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Development of a Semi-Active Learning Support System with Operation Index for the Mathematics of Vectors

Tomohito JUMONJI[a*], Nonoka AIKAWA[a] & Takahito TOMOTO[c]
[a]*Graduate School of Engineering, Tokyo Polytechnic University, Atsugi, Kanagawa, Japan*
[b] *Faculty of Information Science, Chiba Institute of Technology, Narashino, Chiba, Japan*
*tomohito067@gmail.com

Abstract: It is important for learners to understand the relationship between mathematical problems and their answers to those problems. One way for learners to recognize the significance of their answers method for a problem is through trial and error. However, in ordinary mathematics learning, the correct answer is generally fed back from the textbook or teacher after the problem is attempted, which does not give learners an opportunity to reflect on their answers. So far, we have developed a system that encourages trial and error by visualizing the learner's answer for the mathematics of vectors and having them interact with it. However, there was a possibility that some learners would only look at the visualized figures to determine correctness and would not reflect on the errors they were making. In this study, we proposed and developed a system that requires learners to interact with figures on the system side, thereby promoting greater understanding of the relationship between mathematical problems and their answers.

Keywords: high-school mathematics, mathematics of vectors, error visualization

1. Introduction

It is important for learners to understand the relationship between mathematical problems and their answers to those problems (Hayashi et al., 2021; Enomoto et al., 2018; Yang et al., 2014). Trial and error is one way for learners to recognize the significance of their answers method for the problems. However, in typical learning facilitated by textbooks and classroom teaching, if a learner gives an incorrect answer, they are then simply given (e.g., by the teacher or textbook) the correct answer and its explanation, and there is little adaptive feedback for the learner's specific error (Yang et al., 2012). If the instructor gives only correct answers and their explanations, the learner may become a "passive learner" who simply accepts the correct answers implicitly. This reduces the likelihood that they will reflect on their errors, and may encourage them to try to memorize the correct answers presented to them without understand how or why an error was made. Therefore, it is important to provide learners with an environment in which they can proactively reflect on their incorrect answers through trial and error.

To encourage learners to perform trial and error, it is necessary to provide them with information that will serve as clues to the correct answer, rather than just giving them the correct answer. However, just having them focus on the wrong part and giving them clues to the correct answer may make them correct what is wrong and how it is wrong, but it may not make them reflect on why it is wrong.

Hirashima et al. (2002) proposed the framework of error-based simulation (EBS) to enable error visualization. This presents the learner with an opportunity to notice the error in their answer by themselves, which in turn is expected to encourage them to reflect on why it is wrong and to engage in the trial-and-error process. Hirashima et al. developed EBS mainly for dynamics (Horiguchi et al., 2002; Hirashima et al., 1998), and verified its effectiveness through many classroom practice sessions (Hirashima et al., 2009; Horiguchi et al., 2014). In addition Kurokawa et al. (2018a; 2018b) developed and evaluated a system that encourages learners to perform trial and error by visualizing errors in loci in mathematics.

However, conventional dynamics EBS also does not guarantee that the learner will observe the strange behavior that is visualized. If they only check whether their answer is correct or incorrect, and do not fully reflect on their answer, then learning becomes passive. Not reflecting on the relationship between their answers and the visualized behaviors in the figures may result in learners not understanding why their answers are wrong.

Therefore, in this study, we propose a method that explicitly requires the learner to manipulate the figures, with the aim that they will become aware of how their answers relate to those figures. This method is expected to enable learners to "proactively" discover "why their answers are wrong" through operation of figures, rather than just passively accepting correct or incorrect answers. Since the activity itself is required by the system, we refer to it as "semi-active" in this study.

