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Developing a Geometric Proof Problem-Solving Support System Utilizing Card Selection

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Abstract: We think the solving of proof questions is cited as being conducive to the development of the skill of logical thinking. In this paper, we conducted an analysis of the structure of the proof question regarding the congruence of triangles. On the basis of this analysis, we constructed a solutions-support system that utilized the card selection method. As a result, the structure of the proof question consist of “assumptions” and “conclusions”. In this system, the blanks are then increased one after another, and finally, all the proofs are left blank. It is not only assumptions and conditions that are searched. In addition, sorting the given cards themselves and solving the proof promotes gradual learning.

Keywords: skill of logical thinking, Geometric Proof, Card selection, Problem-solving, assumptions and conclusions

1. Introduction

In recent years, the skill of logical thinking has received much attention. It can be said that this is because of an increase in situations that require expressing ideas logically, including interviews for jobs and for continuing on to higher education. Presentations and composition writing are often cited as being conducive to the development of the skill of logical thinking; however, besides these, the solving of proof questions can also be cited. Unlike presentations and composition writing, in the case of proof questions, things (i.e., assumptions or conditions) that one already knows about a given problem lead to conclusions that one wants to know, and the assumptions, conditions, and conclusions are already given. Therefore, this is believed to be easier for nurturing logical thinking when compared with other methods.

The question method of proof questions includes the “description method”, in which the learner describes everything from the beginning to the end, and the “fill-in-the-blank method”, in which a portion of the proof question is left blank and the learner describes the expressions or characters that correspond with the “blank.” However, when learners who use the fill-in-the-blank method suddenly utilize the description method to present a proof, it becomes necessary to think of the whole only from the part. Consequently, things that require thinking increase and give rise to difficulties. Therefore, it was considered possible to learn proofs through utilizing the “card selection method” proposed by Hirashima et al. (2012) and thinking of the structure from the part. Since, in the card selection method, it is essential to examine the cards prepared beforehand, the structure of the proof needs to be clear and it is vital to build a system. In this study, we conducted an analysis of the structure of the proof question regarding the congruence of triangles. On the basis of this analysis, we constructed a solutions-support system that utilized the card selection method.

2. The Structure of the Proof Question

Proof questions consist of “assumptions” and “conclusions,” and it is crucial to discover the conditions that lead to the conclusions and the elements essential to utilize these conditions and to solve them through description in accordance with an order. For example, the proof of the question in Figure 1 is shown as follows:

In the figure on the right, when points D and E, respectively, are taken as points on the line segments AC and AB, if AD = AE and ∠ADB = ∠AEC, prove that AB = AC.

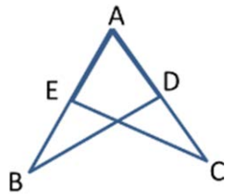

Figure 1. Example of proof question

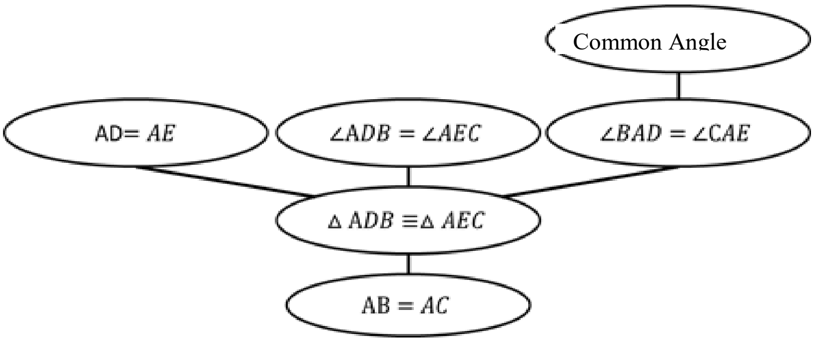

Figure 2. Structure of the Proof in Figure 1

Proof

In △ADB and △AEC

From the assumption AD = AE ……(1)

From the assumption ∠ADB = ∠AEC ……(2)

As the sides are common ∠BAD = ∠CAE ……(3)

From (1), (2), and (3),

as one side and the angles of the two sides, respectively, are equal, we get the following:

△ADB ≡ △AEC and

since, for congruent shapes, the corresponding sides are equal in length, we get the following:

AB = AC

(Q.E.D.)

