

Enabling Students to Seek Computer-based Scaffolds for Solving Mathematical Word Problems

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Abstract: Scaffolding can develop a student's ability to internalize new information on the basis of prior knowledge. However, it assumes that all students are novices and likely limits students' way of thinking. Therefore, this paper incorporates the concept of help seeking in the usage of scaffolds, so that students can think as a whole before using scaffolds. The authors preliminarily designed an activity flow of seeking scaffolds. Furthermore, a trial study was conducted to investigate students' behaviors of choosing and using scaffolds. The result showed that the activity flow encouraged students to make attempts first without the limitation of scaffolds. Yet, it was also found that students lacked the ability to use scaffolds appropriately, suggesting further improvement.

Keywords: scaffolding, help seeking, mathematical word problems, problem solving

Introduction

Scaffolding is widely used as a tutoring strategy, because it can expand a student's capability by linking prior knowledge and new knowledge. In terms of Vygotsky's theory [9], capable people (e.g. parents, tutors, or capable peers) as a form of scaffolding can help students develop their potentials that they cannot reach alone. In a technology supported learning environment, the forms of scaffolding have been shifted and extended from the interaction with capable people to the support of artifacts, resources, and environments [5]. When the students who are scaffolded become capable, the scaffolds should be faded in order to return controls back to the students. Even so, the students still need to be monitored until they can really do it on their own.

When students start to solve a new mathematical problem with computer-based scaffolds, they are usually assumed to be novices in the beginning. However, even if they know nothing about the new problem, they still have an opportunity to solve it by themselves. This is due to the fact that their prior knowledge may be enough to solve the new problem. They are likely able to develop their own solutions even if they are not scaffolded. However, scaffolds may limit students' ways of thinking and students may thus feel annoyed and bored. As if a dish washer takes several years to learn cooking from a chef, students sometimes spend too much time on following what computers do until they meet the requirements of fading. Scaffolds may leave too little space for students' development. This phenomenon refers to 'over-scaffolding' [2][10], students being supported too much, irrespective of students' abilities. Students need to develop higher-level ability besides knowing how to do.

Aleven and his colleagues [1] concerned students' metacognitive ability instead of cognitive ability. More specifically, they aimed at developing students' ability of help-seeking by computers. Their research was based on a finding that students who rarely asked for help usually needed help [3][6][8]. For this reason, they have established a help-seeking metacognitive model [7], which described a student's legitimate activity flow

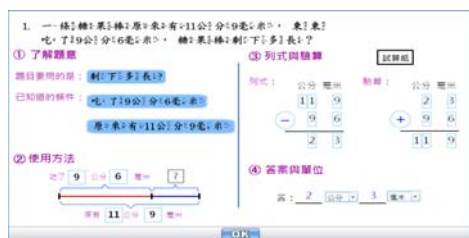
with a large number of production rules. More specifically, before trying to solve a problem, a student had to take reasonable time thinking about the solution plan. If he was not familiar with the problem, he had to ask for a hint. If he was familiar with it but still did not know what to do, he had to search if there was any helpful information.

Although scaffolding and the help-seeking model both support the concept of learning by doing, they have differences from the following perspectives. First, the former asks students to follow parts of a certain procedure to achieve the goal, while the latter allows students to do the whole procedure in the beginning. Second, even if students are scaffolded, they still need to finish the rest of procedure. Yet, when students get helps, they actually get hints and information without paying efforts for the helps, which likely result in abusing the helps [1][7]. For avoiding the situation, when students need helps, system should provide them with a small task that requires students to accomplish instead of hints only. To this end, this paper aims at designing a system that enables students to seek scaffolds for solving mathematical word problems.

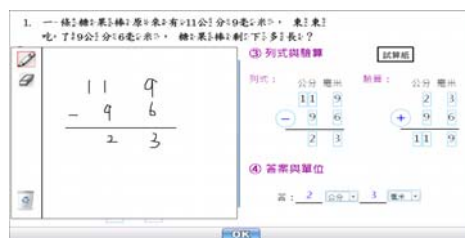
1. System Design

1.1 Background

In an ongoing research project, all students in a third-year class have been equipped with personal tablet PCs. In particular, they received computer-based scaffolds to learn solving mathematical word problems for six months. The scaffolds were designed based on Mayer's cognitive process of solving mathematical problems [4]. Furthermore, students were required to follow four steps to solve a problem: restructuring the problem texts, using a line segment diagram, formulating equations, and checking the solution (see Figure 1(a)). Students were scaffolded in the first three steps. More specifically, in the first step, restructuring the problem texts referred to arranging the pieces of information in the problem in order to determine known variables, unknown variables and even superfluous information, if exists. In the second step, students were supported to use a line segment diagram to reorganize the problem. And then, in the third step, students formulated one or more equations by deciding the operator and operands. Students were allowed to use a stylus to write down and calculate the equations on a digital sheet (see Figure 1(b)). Finally, students contextualized the solution with an appropriate unit in the last step.