2. Error Visualization in Mathematics

2.1 Learning support targeting STEM education

There have been various studies on STEM education. Sengupta et al. (2018) took a discursive, perspectival, material, and embodied phenomenological approach to coding and computational thinking, rather than viewing it as the acquisition of computational logic and symbolic forms. Yang et al. (2012; 2014) supported learning in one-to-one classrooms using variation-based discovery learning and facilitating drawing representations, math representations, and answers explanations through peer instruction among students. Mehanovic et al. (2012) designed and investigated interactive learning environments to support mathematics learning, based on the claim that technological tools appropriately integrated into mathematical tasks can support understanding of a wide range of mathematical concepts. Slater et al. (2018) addressed the gap by investigating how motivational constructs manifest within online learning systems, measures of mathematical identity (self-concept, values, and interest in mathematics), and correlating them with behavior and performance within Reasoning Mind's Elementary Mathematics Foundations system. Wong et al. (2017) reviewed multiple aspects of the structure of motivation in mathematics learning in a technology enhanced learning (TEL) environment, not limited to a single motivational theory. Junus. (2018) investigated students' mathematical misconceptions using concept maps and discussion records regarding inner product spaces. Ya-Jing et al. (2017) investigated the impact of a collaborative peer evaluation system using concept maps on learning effectiveness in programming education. Nakamoto et al. (2021) generated a model using self-explanation and penstroke data to identify the causes of students' stuckness. Enomoto et al. (2018) developed and experimented with a learning support system for problem-posing exercises to promote the interpretation of relationships among mathematical formulas. Hayashi et al. (2021) proposed a practice of conversion between verbal, mathematical, and graphical representations for addition-subtraction sentence problems (Hayashi, Y., 2021).

Thus, learning support in STEM education is highly important, especially in the area of mathematics. In particular, how to provide feedback on math problems is an important consideration. An approach to make learners recognize their own misunderstandings and correct their errors is called "error visualization". Error visualization is a method of making errors visible by converting the learner's answers to other media. Error visualization encourages learners to correct their knowledge by making them aware of their own misunderstandings, thus enabling them to correct their own errors with a certain level of motivation. In this error visualization, the error visualization of dynamics is well known. It visualizes errors by transforming the learner's force inputs into phenomena. Therefore, this study examines the adaptation of error visualization in the mathematical domain. In particular, we consider the transformation of representations in mathematics to be important for error visualization.

2.2 Error visualization for linear algebra

As mentioned above, Kurokawa et al. (2018a; 2018b) developed and evaluated a system that encourages learners to perform trial-and-error by visualizing errors loci in mathematics. They focused on two of the five mathematical representations identified by Nakahara (Nakahara., 1995), namely, symbolic representations, which are representations of mathematical expressions often used in mathematics learning, and graphic representations, which are representations of figures and graphs. They developed a mathematical expression conversion system that encourages trial-and-error by changing the symbolic expression (mathematical equations) or graphic expression (graphs) that were given by learners into other expressions, thereby visualizing errors.

However, unlike in dynamics, the domain of mathematics is not concerned with phenomena, but with figures Unlike phenomena, there is no motion, and therefore it is usually impossible to generate behavior. EBS of dynamics, events are visualized as behaviors in situations such as "a block is pushed and sinks to the floor". However, the mathematical domain is composed of abstract concepts and operations, which must be independent of situations, and therefore cannot be visualized as in conventional EBS of dynamics Unlike in dynamics, the domain of mathematics is not concerned with phenomena, but with figures Unlike phenomena, there is no motion, and therefore it is usually impossible to generate behavior. EBS of dynamics, events are visualized as behaviors in situations such as "a block is pushed and sinks to the floor". However, the mathematical domain is composed of abstract concepts and operations, which must be independent of situations, and therefore cannot be visualized as in conventional EBS of dynamics. which made it more difficult to suggest errors to learners. which made it more difficult to suggest errors to learners. Their so-called mathematical representation transformation system visualizes graphs (graphic representations) according to mathematical rules called "constraints" that exist in the mathematical equations (symbolic representations) constructed by the learner, while at the same time suggesting errors to the learner by allowing operation of figures within the constraints that exist in the mathematical equations constructed by the learner.

