Ogata, H. et al. (Eds.) (2015). Proceedings of the 23rd International Conference on Computers in Education. China: Asia-Pacific Society for Computers in Education

Bridging Model between Problem and Solution Representations in Arithmetic/Mathematics Word Problems

Tsukasa Hirashima[a], Yusuke Hayashi[a], Sho Yamamoto[a], Kazushige Maeda[b]
[a]Graduate School of Engineering, Hiroshima University, JAPAN
[b]Attached Elementary School, Hiroshima University, JAPAN
tsukasa@lel.hiroshima-u.ac.jp

Abstract. In solving arithmetic/mathematics word problems, comprehension phase is the most important and difficult phase. In the comprehension phase, conceptual and quantitative comprehension is formed as connection between problem representation and solution representation. Externalization of the comprehension phase is a promising way to support learners to overcome their difficulty in this phase. In this paper, triangle block model is proposed as a way to externalize the conceptual and quantitative comprehension. We have already developed a learning environment based on the model. In this environment, a learner builds the externalized representation of the conceptual and quantitative comprehension by combining provided components. Through an experimental use by seventy-two sixth grade students in an elementary school, we have confirmed that the learning activity designed based on the model is accepted by the students and the responsible teacher of them.

Keywords: Externalization of Conceptual and Quantitative Comprehension, Arithmetic/Mathematics Word Problem, Triangle Block Model

1. Introduction

A problem that is written by natural language and solved by calculation with quantitative relations is called a "word/story problem". Solving word problems is one of the most important activities in the learning of arithmetic/mathematics, physics and so on because this activity promotes a learner to obtain ability to apply formal knowledge he/she learnt in such subjects to real-world. It is also well-known that word problems are often more difficult than problems written by formal expressions directly usable to produce quantitative solutions. Especially in arithmetic/mathematics word problems, many researchers have already investigated the difficulty of the word problems and they agreed that (1) the problem solving process of the word problem is divided into two sub-processes: (1a) comprehension phase and (1b) solution phase. They have also agreed that (2) comprehension phase is the main origin of the difficulty of the word problems [Polya 1945; Riley 1983; Bell 1984; Cummins 1988; Resnick 1988; Mary 1992; Hegarty 1995]. Therefore, overcoming the difficulty of this comprehension phase is the most important target in the research of technology-enhanced learning environment for arithmetic/mathematics word problems.

The process of the word problem solving composed of comprehension phase and solution phase can be illustrated as Figure 1. In the comprehension phase, problem solvers process the representation of the word problem and create corresponding internal representations of conceptual and quantitative relationships expressed in the problem representation [Nathan 1992; Koedinger 2004]. In the solution phase, problem solvers create an arithmetic or algebraic expression to calculate the answer of the problem based on the internal representations. The internal representation is called "Concept-Quantity representation" (CQ representation for short) in this paper. The arithmetic or algebraic expression is usually represented externally and called "solution representation". Because several investigations indicated that both arithmetic and mathematics word problems have the same difficulty in the process of problem comprehension [Kintsch 1985; Nathan 2000], the two types of problems are not clearly distinguished in this paper.

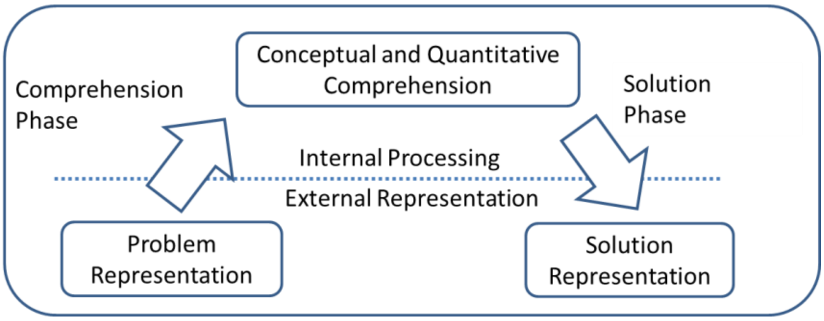

Figure 1. A model of solving process of a word problem.

The illustration shown in Figure 1 suggests that the gap between the problem and solution representations is the main origin of the difficulty of the word problems. Besides, it also suggests that externalization of the conceptual and quantitative comprehension that is created and processed in mind is a promising approach to bridging the gap. This approach is able to be illustrated as Figure 2. We call this approach "externalization of thinking task". Here, it is expected that conceptual and quantitative comprehension is promoted by producing and observing the external CQ representation.

