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Fraction Block as a Tool for Learning & Teaching Fraction and Its Experimental Use in an Elementary School

Akimitsu JOYA[a*], Kazushige MAEDA[b] & Tsukasa HIRASHIMA[a],
[a]*Graduate School of Engineering, Hiroshima University, Japan*
[b]*Attached Elementary School of Hiroshima University, Japan*
*joya@lel.hiroshima-u.ac.jp

Abstract: This paper proposes "fraction block" as an educational reifications of fraction. Fraction block reifies characteristics of "ratio fraction" as a pair of numerator block and denominator block. The length of the blocks can be changed by keeping the ratio of numerator block to denominator block. By using the fraction block, a quantity represented as a length of tape is derived from another quantity represented as another length of tape. This deriving operation is a reification of multiplication or division with a fraction. We have implemented a learning environment where a learner is able to directly operate the fraction block in order to derive a quantity from a quantity. Experimental use of this environment in an elementary school is also reported in this paper.

Keywords: Fraction Block, Visual & Manipulatable Model, Ratio Fraction, Multiplication & Division.

Introduction

In order to comprehend a certain difficult or complex subject, learning by operating a model that includes all features of the subject is effective way for a learner to promote intuitive and constructive understanding (Papert, 1982). Papert advocated a computer-based learning environment where a learner was able to experience mathematic operations and the results of the operations, and then he called it "the country of mathematics". "Logo" is a realization of such learning environment for geometry. Our research that is inspired by his researches has targeted on promotion of conceptual understanding of fraction that is well-known as one of the most difficult subjects in arithmetic in elementary school.

Students around the world have difficulties in learning about fractions. In many countries, the average student never gains a conceptual knowledge of fractions. Conceptual knowledge of fractions is defined as knowledge of what fractions mean, for example their magnitudes and relations to physical quantities. Student's difficulties with fractions often stem from lack of conceptual understanding. Many students view fractions as meaningless symbols or view the numerator and denominator as separate numbers rather than a unified whole (Warrington & Kamii, 1998, Cramer et al., 2002, Bulgar, 2003, Fazio & Siegler, 2011). Also in Japan, fraction is known one of the most difficult subjects in elementary school curriculum of arithmetic. Teaching conceptual understanding of fractions is assumed to be important problem in pedagogy even now. Although almost Japanese children can correct calculation problem, many of them can't justify the arithmetic calculation procedures of fractions that they used to derive correct answers (Naito et al., 1990). In the academic ability survey all over Japan in 2010, children were questioned a word problem "2 liters of juice is divided into 3, what liters is 1 part? You have to answer by fraction", which is well known problem that students often fail to distinguish the meaning of ratio from quantity in fraction. Unfortunately, the correct answer rate is 40.6% in sixth grade students. This result suggests that not a few children don't understand the meaning of a fraction well. Besides, in the academic ability survey all over Japan in 2008, children were questioned calculation problem "2÷3 (You have to answer by fraction)" and the correct answer rate was 73.8%. This result suggests that they know how to calculate fractions. Since, teaching conceptual understanding of fractions is more important than formal calculation.

In this research, “fraction block” is proposed as an educational reification of “ratio fraction”. The fraction block reifies a fraction as a pair of numerator block and denominator block. The length of the blocks can be changed by keeping the ratio of numerator block to denominator block. Then, by using the fraction block, a quantity represented as a length of tape is derived from another quantity represented as another length of tape. This deriving operation is a reification of multiplication or division with a fraction. We have implemented a computer-based environment where a learner is allowed to directly manipulate the fraction block in order to derive a quantity from another quantity. In the process, the learner should modify the length of the fraction block.

In this paper, the idea of fraction block and its implementation as an educational tool are described. Experimental use of this environment in an elementary school is also reported in this paper.