As an example, suppose the correct answer to a question is “point P moves on $y = 2x + 3$,” but the learner's answer is “point P moves on $y = -2x + 3$.” When visualizing the correct answer, point P will move along the graph generated by the correct equation, but point P as determined by the learner's answer will move along the different graph $-2x + 3$. The range of motion of a point P on the graph of the system varies depending on the equation. The learner can check the range of movement by actually moving it on the system. Thus, by adjusting point P on the generated graph, the learner can confirm that the behavior of point P in the graph operating the correct answer differs from the behavior of point P in the graph operating their own answer. By showing the learner this difference in behavior through operation, the system helps them to recognize their error, and we expect the learner to engage in trial and error.

2.3 Error Visualization for the mathematics of vectors

We have previously developed a learning support system based on work by Kurokawa et al. (2018a; 2018c), using error visualization for mathematics of vectors and verified its learning effectiveness (Jumonji et al., 2022). In that study, we succeeded in visualizing errors by converting the direction, which is a characteristic of vectors, into a figure as an arrow.

However, as stated previously, since vectors involve only direction and magnitude, it is often not possible to uniquely plot a vector on a figure. For example, the component vector a=(3,4) is a vector with a magnitude and direction of 3 on the x-axis and 4 on the y-axis, but when placing it on a graph, it cannot be plotted uniquely because it has no restriction on the coordinate at which it is placed and can be placed at the origin O or at any other coordinate. However, it is important to understand that in the relation AB=AC+CB, the three vectors can be arranged in a triangular shape. Therefore, it is important to think about how to place the vectors that cannot be uniquely placed, making it more important for the learner to be able to "operate" them themselves. First, the system presents a problem statement and a figure that is visualized when the correct answer. The learner observes the presented items and constructs an equation that he or she thinks is correct. The system visualizes the figure based on the equation constructed by the learner. Learners The learning support systems developed by the authors to date have shown a certain level of learning effectiveness.

Although the methods of Kurokawa et al. and our previous study allow learners to manipulate figures, those systems only present the figures that represent the answers (mathematical formulas) constructed by the learners, and learners are not required to actually perform the operations. Therefore, although we were able to visualize and present the learner's answer as a figure to the learner, the reflection of the answer using the operation was learner dependent. By giving the learner feedback on the correctness of their answer only, the student may simply accept the feedback and not fully reflect on their own answers, and thus learning becomes passive. If the learner does not fully recognize the relationship between their own answers and the visualized behaviors and figures, they may not understand why their answers are wrong. It is therefore important to make the learner aware of this relationship by providing specific interactions that highlight how their answers are incorrect. This problem has been discussed in past studies on error visualization. Free descriptions and concept maps were proposed as methods for making learners aware of how their answers relate to the visualized behaviors, and not understanding this will prevent them from fully reflecting on their errors. We hope to overcome this difficulty with the activity proposed in this paper, which we will elaborate upon in the next section.

3. Methodology

As stated previously, we are proposing a system that requires learners to explicitly manipulate figures, thereby allowing them to perform semi-active learning activities.

The operations required in our method are designed based on the model that is to be learned in the learning activity. For example, if we want the learner to understand a relationship like "in a vector addition-subtraction formula, a triangle can be drawn using the group of vectors composed by the formula," then the operation must be able to draw such a triangle. When the learner makes an error, the incorrect response is visualized in a strange figure. The learner then superimposes components from their answer onto those for the correct answer. In the vector addition/subtraction example, the learner visualizes each of the terms in the equation for their answers as a vector, then draws the triangles that are possible from that equation. By superimposing these onto the triangle associated with the correct answer, the learner verifies that their answer is incorrect. This then encourages them to proactively reflect on their answers.

Although we focus only on vector addition in this study, we expect that this method will be generally applicable regardless of field. Previous studies have left it up to the learner whether to interact with the figures, but our system requires it as part of the problem solving process, thereby promoting trial and error.