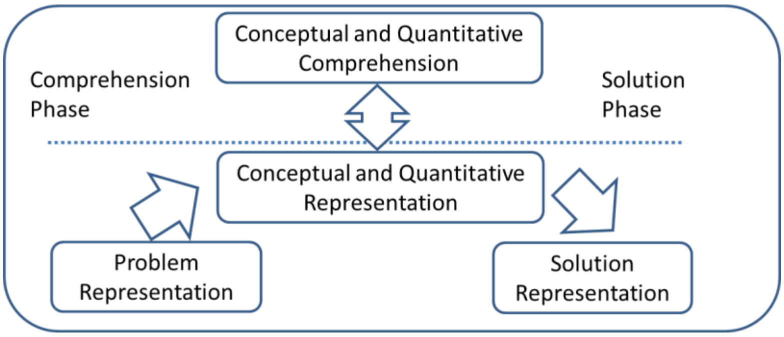

Figure 2. Externalization of Conceptual and Quantitative Representation.

There are several researches that have attempted the externalization of the conceptual and quantitative comprehension. In a learning environment named ANIMATE [Nathan 1992], the CQ representation was formed by connection of several schemas. The schema was described as operational relations among three concepts, for example, "Distance=Rate*Time", or "Distance1+Distance2=Disntance3". Here, "Distance/Time=Rate" was dealt with as a difference schema with "Distance=Rate*Time". A learner was allowed to make a network (that is, CQ representation in ANIMATE) corresponding to a problem representation by connecting several schemas in ANIMATE. Based on the network, the system generated an animation and a learner who made the network was able to confirm the correctness of the network by observing the animation. The system didn't have an ability to diagnose the network. Although ANIMATE was a pioneer of learning environments attempting the externalization of thinking task, its CQ representation was not enough to bridge the gap in the following two points. (1)The CQ representation didn't use the concepts directly appeared in the problem. The CQ representation in ANIMATE is composed of the abstract concepts used in the schemas. (2) Because the CQ representation couldn't be diagnosed by the system, it was impossible to design adaptive support to build the CQ representation. MathCal [Chang 2006] provided a learner with a series of operational relations with several blanks and requested him/her to fill in the blanks with concepts included in the problem representation. Although the completed relations (that is, CQ representation in MathCal) could be diagnosed, the shape of the CQ representation was specified beforehand and a learner was not allowed to build it by him/herself. HyperGraph [Arnau 2013] was a kind of the CQ representation and it was possible to explain several correct algebraic solutions based on one CQ representation. MIPS [Hirashima 1992] was also a kind of the CQ representation to explain the way to solve complex arithmetic word problems without algebraic way. Although these representations were promising to support the solution phase, no usage to support comprehension phase was developed.

Based on these considerations, we propose “Triangle Block Model” as a CQ representation that satisfies following requirements: (1) a learner is allowed to build the CQ representation, (2) concepts constituting both the CQ representation and the problem representation are the same ones, and (3) the CQ representation is able to be diagnosed. Based on the triangle block model, we have already developed an interactive learning environment where a learner is allowed to build a CQ representation corresponding to a problem representation by using provided components. Then, the CQ representation is diagnosable. The learning environment was experimentally used in an elementary school. Seventy-two 6^{th} grade students in two classes used two class times (one class time was 45 minutes). Through this use, we have confirmed that the elementary school students were able to build the representation smoothly and felt the activities were useful for learning. The responsible teacher of them also accepted this confirmation. Moreover, we found that most of the representations built by the students were categorized into three types: (I) story type, (II) solution type and (III) addition-multiplication type, even though there were several other types of representations corresponding to the problem representation, and then, the three types of representation was reasonably explained from the viewpoint of problem solving process. These results suggest that the CQ representation with the triangle block model is a promising representation to bridge between problem and solution representations.

In this paper, in the next section, the framework of the triangle block model is described. Then, an interactive learning environment based on the triangle block model called “MONSAKUN Triangle-Block” is described. The results and analysis of experimental use of MONSAKUN Triangle-Block are also reported.