(a) Four steps to solve a problem



(b) A digital sheet

Figure 1. Original computer-based scaffolds

Although the way to scaffold students seemed reasonable to help them learn, it actually assumed all students were beginners who learned from an experienced tutor. The scaffolds would be faded only if there was enough evidence to show they have learnt. Such philosophy may look down on the students. Furthermore, a preliminary finding of the project showed that when the students used these scaffolds to solve mathematical word problems for a long time, they thought problem solving as a repetitive job and became bored in the end. The finding suggested a reverse philosophy that all students should be assumed

to be able to solve problems in the beginning. In other words, they were provided with scaffolds only when encountering difficulties.

1.2 Seeking scaffolds

Figure 2 illustrates the activity model of seeking scaffolds. When a student starts to solve a mathematical word problem, he can choose to solve the problem on his own or choose a scaffold, depending on whether he can solve it or not. If he chooses the former, he is free to use his stylus to draw representations, to formulate equations, and to do calculation on his digital sheet on the screen (Figure 3(a)). Otherwise, he can choose one from three kinds of scaffolds. The three scaffolds are modified from the aforementioned project, i.e. restructuring the problem texts, using a line segment diagram, and formulating equations.

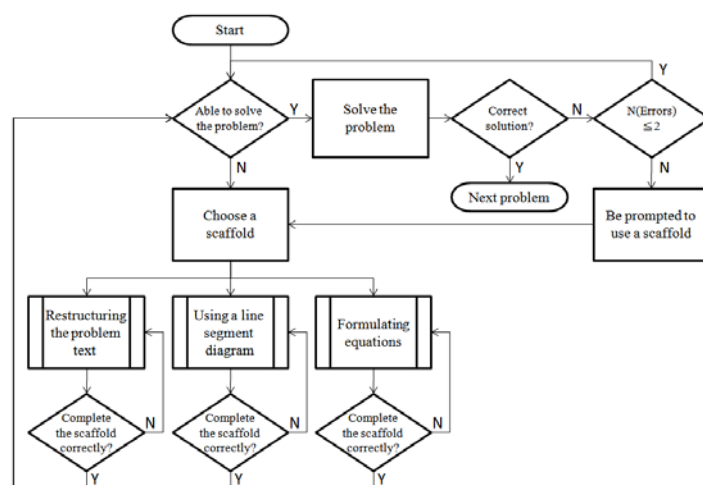
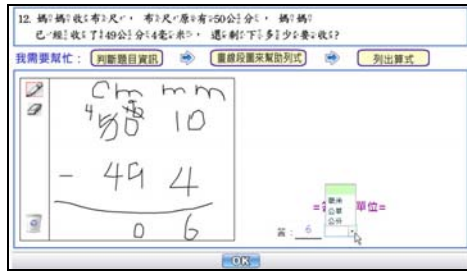


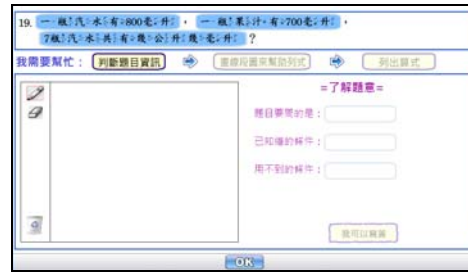
Figure 2. The activity model of seeking scaffolds

In the first scaffold (Figure 3(b)), the student is required to categorize the problem texts into a goal, useful information and superfluous information. He has to drag texts colored blue in the problem and drop them in an appropriate category. The goal represents an unknown variable, which may derive from known variables in the useful information. The task of categorization may simplify the problem, so that students can focus on the important part. In the second scaffold (Figure 3(c)), the student is required to complete a line segment diagram representing the problem. In the diagram, there are several pieces of key information missing. For entering the missing pieces, he has to read the problem again. The line segment diagram presents the problem in a visual way, so that students can easily understand its meaning. In the third scaffold (Figure 3(d)), the student is required to formulate one or more equations. He has to enter the operands and decide the operator. Besides, he is also asked to point out which statement determines the operator.