Operation index were presented to the learner as explicit demands for manipulating the figure. These index represent the goals to achieve by manipulating the figure, and it is left to them to experiment with the figure and deduce how to achieve them. This requires the learner to be proactive and encourages them to independently discover why their answer is wrong.

An operation index is presented in the form of a model of a certain activity that represents what is required for solving a given problem. In the addition and subtraction of vectors, which is the subject of the present study, a triangle can be formed from vectors on both sides of a given expression. As an example, suppose that we are given a regular pentagon ABCDE and are told to find XX for AC = XX + XX using point B. If the correct answer to this problem is AC = AB + BC, then a triangle can be formed from the vector AC on the left and the two vectors AB and BC on the right, as shown in Figure 1. The operation index for this problem would be the triangle consisting of these three vectors. By manipulating the visualized vectors in such a way that they fit into this triangle, the learner can learn the rule that the visualized vectors form triangles via the addition and subtraction of the vectors.

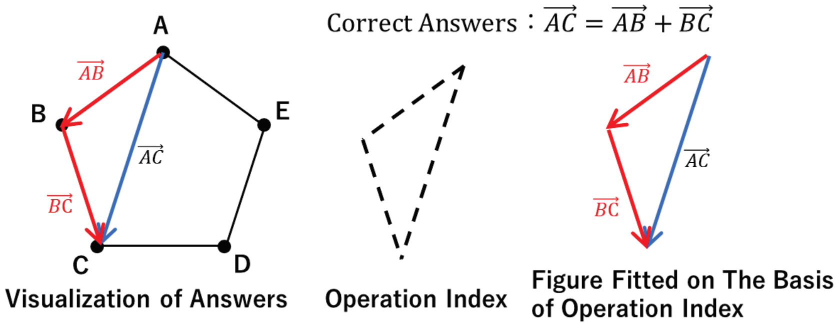

Figure 1. The operation index for vector addition.

Furthermore, if the learner gives an incorrect answer, then it will not be possible for them to form a triangle that matches the operation index associated with the correct answer. For instance, assume the learner incorrectly answers AC = AB + CB, then the resulting visualization and operation index will be shown as in Figure 2. It is possible to form this triangle using AC, AB, and CB by manipulating the visualized vectors (upper portion of Figure 2), but it is not possible form the triangle using AC, AB, and BC (lower portion of Figure 2). Trying to reproduce the operation index makes the learner aware of the errors in their answer and helps with knowledge retention.

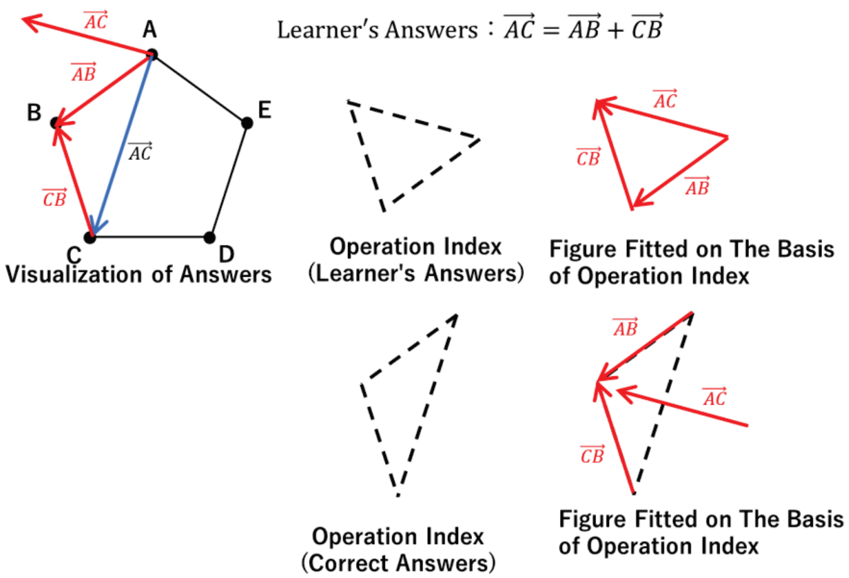

Figure 2. Operation index in vector addition for an erroneous answer.