2. Concept-Quantity Representation with Triangle Block Model

2.1 Overview of Building of CQ representation with Triangle Blocks

In this subsection, an overview of building the CQ representation with the triangle blocks is introduced. Figure 3 shows examples of (1) problem representation and (2) CQ representation with triangle blocks. The CQ representation in Figure 3 is composed of two triangle blocks that are shown in Figure 4. These triangle blocks can be decomposed into several components shown in Figure 5. Therefore, by combining these components, it is possible to build the CQ representation shown in Figure 3 again. Based on the CQ representation, calculation procedure to answer the problem can be derived as shown in Figure 6. Details of them are described in the remaining section.

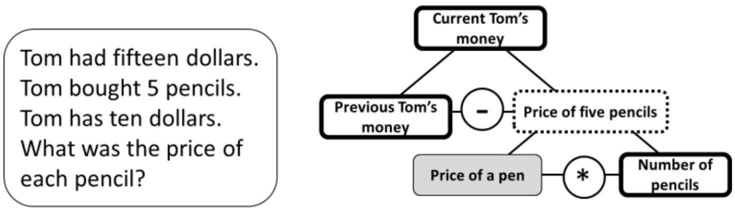

Figure 3. Arithmetic Word Problem and CQ Representation with Triangle Blocks.

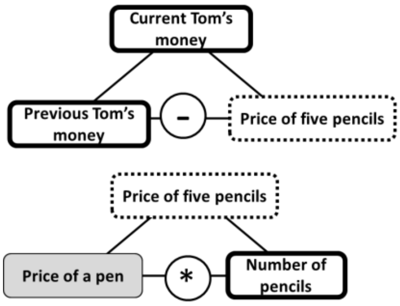

Figure 4. Triangle Blocks.

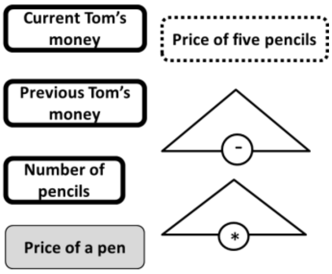

Figure 5. Components of Triangle Blocks.

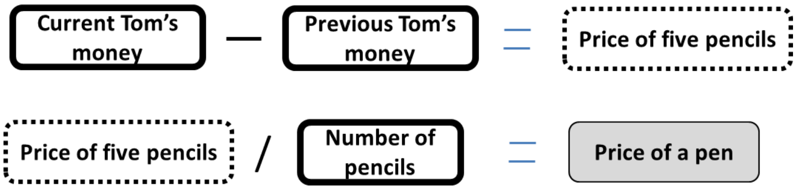

Figure 6. Solution Representation.

2.2 A Triangle Block

A basic unit of arithmetic word problem is composed of three arithmetic concepts connected with a quantitative relation expressed by an equation with one of four operations. The set of three concepts and a quantitative relation are expressed in a triangle block. In the triangle block, the three concepts are arranged in the three apexes. The operation placed in the base edge of the triangle as an expression of an operational relation between two concepts arranged in both ends of the base edge, and the result of the operation is expressed by the concept placed at the remaining apex opposite to the base edge. An example of the basic unit of arithmetic word problem (basic problem for short) and a corresponding triangle are shown on the left side of Figure 7. In this example, "price of a pen", "number of pencils" and "price of five pencils" are the three arithmetic concepts constituting the basic problem and the corresponding triangle block.

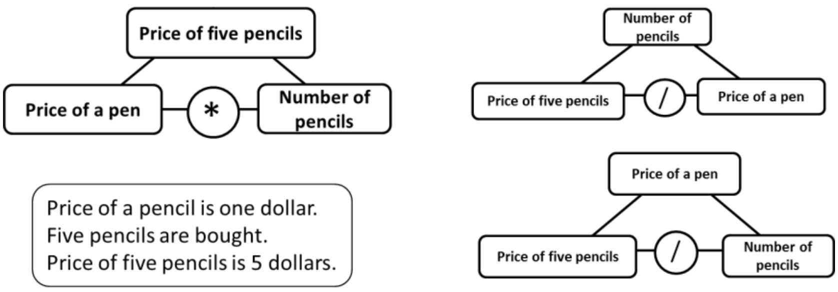

Figure. 7. Three Triangle Blocks corresponding to a Basic Problem/Story.