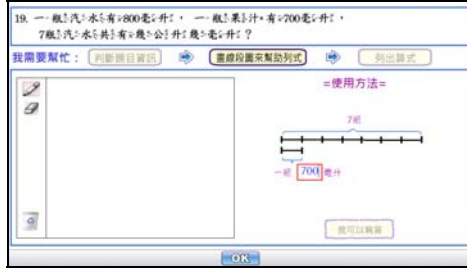
After the student completes a scaffold, he may click the “OK” button to check his answers in the scaffold. If the answers are wrong, the system colors the frames of the wrong answers red immediately, indicating which answers the student needs to correct. When he finishes a scaffold successfully, he still can choose another one or solve the problem by himself. After solving the problem, he may check his solution by clicking the “OK” button. Similarly to the feedback in scaffolds, the system colors the frame of the wrong solution red. If the solution is wrong more than twice, the system would force him to choose a scaffold.



(a) Solving a problem on one's own



(b) The scaffold of restructuring the problem texts



(c) The scaffold of using a line segment diagram



(d) The scaffold of formulating equations

Figure 3. Snapshots of the system

2. Trial Study

The purpose of the trial study was to explore participants' behaviors of seeking scaffolds. The participants were selected from the aforementioned project. As mentioned earlier, all of them have been trained to use computers to learn mathematics. They particularly received computer-based scaffolds to learn solving mathematical word problems. When they used the new activity model of seeking scaffolds, their perceptions and behaviors may change. The behaviors could be observed when they used the system; and their perceptions could be revealed by querying after they did an action.

2.1 Participants selection

Three participants were selected according to the following procedure. The authors conducted a paper-and-pencil test to the third-grade class who participated the project ($N=28$) for examining students' abilities to solve mathematical word problems. The test was consisted of 20 multiple-choice questions (2.5 points per question, 50 points in total) and 10 word problems (5 points per questions, 50 points in total). The 20 multiple-choice questions were used to test five sub-abilities to solve word problems, which are described as follows: (1) to eliminate superfluous information, (2) to locate known variables, (3) to locate unknown variables, (4) to identify correct line segment diagrams, and (5) to use appropriate operators. Each sub-ability included four questions. The 10 word problems were used to test their synergetic ability to use the five sub-abilities.

By the scores in the test, all students were divided into three groups: high-ability, medium-ability, and low-ability groups. The authors selected one participant in every group (see Table 1). Owen (pseudonym) was selected from the high-ability group. In the test, his answers were almost right except a multiple-choice question about line segment diagrams and a word problem because of his wrong calculation. For the medium-ability group, Vicky was selected. Although she was not good at finding information in word problems and, in particular, identifying line segment diagrams, she could solve word problems 80% correctly. This may imply that she tended to pay attention only to the numbers in the problem without understanding its meaning. Finally, Sean was selected from the low-ability group. His weakness was mainly locating known variables, identifying line segment diagrams, and

using operators. Although he scored almost the same as Vicky in multiple-choice questions, he could solve word problems only 50% correctly. The test showed that he had a worse synergetic ability than Vicky.

Table 1. The participants' scores of an ability test in mathematical word problem solving

	Eliminating Superfluous Information	Locating Known Variables	Locating Unknown Variables	Identifying Line Segment Diagrams	Using Appropriate Operators	Solving Word Problems	Total
Owen	10	10	10	7.5	10	45	92.5
Vicky	5	5	5	0	7.5	40	62.5
Sean	7.5	2.5	7.5	5	2.5	24	49.0

2.2 Setting

The three participants were separately invited to use the system. The trial study took place in the library of their school. Every participant was requested to sit on a seat with a tablet PC on a desk. To lessen the pressure to use the system, before the participant used the computer, he/she was told that the trial study was not to test his/her ability; rather, to help us to improve the system. The participant was also told that when using the system to solve problems, he/she was likely queried about his/her actions. Two researchers accompanied and queried the participant during the process. The queries were based on direct observation of the behaviors of seeking scaffolds, especially when and how they chose a certain scaffold. The participant was queried only after they took an action. Besides direct observation and queries, their operations on computers were also recorded and logged. During the process, the researchers would not give any instructions about how to solve the problem. However, when a participant spent more than one minute on thinking without any actions, the researchers may prompt him/her to use scaffolds.