4. Development Systems

Figure 3 shows a screenshot of what is actually displayed by the system. The target range of this system is vector composition in mathematics. This allows the system to request explicit operation from the learner as a presentation of the operation index, since the model to be presented as the operation index is fixed to one model. It first provides the learner with a problem statement and asks them to find its answer. In previous learning support systems, the figure associated with the correct answer was presented at the same time. In contrast, the present system gives the triangle corresponding to the correct answer as an operation index. Once a learner inputs an answer, the system presents a visualization of their answer along with the operation index for the correct answer.

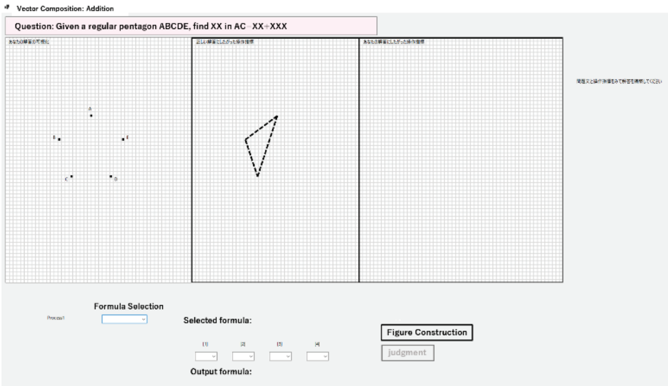

Figure 3. Screen output from the proposed system.

In the example shown in Figure 3, the learner presented with a regular pentagon ABCDE and asked to determine XX for AC = XX + XX using point B. Then, the triangles formed by the vectors AC, AB, and BC, which are generated when the correct answer in this question is AC = AB + BC, are presented as the operation index for the correct answer. We believe that the learner's search space can be expanded more with this system than with conventional learning support systems, which provide feedback based on the correct figure only.

The learner constructs the correct answer by looking at the question text and the operation index, then using a combo box from the answer input form at the bottom of the screen (Figure 3). The system visualizes errors by converting the learner's answers (mathematical expressions) into vectors (figures) (left portion of Figure 4). At the same time, the system also presents the triangles that can be formed by the vectors in the learner's answer as operation index (right portion of Figure 4).

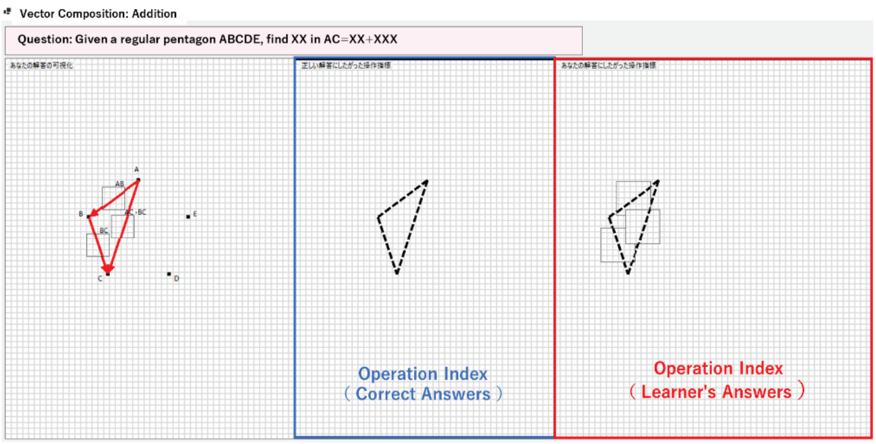

Figure 4. The learner’s operation index (correct answers).

Figures 4 and 5 show the operation index presented when the learner answers the question in Figure 3. Figure 4 shows the operation index for the correct answer, AC = AB + BC, while Figure 5 shows the one for the incorrect answer, AC = AB + CB. These visualizations make the difference between the correct answer and the learner's answer clear.

