving, problem posing has also been identified as an important activity in mathematics education [6]. Many benefits are gained from problem posing, and problem posing itself is a critical skill to be acquired through mathematical learning. However, because problem posing is a production task that requires idea generation in some ways, it imposes heavy cognitive load on learners. It is important but also difficult for novice learners to pose diverse problems. Novice learners particularly have difficulty in composing mathematical structures of solutions when posing new problems. Thus, to engage such learners in appropriate and effective learning of problem posing, their problem posing has to be supported. To do so, it is considered to be crucial to facilitate generation of ideas used in composing new solution structures. This paper therefore discussed and experimentally studied an approach to support learning of problem posing.

1. Support for Learning of Problem Posing

1.1 Difficulties in Learning of Problem Posing

Problem solving and problem posing are not entirely different cognitive activities but are closely related. On the other hand, they are of course different in features and formats of their tasks. Problem solving is a convergent task, in which a learner extracts a mathematical structure from given information and reaches a correct answer. In contrast, problem posing is a synthetic activity and a divergent task that fundamentally has multiple answers. Here, we call the former task a comprehension task, and the latter a production task. In problem posing, a learner has to generate novel ideas in some ways because new problems cannot be generally composed only from given information in the task.

In learning of problem posing, it is not useful for learners to repeatedly generate similar problems. To pose effective problems and gain benefits of problem posing, they have to generate and combine various ideas that includes surface features such as contextual settings in problem texts (e.g., purchase of goods or transfer by vehicle), and structural features such as mathematical structures of solutions. We refer to those two attributes as *situations* and *solutions*. Although it is important for learners to generate diverse problems by extracting several solutions from one situation or by recalling multiple situations to which one solution can be adapted, such diverse problem posing is considerably difficult for them. It has been confirmed that problems generated by novice learners lack diversity because of narrow associations between situations and solutions [1, 5].

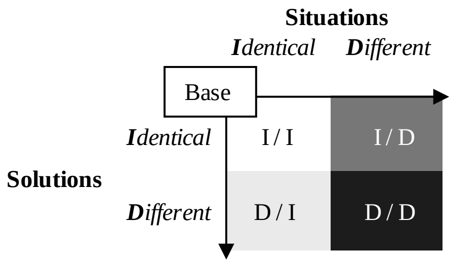

Figure 1. Categories for evaluating the variety of problems

We had investigated the variety of problems posed by novices to understand difficulties in their problem posing [4]. Undergraduate students were asked to generate new problems in domains of initial problems presented as bases. The undergraduate students were encouraged to generate problems as varied and unique as possible. The variety of problems they posed was evaluated according to four categories shown in Figure 1, which indicate similarities in situations and solutions between their problems and the bases. The results of the investigation confirmed that the undergraduate students posed many problems in I / I and D / I, and few problems in I / D. They also revealed that problems in D / I that had situations different from the bases were appropriately composed. On the other hand, problems in I / D and D / D, which had solutions different from the bases, were relatively simple and inappropriate. Those facts indicate that they could generate novel situations but failed to alter solutions in problem posing. They may not have had sufficient ideas to adequately formulate new solutions. Thus, to enhance problem posing by novice learners, we need to design methods to support learners' idea generation for formulating solutions.

1.2 Support for Learning of Problem Posing as a Production Task

Learning from examples is necessarily adopted in initial skill acquisition. In mathematical learning, learners generally learn many problems as examples by solving them. In other words, they learn examples through a comprehension task. In learning of problem posing, however, such learning activity used in problem solving may not be sufficiently effective because cognitive processes used in the comprehension and production tasks are different.

There are certain gaps between comprehension and production tasks. Thus, skills of comprehension and production may not be simultaneously developed and improved through a common activity. In fact, is has been reported that learning of such different tasks as comprehension and production had no influence on each other [7]. Mathematics problems are often solved without deep understanding of solution structures, even though, problems cannot be appropriately posed without the structural understanding, particularly when composing new solutions. Therefore, to improve productive performance by learning from examples, it is necessary to adequately learn solution structures of the examples, and it is effective to do that through a productive activity.

Hirashima and his colleagues developed computational learning environments which support problem posing by changing instance problems [8]. In the computational environments, learners can compose new problems by altering settings of given problems through operations in graphical user interfaces. They verified the effectiveness of problem posing with their environments through comparison with learning by problem solving. The learning effects were evaluated with a test of associating problems by extracting similar and different elements in the problems. They reported that learning by problem posing improved performance of the test more effectively than learning by problem solving. They also found that problem posing particularly facilitated associations of problems based on features of their solutions. Therefore, it is considered that learning problems through productive activities can facilitate learners in carefully examining solution structures.

According to this insight, we implemented a supporting system for learning examples through production in problem posing. In our system, learners learn the examples by imitation. Imitation here is to reproduce a problem identical to a given example. The learning activity of imitation is generally adopted in productive task domains (e.g., art and music), and the effects of imitation have been documented. For example, Ishibashi and Okada [2] argued that imitating examples can prompt imitators' understanding of conceptual background of the examples, and experimentally confirmed that imitation facilitates creative performance by imitators.

2. Supporting System for Learning Examples through Production

We implemented a system that supports a learner in a problem-posing task [3]. It can present a variety of problems as examples while controlling similarities in situations and solutions, and aid to reproduce the examples. Support by our system provides a learner with various ideas in problem posing through learning by imitation of examples.