In a basic problem, the quantitative relation can be expressed by three ways in logically. If a problem has a quantitative relation expressed "X+Y=Z", it includes two more quantitative relation expressed as "Z-Y=X" and "Z-X=Y". In the case of "X*Y=Z", it includes two more quantitative relations expressed as "Z/Y=X" and "Z/X=Y" in the same way. On the right side of Figure 7, two other triangle blocks corresponding to the same basic problem shown at the lower left of Figure 7. A triangle block is an expression that explicitly expresses one operational relation with the base edge. The triangle block also implicitly suggests two remaining operational relations with the two oblique edges of the triangle. The two other triangle blocks are created by changing an oblique edge to the base edge and making the implicit operation on the oblique edge explicit one. One of the most important characteristics of the triangle block is that the two oblique edges visualize the existence of two other relations between the other two pairs of concepts.

Sentences shown in Figure 7 don't include any unknown value. Therefore, strictly speaking, it is not a problem but a story. However, even in a story, it is possible to derive a value by using other two values by using its quantitative relation. In short, a basic story includes three basic problems. We don't strictly distinguish a story from a problem in this paper.

2.3 Combination of Triangle Blocks

A complex problem is expressed by combination of several triangle blocks as shown in Figure 3. We call such combination "combined triangle block". Two triangle blocks are able to be connected by overlapping two apexes with the same concepts. The two triangle blocks shown in Figure 4 are connected by using "price of five pencils" in Figure 3. Here the combination of triangle blocks shown in Figure 3 is explained as an example. A bold solid rectangle expresses a concept of which value is given explicitly in the problem. The thin solid rectangle with gray background expresses a concept of which value is required to derive in the problem. A rectangle with broken line expresses an intermediate concept that doesn't appear in the problem but is necessary to derive the required value from the given values.

A triangle block can be transferred to two other quantitative equivalent triangle blocks as shown in Figure 7. Therefore, there are several combined triangle blocks corresponding to a problem. In the case of Figure 3, three other combined triangle blocks can be made as shown in Figure 8. These combined triangle blocks are alternatives of the CQ representations corresponding to the problem. Categorization of the CQ representations and frequencies of their appearance are discussed in the analysis of the experimental use described in Section 4. Here, it is constrained to connect two blocks only at "one of two apexes of the base edge" and "an apex opposite to the base edge". This means that it is not accepted to connect the base edge to another base edge. By this constraint, the combined triangle block forms a tree structure and is able to be converted to a series of numerical expression. The numerical expression can be solved arithmetically.

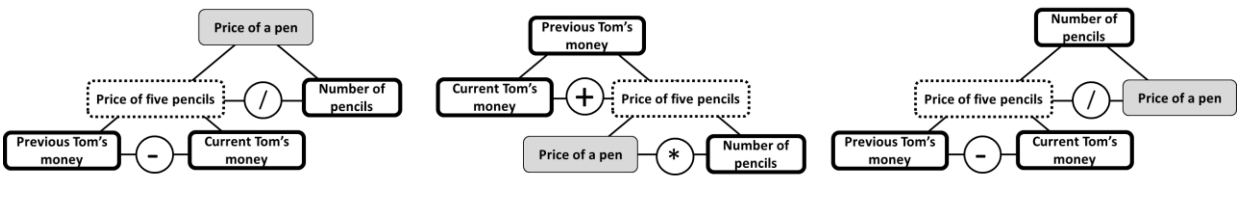

Figure 8. Other variations of Combined Triangle Block to the Problem Shown in Figure 3.

3. MONSAKUN Triangle-Block

3. 1 Kit-Build Method to Realize Building Combined Triangle Blocks by Learners

In order to realize the activity to build the combined triangle block as practical one for both learners (that is, it is not so hard to build) and the system (that is diagnosable), we have adopted kit-build method [Hirashima 2011; Hirashima 2015]. In the kid-build method, a correct structure is prepared at first. The structure, then, is decomposed into several components. The components are provided to a learner and he/she is required to build a structure by combining the components. By providing the components, the task to build a structure becomes a clear task for the leaner. As for the advantage for the system, the structure built by a learner can be automatically diagnosed by comparing it with the correct one because all components are the given ones.

Two kinds of components are provided to a learner as the components of the combined triangle block, one is an arithmetic concept and the other is an arithmetic operation. The arithmetic concepts are categorized into following three regarding their values, (1) given concept, (2) answer concept, and (3) intermediate concept. The value of given concept can be derived from the problem sentence directly. The value is specified as the answer in the problem sentence. The value of the intermediate concept cannot be derived from the problem sentences directly. As for arithmetic operations, four basic arithmetic operations, that is, addition, subtraction, multiplication, and division are dealt with. The interface where a student carries out this task is explained in the next section.