Figure 6 shows how the system specifies the vectors and allows the learner to select and manipulate them. This necessitates that the learner think more deeply about the individual vectors they are manipulating.

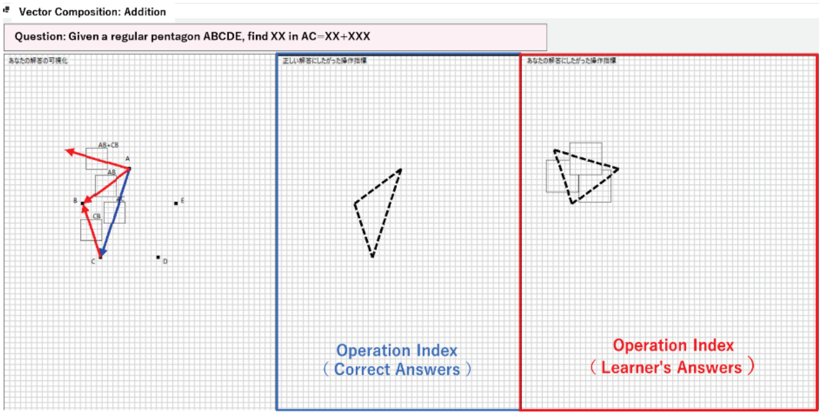

Figure 5. The learner's operation index (error).

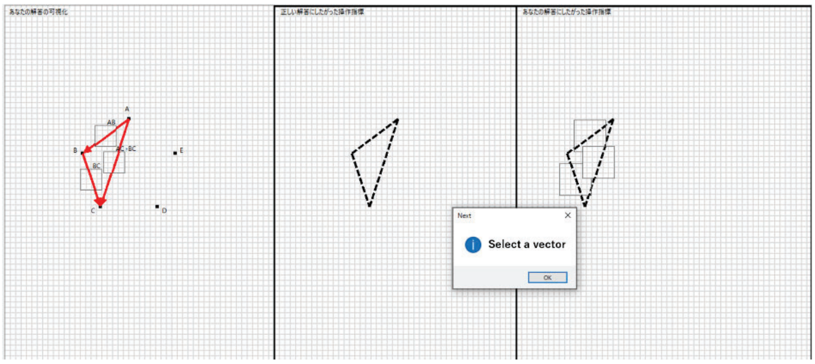

Figure 6. Vector selection.

The learner is then asked to use the vectors to reproduce the operation index from their answer and the correct answer, an example of which is shown in Figure 7. We believe that this allows the learner to recognize the relationship between their answer given as a mathematical formula and the graphical representation. The learner will not be able to reproduce the operation index for the correct answer if their answer is incorrect (Figure 8). This process allows the learner to proactively discover why their answer is wrong and to reflect on it. Moreover, since the learner can only move on to the next answer by forming a triangle, we expect this to solve the problem found in conventional learning support systems in which the learner neither checks the visualization of the error nor reflects on it.

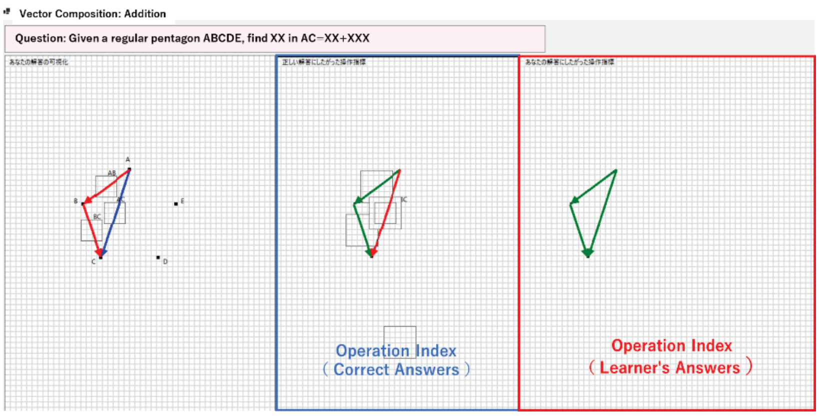

Figure 7. Output after vector manipulation (correct answer).