When this proof is structured, it looks like Figure 2. However, the assumptions in the problem sentences have been omitted.

3. Problem-Solving Support System for Proofs

The screen of the proposed system is shown in Figure 3. Complete the proof question by moving the cards from the card group on the lower right-hand side to the blank part of the proof on the left-hand side using the "drag and drop" function. When all the blanks have been filled with the cards, you will be able to press the "answer" button. When the learner presses this button, the system determines whether the answers are correct or incorrect and provides the learner with an evaluation of right and wrong answers.

The card selection method refers to a method in which cards with simple sentences written on them are matched with blanks. In this method, as shown in Figure 3, it is only the part that can be understood from the problem sentences (including from assumptions and conditions) that is kept blank. Hence, the method is close to the fill-in-the-blank method and is not as difficult as the description method. It is easy even for beginners. Furthermore, as per Figure 4, if parts that form the sentence (such as conclusions or "from these") are kept blank and cards that can be selected are increased, then, rather than just exploring the essential portion, one is required to think of the composition of the sentence. This results in a question format that is closer to the description method than it is to the fill-in-the-blank

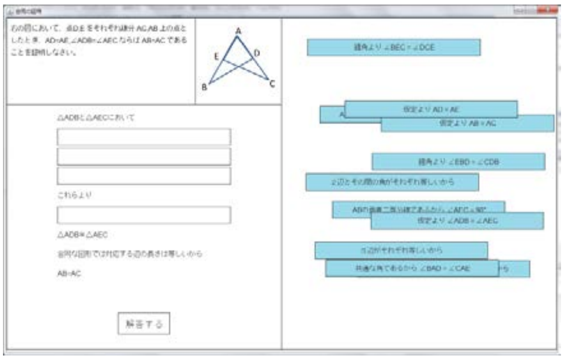

Figure 3. System Screen in Japanese

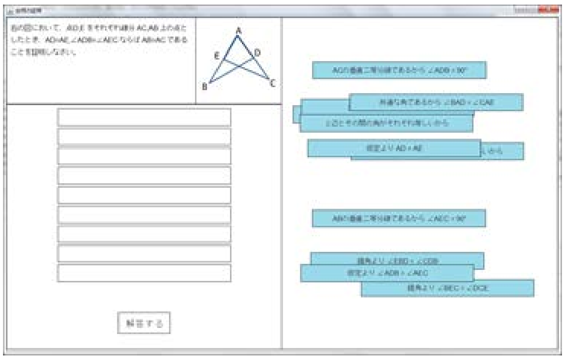

Figure 4. Question format that is closer to the description format in Japanese

method. At first, it is only the assumptions and conditions that are kept blank and simple problems are solved. As learners advance, they are first led to the conclusion that "the given figures are congruent," and then the conclusions drawn are left blank. The blanks are then increased one after another, and finally, as shown in Figure 4, all the proofs are left blank. It is not only assumptions and conditions that are searched. In addition, sorting the given cards themselves and solving the proof promotes gradual learning.

4. Summary

In this study, we discussed the addition of blanks and cards in order to improve the proof problem-solving support system and the level of difficulty of the problems. Future challenges include the essential improvement of the system and having in the database information regarding the level of difficulty on the basis of the distribution map created from the results of the structural analysis of proof questions conducted in previous studies. Furthermore, it is important for problems corresponding to the level of difficulty to be created by gradually increasing and decreasing the blanks and the cards. Finally, it is essential to examine whether it is actually possible to advance learning through the use of the system.

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