Every participant was required to use the system to solve mathematical word problems in 30 minutes. The word problems included two levels of difficulty. The first level had 12 questions (denoted by P1-1 to P1-12) involving addition and subtraction, and the second level had 13 questions (denoted by P2-1 to P2-13), further involving multiplication. If a participant performed well at the first level and never used scaffolds in solving the first three problems, which were representative of the level, the researchers may suggest him/her to solve the second level of word problems. After using the system, the participant was required to answer several questions, such as, when and why he/she decided to use scaffolds. In particular, he/she was also required to compare the preference between the system of seeking scaffolds and the system of using scaffolds by force.

2.3 Results

Figure 4 illustrated the three participants' process of solving word problems and using scaffolds. Every participant solved problems from left to right (i.e. from P1-1 to P2-13). For solving a problem, each action was illustrated as a box in a sequence from top to bottom. The color of the box represented different types of actions. For example, a gray box followed by a white box meant that the participant first attempted to solve the problem and the solution was correct. The number labeled in the box showed that the participant use a certain scaffold. For example, when Vicky solved P1-4, she chose the first scaffold and then solve the problem correctly.

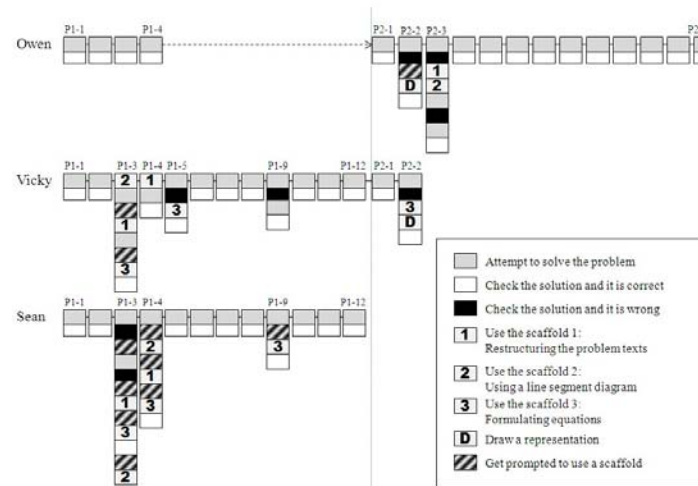


Figure 4. The participants' process of using the system

As indicated in Figure 4, Owen could correctly solve the problems of the first level without using any scaffolds. For this reason, he was suggested to solve the problems of the second level. He appeared very confident and thus decided to solve harder problems after solving P1-4. He finally solved 17 word problems, including 13 of the second level. Among these problems, he only used scaffolds for solving P2-3. Additionally, when solving P2-2, he drew a diagram on a digital sheet to help him solve the problem (Figure 5(a)).

As for Vicky, she solved 14 word problems, including 2 of the second level. Among these problems, she solved four problems by using scaffolds. In the beginning of solving P1-3 and P1-4, she had no idea to solve them and decided to use scaffolds. For P1-5 and P2-2, after her failed attempt to solve the problem by herself, she decided to use the scaffold of formulating expressions. Like Owen, when solving P2-2, she also drew a diagram to solve the problem after she used the third scaffold (Figure 5(b)).

Sean solved 12 word problems of the first level, and none of the second level. When solving P1-3, P1-4, and P1-9, he usually spent a lot of time on thinking without doing any actions. The researchers thus followed the rule to prompt him to use a scaffold. In addition, it was also noteworthy that he attempted twice to solve P1-3 on his own and finally decided to take researcher's prompts to use the scaffolds. However, after solving it correctly by the scaffold of formulating expressions, he still did not understand the problem visibly. Although his answer was correct, the researchers use the scaffold of using a line segment diagram to help him understand the problem.

Although the participants were few, their behaviors could still carry several implications, which described as follows.

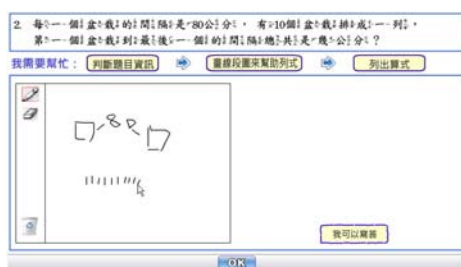
Attempts first instead of scaffolds first

It was observed that all of the participants tended to solve problems at the start without using scaffolds. Owen was very self-confident. Even if he met a new problem he had never learnt, he said "[I] will try to solve it at first; unless [I] cannot do it, [I] will ask for helps". For Vicky, although she bashfully said that "I don't believe I can solve it (a mathematical word problem) correctly", she was still willing to make attempts. Sean was also observed to solve a problem by himself even though the problem is obviously hard for him. Such behavior was usually labeled as 'help avoidance' [1]. However, it could be regarded as a behavior of active attempts before the request of scaffolds.