From a combined triangle block, all variations composed of the same concepts and corresponding quantitative relations as shown in Figure 8 can be generated automatically. Therefore, if a correct combined triangle block is given, by generating all variations from it, it is possible to judge whether a combined triangle block built by a learner is correct or not.

We have already developed a learning environment where a learner can build a combined triangle block by using provided components and receive feedback for the block he/she built. We call the subsystem that a learner directly uses “MONSAKUN Triangle-Block”. MONSAKUN Triangle-Block is implemented as Web application and a learner uses it by his/her own 10-inch tablet PC. Therefore, it can be used in a usual classroom. A learner builds combined triangle block by using MONSAKUN Triangle-Block, then it sends learner’s learning data to sever via wireless LAN. A teacher can examine the learning data with visualization tool on teacher’s tablet PC.

3.2 Interface to Build Combined Triangle Block

Figure 9 shows two pictures of the interface of MONSAKUN Triangle-Block. Because currently we have only Japanese version, words in the interface are translated into English for this paper. The left picture shows an initial screen. An arithmetic word problem is set in the left column. Nodes provided at the left side of the right column (building field) express the concepts that are components of the correct combined triangle block. The nodes can be moved by single finger touch & drag. At the bottom of the building field, four operations are provided. By touching one operation in the four, a triangle with the operation at the base edge is appeared in the building field. By setting three arithmetic concepts on the three apexes of the triangle, a basic triangle block is made. In this problem, combined triangle block shown in Figure 3 and its variations shown in Figure 8 are correct ones. When a combined triangle block is built and the answer button is pushed, the combined triangle block is diagnosed by comparing it with the correct ones.

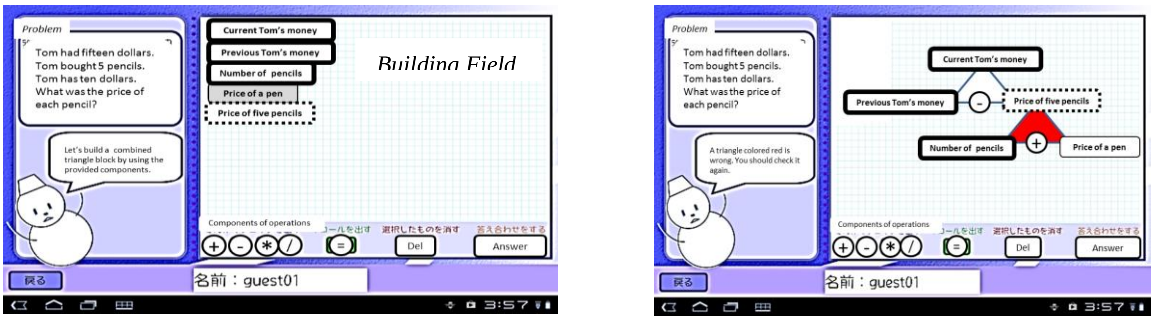

Figure 9. Interface of MONSAKUN Triangle-Block

By clicking “answer button” at the right bottom corner, the combined triangle block is diagnosed. This diagnosis is carried out with following three steps: (1) finding of unused components (2) finding of an isolated triangle block, (3) finding of inadequate basic triangles. The step (1) & (2) are syntactic diagnoses. The step (3) is more important diagnosis. The step (3) is carried out by comparing with the basic triangle blocks consisting of the correct combined triangle block. Incorrect patterns of the basic triangle block are categorized into the following two: (error-1) error in the combination of the three arithmetic concepts, and (error-2) error in the operation of the triplet. Because only necessary and sufficient concepts are provided, all correct combined triangle blocks are composed of the same triplets of concepts although their base edges and operations might be different. In the current diagnosis function, error-2 is examined only when the triplet of the concepts is correct. When the error-1 or -2 is detected, the erroneous basic triangle block is specified by changing the color as shown in right side picture in Figure 9. In the case of error-1, a student is requested to reconsider the nodes of the block. When the error-2 is detected, a student is requested to reconsider the operation of the block. Therefore, the environment can generate feedback depending on learners' errors

As for preparation of the correct combined triangle blocks, at first, a correct one should be prepared to a problem beforehand. Variations of it can be automatically generated. By comparing learner's combined triangle block with them, the type of the combined triangle block is also detected. The detected types are used in the analysis of the results of the experimental use in the next subsection.