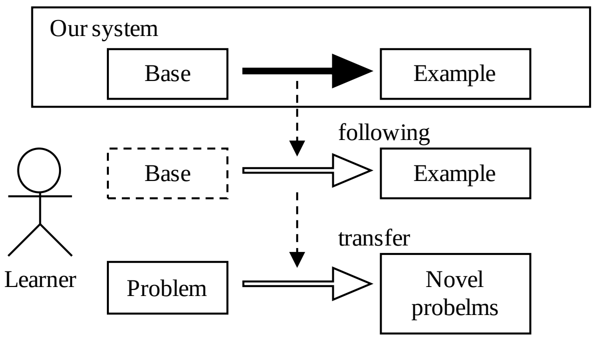

Figure 2. Basic framework of learning from examples through imitation

In learning with our system, a learner is required to pose new problems in the domain of a problem initially given as a base. The learner is also presented problems as examples, each of which can be generated by altering the base. Figure 2 indicates the basic framework of learning from examples through imitation. When a learner studies a presented example, the system hides the example itself and shows its generation process information, which indicates how the example is generated from the base. The generation process information also includes sufficient information for reproducing the example. The learner composes a problem identical to the example by following the generation process information. This learning activity must facilitate understanding of essential ideas used in generation of the example, particularly those for formulating its solution structure. S/he then transfers what is learned through the imitation into new problem posing. (For details on support of learning from examples through imitation, see [3].)

3. Experimental Studies

We experimentally evaluated the effects of learning from examples through imitation. In the evaluation, we confirmed the following two viewpoints: (1) whether learners can understand ideas used in composing an example and can transfer the ideas into their problem posing, and (2) whether learners can pose diverse and appropriate problems after learning of the example through imitation.

3.1 Method

We conducted two studies to confirm the two view points. In each study, undergraduate students were engaged in a problem-posing test after learning with our system. Procedures of experimental tasks were as follows.

1. Training of system operation
   The participants practiced operation of the system through input of a base given in the next learning phase.
2. Learning phase
   Each participant learned a presented example by reproducing it. The example was an I / D problem generated by altering the solution of the base by adding two operations. To test whether the participants could understand structural features in a solution of an example, we adopted the I / D. The domain of the task was word problems solved by simultaneous equations.
3. Post test
   The participants were given another problem posing test in the domain of word problems solved by a unitary equation.

Prior to start of the post test, each participant was instructed to pose a problem by applying what they had learned in the learning phase to their problem posing in Study 1. In Study 2, participants were instructed to pose problems as many, varied and unique as possible. The task of the post test in Study 2 was entirely the same as that in the previous study [4], although its participants were given no learning task.
Problems posed by the participants in the post test were analyzed based on two viewpoints: the variety and strategies to alter the solution of the base. The variety was measured with the four categories shown in Figure 1. The solution altering strategies were labelled based on differences in solutions between the posed problems and base. First, all of the posed problems were divided into two groups based on whether their solutions were altered from the base or not. The altered problems were then divided into two groups based on whether their solutions were partially altered by adding operations to the base or removing operations from it, or altered overall by composing new structures.

3.2 Results and Discussion

Twenty-one undergraduate students participated in Study 1, and ten in Study 2. Figure 3 indicates the proportions of the posed problems in each category in each study and the previous study. Many problems in I / D, which category was identical to the example, were posed in Study 1. As shown in the previous study, the I / D problems are rarely posed by novices without intervention or support. On the other hand, participants in Study 2 didn't pose many problems in I / D.

Figure 4 indicates the proportions of the posed problems composed with each solution altering strategy. As the figure shows, half of the participants' problems were posed without solution-alteration and the others were with alteration in Studies 1 and 2. In the altered problems, most of them were posed by partial alteration in Study 1. In contrast, most were posed by overall alteration in Study 2.

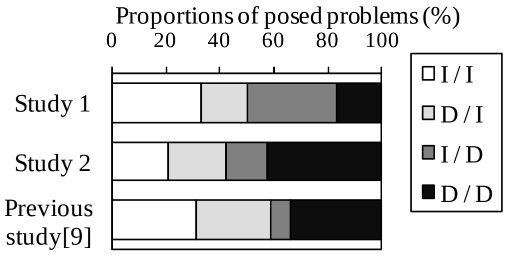

Figure 3. Proportions of posed problems in each category

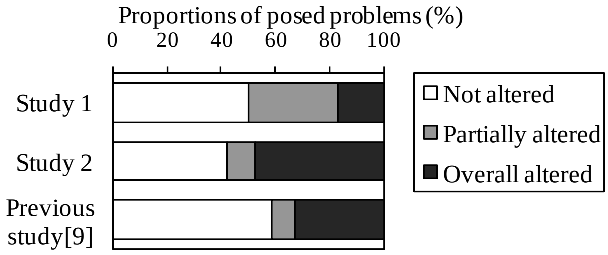

Figure 4. Proportions of posed problems with each solution altering strategy

The results above demonstrated that learning through imitation facilitated understanding of ideas used in generation of the example and adaptation of it into participants' problem posing. In Study 1, learning of the example increased posing of problems in I / D. That also increased problems posed with partial alteration. The example that the participants learned had been also generated by partial alteration. On the other hand, problems in I / D didn't increase in Study 2, which fact indicates that participants' problem posing was not sufficiently diversified. They posed many problems with overall alteration, however, solution structures of the problems were relatively simple like the previous study [4]. Thus, we need further studies to examine the support for diverse and excellent problem posing.

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