Compared to the previous learning model of passively receiving scaffolds in the last several months, what students had to do was only to follow what the scaffolds had defined in advance until they were faded. They did not have to think why they need the scaffolds in the beginning. Although Vicky and Sean did not have any comments about it, Owen particularly complained about the previous learning model: *“It was annoying! ... In the beginning [of learning a new lesson], it was fine, but [I] feel annoyed later”*. He preferred the model of seeking scaffolds, because of its way to *“choose which help I wanted”*. Therefore, seeking scaffolds may provide students with an opportunity to explore how to solve a problem instead of mimicking the scaffolds.

Relieving the constraint of scaffolds

Because the activity model did not limit students' actions, they likely exhibited some behaviors that the researchers did not expect in advance. In this trial study, Owen and Vicky were found to draw their own representation on screens to solve P2-2. The problem was to calculate the distance from the first plants to the last plants, given that there were 10 plants in a row with the same interval of 80 centimeters. Ideally, students had to calculate the number of intervals ($10-1=9$) first, so that they can get the total distance ($80 \times 9 = 720$). However, both Owen and Vicky had the same wrong answer ($80 \times 10 = 800$) for the first time. The reason may be that they only paid attention to the numbers within the problem rather than read it carefully. In the case of Owen, later when he drew ten bars for the ten plants on the screen (Figure 5(a)), he finally figured out the relationship between the numbers of plants and intervals. This was an example that students did not have to follow the scaffold to solve the problem; rather, they could develop their own representations and solutions. On the other hand, although Vicky drew a similar representation, her case showed a different story. When she chose the scaffold of formulating equations, the scaffold suggested her to calculate the number of intervals first. Then she drew the ten bars to represent the ten plants (Figure 5(b)). By counting the intervals, she could finally get the answers. The case implied that even though students used a scaffold, they still need to express their thinking. The digital sheet encouraged students not only to do calculation but also to draw representations by pen-based interface.



(a) Owen's diagram



(b) Vicky's diagram

Figure 5. Drawing representations by pen-based interface

Lacking the ability to choose appropriate scaffolds

Although Owen chose advanced exercises, he still thought the exercises were easy and thus rarely asked for scaffolds. By contrast, Vicky and Sean used the scaffolds more often. It was noticed that they may have a common behavioral pattern in choosing scaffolds. When Vicky tried to solve P1-3 but got stuck, she chose all of the three scaffolds. But when she encountered difficulty again in P1-4, she only chose the first scaffold and solved the problem successfully. Later in P1-5 and P2-2, she tended to use the third scaffold. Similarly,

Sean chose almost all scaffolds in P1-3 and P1-4, but finally chose the third scaffold in P1-9. Such changes could be viewed as a consolidation of seeking-scaffold behaviors. After several time of using scaffolds, they may eventually find their preferred ways.

Nevertheless, what students liked may be not what they really needed. Vicky and Sean both preferred the scaffold of formulating equations and chose it when meeting difficulty. The scaffold only helped them write down the equation, not understand the problem. In Sean's case, after he used the scaffolds of restructuring the problem texts and formulating equations to solve P1-3, he changed the operator from addition to subtraction and solved it correctly. Even if he found the answer, however, he still did not understand the reason for subtraction. For this reason, the researchers asked him to use the last scaffold of using a model and explained why using subtraction rather than addition. However, the successful experience using the scaffolds of formulating equations led him to choose it again. Similarly, Vicky also showed the tendency to use the scaffold of formulating equations in solving later problems. These excerpts showed that some students may not learn how to choose appropriate scaffolds unless they received further instructions.

3. Concluding Remark

This paper designed an activity model to support the concept of seeking scaffolds. The result of the trial study showed that the model encouraged students to solve mathematical word problems by themselves rather than follow scaffolds. Additionally, pen-based interface allowed students to express their thinking without the limitation of scaffolds. Although these features also meant that students may make mistakes in the process, they were provided with more opportunities to correct those mistakes and experience the change. However, the result also revealed that students lacked the ability to choose appropriate scaffolds, suggesting further improvement in the model.

Acknowledgements

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