After the completion of the combined triangle block, the system requests a student to calculate the required value on the block expression. Depending on the type of the block expression, the calculation procedure becomes difference one. This phase corresponds to "solution" phase in the model shown in Figure 2. In the case of the problem and combined triangle block shown in Figure 3, at first, known values in the problem are assigned to nodes in the combined triangle block as shown in Figure 10. A learner is requested to calculate unknown values one by one to make them known. In this case, the value of "price of five pencils" can be calculated by using the values of "previous Tom's money" and "current Tom's money". The operation is subtraction. After that, the value of "price of a pen" can be calculated by using the values of "price of five pencils" and "number of pencils" with division. The operations in this calculation are not same with the operations in the combined triangle block. In this problem, the operations in the combined triangle block in the left side of Figure 8 are same with the calculation operations.

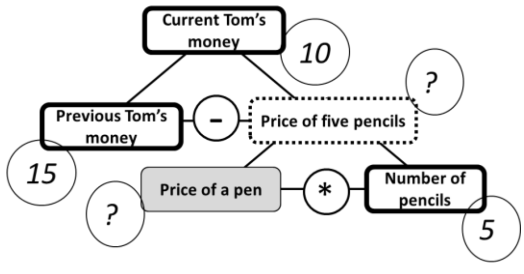

Figure 10. An Example Solution Phase

4. Experimental Use and Analysis of the Results

In this experimental use of MONSAKUN Triangle-Block, we examined that whether 6th grade elementary school students could build the combined tringle blocks corresponding to arithmetic word problems and derive solutions by using them. The problems used in this experiment are composed of from two to four triangles. A responsible teacher of the students confirmed that the problems used in MONSAKUN Triangle-Block are not difficult to solve for most of his students. Then, the teacher predicted that most of students could build the combined triangle blocks to the problems and derived their answers. Through this experiment, therefore, we tried to confirm whether the students could carry out the comprehension phase correctly with the triangle block model as external CQ representation. Because this representation is novel one for the teacher and students, this confirmation is indispensable. Investigation of the effect to promote learners to comprehend word problems is our future work.

3.1 Procedure of Experimental Use

Seventy-five 6^{th} grade students in two classes at an elementary school used MONSAKUN Triangle-Block for two lessons (one is 45 minutes). Due to their curriculum schedule, the interval of the two lessons was a week. At the first lesson, the responsible teacher of both classes took 15 minutes to explain that the students were requested to solve arithmetic word problems with the system and how to use the system, that is, the way to build a combined triangle block and to calculate the required values by using the block expression. They had not received any lectures about the triangle block model before to use this system. The teacher expected that because the model was reasonable and the level of problems was adequate for the students, the students could use the system without such lecture. Remaining 30 minutes, the students continuously used the system. At the second lesson, the first 30 minutes was used to use the system and 10 minutes was used to complete a questionnaire. The remaining 5 minutes was used for usual class matters. In the system, 9 problems have been prepared.

3.2 Analysis of the Results
3.2.1 Activities

In the first lessons, 74 students used the system and completed 6.0 problems in average. Nine students completed the all 9 problems. These students were requested to solve the problems again from the first. The number of problems of the second solving is not counted in the average. In the second lesson, 72 students used the system and completed 7.5 problems in average. Thirty-nine students completed the all 9 problems. The responsible teacher commented that the students had engaged in solving problems with the system very eagerly in comparison with problem solving exercises on paper. The teacher also commented that most of the students built the combined triangle blocks smoother than he had predicted. Although the problems themselves were not difficult for the students, the teach thought the experience to build the explicit structure was worth for them. He concluded that the classes using MONAKUN Triangle-Block had realized high quality activities for arithmetic learning.

3.2.2 Questionnaire

We conducted a questionnaire for the students about the activity using MONSAKUN Triangle-Block at the end of the second class. Main questions are as follows: Q1: “when you solve a word problem, you usually do similar activity like building the combined triangle block"; Q2: “building the combined triangle block was useful to understand the meaning of the problem”. Answers for them are as follows: answers for Q1: {Strong agree: 24, Agree: 35, Disagree: 13, Strong disagree: 0}. Answers for Q2: {Strong agree: 33, Agree: 34, Disagree: 5, Strong disagree: 0}. These results suggest that the triangle block model and activity with it are accepted by most of the students as meaningful ones. This is important evidence that the combined triangle block is a suitable representation to externalize the CQ representation.