1. Our Idea and Method

Although children well know they can make some parts to divide certain thing like a cake, there are three lacking concepts to comprehend the meaning of fraction: that is, (1) by using a fraction, a quantity is derived from another quantity, (2) in order to derive a quantity with fraction, it is necessary to specify a base quantity, (3) an input quantity is assigned to a denominator (in multiplication) or a numerator (in division), and then output quantity is assigned to a numerator or a denominator respectively. Actually, these 3 points is not taught clearly in elementary schools. In an introduction phase, children are taught fraction as part-whole ratio by dividing concrete things like a square paper. In this time, the size of a square paper is not often discussed by a teacher or children. They don’t scale or compare size of the paper. In addition, part and whole are not appear simultaneously then they can’t compare the part size and the whole size. That is, (I) there is no activity to relate dividing operation to quantity. As the next step, children are taught fraction as quantity by dividing 1 meter tape. With a story that is "we cut 1 meter tape into 3 then a part is 1/3 meters", a teacher usually only uses unit quantity for base quantity. Thus 2 fractions, that is, "1/3meters" and "1 dividing into 3" are recognized by learners as the same numbers. Thus, (II) children can’t perceive the importance of defining base quantity and (III) relation between base quantity and part quantity. Additionally, a tape diagram doesn’t present the part quantity and the whole quantity in the same way, and the part tape can be seemed to quantity and ratio (Figure 1). We think it is too hard to understand the relation between dividing operation and two quantities. Several teachers reported that children considered 1 meter was not quantity but whole and part tape was not quantity but part on tape diagram (Komabayashi & Karihara, 1990).

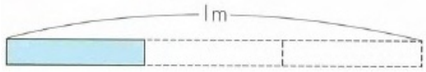

Figure 1. Tape Diagram

Fraction of ratio is able to be explained with 2 ways, (i) dividing concrete thing as part-whole, (ii) calculation methods in word problem. (i) We use concrete things to teach part-whole fraction. But concrete things have quantity necessarily then divided one has also quantity. Therefore ratio and quantity are not separated on diagram. There is a need to separate ratio and quantity, or the method to represent only ratio. (ii) Teachers don’t explain enough why we calculate multiplication and division such as methods. Almost children can calculate multiplication and division but not explain why they calculate by the method. These lacks of understanding will cause trouble on solving word problem.

For the reason, we believe firmly separating ratio and quantity on diagram foster the acquisition of fraction conception and propose fraction block which is visible, direct-manipulatable and assessable. In this model, we separate the representations of quantity and ratio. Quantity is represented by tape (Figure 2) and ratio is represented by fraction block (Figure 3). Learner can change the fraction block’s length to match various tapes as base quantity and pick up related tape. Such as operations about ratio are only usable in our mind, but our idea might be let ratio operations become concrete operations. We expect that children will understand fraction conception by using this concrete operation, and our first step is letting children can distinguish ratio and quantity, and understand the relation.

Figure 2. Tape

Figure 3. Fraction Block (Two Third)

2. Fraction Block Model

We propose that fraction block is visual & manipulatable model of ratio fraction. In this section, we explain the features and why we think children will be understand ratio fraction of by the model.

Features

Fraction block has following 3 features, "visible", "direct-manipulatable" and "assessable". Figure 4 shows an example of fraction block that expresses "2/3". Fraction block is composed of 2 blocks that are located up and down. The upper block is called "numerator block" assigned to numerator. The numerator block is composed of several unit blocks and the number of the unit blocks represents the numerator number. The lower block is called "denominator block" and the number of blocks represents denominator number. In the basic meaning of fraction, denominator number specifies how many parts the fraction breaks a quantity into. Then, numerator number specifies how many parts the fraction gathers as the output. Fraction block can be changed any length horizontally following the length of base quantity (Figure 4). Ratio is applied any base quantity but quantity's base quantity is fixed by unit quantity, or changing base quantity is determinant difference between ratio and quantity. This importance is visible and manipuratable on fraction block model. By the way, Arithmetical fraction expression is composed to 2 numbers vertically, matches fraction block representation.

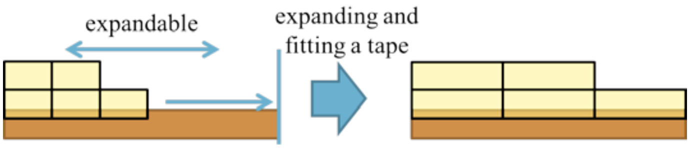

Figure 4. Changing Length to Fit a Tape

On fraction block model, learner can manipulate ratio operations directly. Ratio operations are multiplication and division. Multiplication is the operation that we consider known quantity as base quantity and find related quantity by times ratio. On fraction block model, learner fits denominator block to the tape and pick up numerator block's length tape (Figure 5). Division is the operation that we consider unknown quantity as base quantity, or known quantity is related to numerator number, we find base quantity by divide into ratio. On fraction block model, learner fits numerator block to the tape and pick up denominator block's length tape (Figure 6). From the above, the relation between ratio and quantity is visible and direct manipulatable on fraction block model.