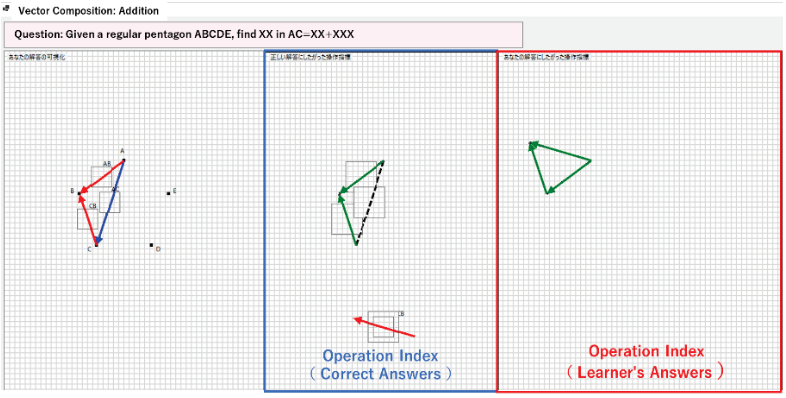

Figure 8. Output after vector manipulation (error).

5. Conclusion

In conventional learning such as learning from textbooks or classroom teaching, even if a learner gives an incorrect answer, they are often given only the correct answer and the corresponding explanation, and there is little adaptive feedback for the learner's specific error. This runs the risk of the learner will becoming a passive learner who only implicitly accepts correct answers, and the activity may encourage the learner to simply try to memorize correct answers without fully reflecting on their own wrong answers. Therefore, it is important to create an environment in which learners can reflect and fully consider why their answers were incorrect.

In the previously developed learning support system, errors were visualized by converting learners' answers into graphs. This provided an environment that allowed learners to think consider where they made mistakes and encouraged trial-and-error engagement.

However, in these systems, learners were only presented with figures, and the system did not require that they reflect on their own answers from the presented figures. If a learner only checks the given feedback on whether their answers are correct and does not fully reflect on their own answers, there was a risk of passive learning. In such a case, the learner may not understand why their answer was wrong, and they are likely to make the same mistake in similar problems.

In order for learners to fully reflect on their own answers, they need to recognize the relationship between their answers and the visualized figures. Therefore, it is important to provide specific activities that require learners to recognize the relationship between their answers and the figures.

In this paper, we proposed a learning support system for vector addition that requires learners to manipulate figures explicitly, thus encouraging them to perform semi-active learning activities. By explicitly requesting operation of figures from the system, learners can independently discover why their answers are wrong through the operation of figures and can reflect on their own answers.

In the proposed system, errors are first visualized by converting the learner's input answers into figures. Then, a triangle is formed according to the learner's answer and is presented to the learner as an operation index. This operation index is a model of the rules governing the problem given to the learner, and it is a criterion for how to perform the operation. In the addition of vectors, there is a rule that a triangle can be formed from the subtraction formula on the right-hand side and the vector on the left-hand side, so a triangle is presented as an operation index.

The learner manipulates the vectors generated by the system by dragging them and places them to form the triangle that represents the learner's answer, presented as the operation index, through trial and error. We believe that this allows the learner to recognize the relationship between their answers and the graph.

Next, the triangle formed by the correct answer is presented as an operation index. If the learner's answer is incorrect, when the vectors generated by the learner's answer are superimposed on the triangle formed by the correct answer, the correct triangle is cannot replicated. This can help the learner to proactively discover why their answer is wrong through the operation of the figure, and to reflect on it.

We believe that this method is a new way to make learners aware of the relationship between their own answers and strange phenomena, which has been discussed in the field of error visualization.

As a task for future work, we will conduct evaluation experiments to verify the learning effectiveness of this system. We will also explore the extent to which the method of presenting operation index can be applied.

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