3.2.3 Categorization of the Combined Triangle Block Built by the Students

We analyzed 446 combined triangle blocks correctly built by the students. Then we found that 94% of the combined triangle blocks were categorized into the three meaningful types: story type (40%), calculation type (44%), and addition-multiplication type (10%). The story type is composed of triangle blocks with operations suggested by sentences in the problem. The combined triangle block shown in Figure 3 is an example of the story type. In the problem, "bought" suggests "subtraction", and both "price of each pencil" and "five pencils" suggest "multiplication". Because the story type directly corresponds to the problem representation, appearance of this type is reasonable. The combined triangle block at the left of Figure 8 is an example of the calculation type. The operations of this type of combined triangle block are the same ones with the calculations to derive the answer. Appearance of calculation type is also reasonable because the structure is corresponding to the calculation to derive the answer of the problem. The combined triangle block at the middle of Figure 8 is an example of the addition-multiplication type. The operations used in the addition-multiplication type are only addition and multiplication. The addition-multiplication type has the same shape regardless of the words in the problem representation or answer quantities in the problem. In arithmetic/mathematics, addition and multiplication are taught as the basic operations, and then subtraction and division are taught as reverse operations for addition and multiplication. So, to think only with addition and multiplication is sometimes recommended as an advanced way. Therefore, this type is also a reasonable one. The combined triangle block shown in right side of Figure 8 is an example of ones that don't belong to the three types. This is not wrong but the reason to build it is not clear in comparison with the three types. Because the average number of variations of the combined triangle block to a problem in this experiment was 6.2, if the forms appeared randomly, it was predicted that the other types except the three types appeared more than 50%, although the appearance rate in the experiment was only 6%. These results suggest that the three types of the combined triangle block has meaningful one and the learners didn't build the blocks by trial and test.

We also found that the students were categorized by the types they built. Besides, it was found that their average scores of an achievement test of arithmetic were difference by the categories, that is, story type (14 students, 78.7 marks (SD=13.8)), calculation type (16, 68.1(18.8)), addition-multiplication type (5, 83.2(10.2)). As another group, 26 students used "calculation type" at first several problems, and then they changed their types to "story type. In this use, the latter half problems are more complex ones with three or four basic triangle blocks. The average score of this group is 78.5(14.3). Although there is no significant difference between the groups, we will analyze the relation between scores and their forms in mode details. As for the scores of an achievement test of arithmetic, we found significant correlation between "the number of solved problems with the system" and the score (r=0.533, p=8.33E-7). These results also suggest that the triangle block model and activity with it are useful to investigate the students' comprehension process.

5. Considerations and Remarks

Through an experimental use of the system designed based on triangle block model, it was confirmed that the students could build the combined triangle block actively, and they thought that the activity with the system was similar to the activity of usual problem-solving. Then, they thought that the activity with the system was useful to understand the problems. These results suggested that the triangle block model is promising as a method to externalize the CQ comprehension in solving arithmetic word problems. Besides, the analysis between the types of the combined triangle block and the scores of the achievement test has suggested that the types reflecting students' structural thinking.

In the current stage of this research, we have only experimentally confirmed that the triangle block model was acceptable for a teacher and learners as CQ representation. It is necessary to evaluate the learning effect to use the model as an important future work. One more important task is to use the triangle block model to learn the basic problems. We have been investigating interactive learning environment by problem-posing targeting the basic problems. The learning environment has been practically used it by the first, second and third grade students at several elementary schools [Hirashima 2007, Hirashima 2001, Yamamoto 2012, Yamamoto 2013, Yamamoto 2014]. For targeting higher than third grade students, the triangle block model was developed from the triplet sentence model [Hirashima 2014] for the basic problems used in these researches. Therefore, the way to connect or integrate learning activities of the basic and combined word problems is also our important research topic.

Acknowledgements

This work was supported by JSPS KAKENHI Grant Number 15H02931.

References

Arnau, D., Arevalillo-Herráez, M., Puig, L, Antonio González-Calero, J. (2013). Fundamentals of the design and the operation of an intelligent tutoring system for the learning of the arithmetical and algebraic way of solving word problems, Computers & Education, 63, 119-130.

Bell, A., Efraim Fischbein, Brian Greer (1984): Choice of operation in verbal arithmetic problems: the effects of number size, problem structure and context, Educational Studies in Mathematics, Vol.15, No.2, pp.129-147.

Chang, K., Yao-Ting Sung, Shiu-Feng Lin (2006). Computer-assisted learning for mathematical problem solving Solution Path with Semantics, Computers & Education 46,140–151.

Cummins, D., Kintsch, W., Reusser, K., Weimer, R.(1988). The role of understanding in solving word problems. Cognitive Psychology, 20, 439-462.

Hegarty,M.,Mayer,R.E.,&Monk,C.A. (1995): Comprehension of arithmetic word problems: a comparison of successful and unsuccessful problem solvers, Journal of Educational Psychology，Vol.87，No.1, pp.18-32.

Hirashima, T., Kashihara, A., Toyoda, t. (1992). Providing Problem Explanation for ITS, proc. of Intelligent Tutoring Systems 1992, 76-83.

Hirashima, T., Yokoyama, T., Okamoto, M., Takeuchi, A.(2007). Learning by problem-posing as sentence-integration and experimental use, AIED 2007, 254-261.

Hirashima, T., Kurayama, M.(2011). Learning by problem-posing for reverse-thinking problems, Proc. of AIED2011, pp.123-130.

Hirashima, T., Kazuya Yamasaki, Hiroyuki Fukuda, Hideo Funaoi (2011). Kit-build concept map for automatic diagnosis, Proc. of AIED2011, pp.466-468.

Hirashima, T., Hayashi, Y., Yamamoto, S. : Triplet Structure Model of Arithmetical Word Problems for Learning by Problem-Posing, Proc. of HCII2014(LNCS 8522), pp.42-50(2014).

Hirashima, T., K. Yamasaki, H. Fukuda, H. Funaoi.(2015). Framework of Kit-Build Concept Map for Automatic Diagnosis and Its Preliminary Use, Research and Practice in Technology Enhanced Learning (RPTEL).

Kintsch, W., Greeno, J.G.(1985). Understanding and solving word arithmetic problems. Psychological Review, 95, 109-129.

Koedinger, K.R., Anderson, J.R. (1988). Illustrating principled design: The early evolution of a cognitive tutor for algebra symbolization. Interactive Learning Environments, 5, 160-180.

Koedinger, K. R., & Nathan, M. J. (2004). The real story behind story problems: Effects of representations on quantitative reasoning. Journal of the Learning Sciences, 13(2), 129–164.

Nathan 1992, Koedinger 2004Mary，H., Mayer, R. E.，and Green, C. E. (1992). Comparison of Arithmetic Word Problems: Evidence From Students' Eye Fixations"，Journal of Educational Psychology，Vol.84，No.1, 76-84.

Nathan, M., Kintsch, W., Young, E. (1992). A theory of Algebra-Word-Problem Comprehension and Its Implications for the Design of Learning Environments, Cognition and Instruction, Vol. 9, No.4, 329-389.

Nathan, M.J., Koedinger, K.R. (2000). An investigation of teachers' beliefs of student's algebra development, Cognition and Instruction, 18(2), 209-237.

Pólya, G. (1945). How to Solve It. Princeton University Press.

Resnick, L.B. (1988). Treating mathematics as an ill-structured discipline. In Charles, R.I. & Silver, E.A.(eds.), The teaching and assessing of mathematical problem solving, 32-60, LEA.

Riley,M.S, Greeno,J.G, Heller,J.I. (1983). Development of children's problem solving ability in arithmetic "H.P.Ginsburg(Ed) The development of mathematical thinking.New York:Academic Press.，pp.153－196.

Yamamoto, S., Kanbe, T., Yoshida, Y., Maeda, K., Hayashi, Y., Hirashima, T. (2012). A Case Study of Learning by Problem-Posing in Introductory Phase of Arithmetic Word Problems, Proc. of ICCE2012, Main Conference E-Book, pp.25-32.

Yamamoto, S., Kanbe, T., Yoshida, Y., Maeda, K., Hayashi, Y., Hirashima, T. (2013). Interactive Environment for Learning by Problem-Posing of Arithmetic Word Problems Solved by One-step Multiplication, Proc. of ICCE2013, pp.51-60.

Yamamoto, S., Akao, Y., Murotsu, M., Kanbe, T., Yoshida, Y., Maeda, K., Hayashi, Y., Hirashima, T. (2014). Interactive Environment for Learning by Problem-Posing of Arithmetic Word Problems Solved by One-step Multiplication and Division, ICCE2014, pp.89-94.