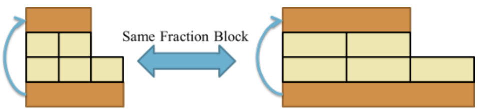

Figure 5. Multiplication (Multiplying 2/3)

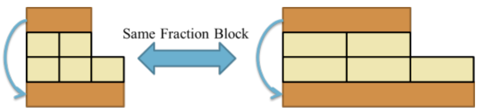

Figure 6. Division (Dividing into 2/3)

Fraction block model is also assessable. Because it is identify length of tape and operator, manipulation of fraction block relates to arithmetic formula. Implementing on Computer, computer can assess learner's process of using fraction block as arithmetic formula and feedback correct or not. Also, computer store learner's process as log data and we can analyze it.

Necessity of Experimental Use

In order to confirm that our model is an adequate representation of fraction of ratio, we used it in educational context and examined that it is usable for students. In order to this confirmation, we have implemented an interactive learning environment where a student is able to directly manipulate the fraction block. Through discussion with our cooperative elementary school teacher, we decided target students as 6th grade. Therefore, in this experimental use, we examined whether students who have already learned the fraction can use the fraction block smoothly or not. If they can use smoothly, we can guess that the fraction block is fit for their understanding of fraction. Because our goal is to use the fraction block in introductory phase of fraction, this is the primitive but indispensable step of this research.

3. Learning Environment

We have implemented a computer-supported learning environment based on the fraction block. The environment is able to use on tablet computer and it is connected to server through wireless LAN. A learner can directly manipulate fractions of ratio, and multiply and divide a length of tape from a length of tape by using a fraction block. An example of screen of the environment is shown in Figure 7. The right side of the screen on green background is the filed to manipulate fraction blocks. There are several yellow rectangles and semitransparent red rectangles. Yellow ones are tapes which represent quantities. Red ones are fraction blocks or integer blocks (we will explain it later) which are representations of ratio. Leaner can pick up related tape by fitting a fraction block to a tape. As an aside, leaner use only touching or dragging with his/her finger on the learning environment.

Figure 7. An Example of Screen

Two kinds of representation and transforming practices

Computer shows learner two kinds of representation, one is arithmetical formula on left hand screen, another is fraction block model on right hand screen. Leaner has two practices, (1) transforming formula to fraction block and (2) transforming fraction block to formula. (1) In this practice, computer shows one formula first, learner makes correct length tape in the same way as formula. The formula defines length and how to make. If leaner mistook a length, computer feedbacks length of the tape, else if tape was made in the different way, computer feedbacks formula as the way of leaner’s operations. (2) In this practice, computer shows four formulas which all are same length but made by different way (Figure 8) and present the animation that making answer tape by using fraction block in the certain way. Learner watches the animation and selects to answer one formula.

$$\left\{ \ 2\mathrm{m} \times \frac{1}{3} \quad 2\mathrm{m} \div 3 \quad 1\mathrm{m} \times \frac{1}{3} \times 2 \quad 1\mathrm{m} \div 3 \times 2 \ \right\}$$

Figure 8. Four Formulas as 2/3 Meters

Tools in environment of fraction block model

Leaner can use following items.

- Fraction block : The reification of fraction of ratio. The shape is a pair of red semitransparent blocks ordered vertically. It is expandable optionally and fitted to any tape to make related tape.
- Integer block : The reification of integer of ratio. The shape is a red semitransparent block. It is expandable optionally and fitted whole block length or one block length to any tape to make related tape. Fitting whole block length is division and one block length is multiplication. Leaner can compare fraction operations to integer operations.

- Tape : The reification of quantity. The shape is a yellow rectangle. It is not expandable. The length is either integer or faction. Leaner can cut off integer length tape from roll of tape and make fraction length tape from fraction block, and check the length by measure arranged above.

Process of making the tape for answer

Learner answers the tape by following process.

i. Cut off a tape from roll of tape.
ii. Fitting fraction block or Integer block to tape, and make related tape.
iii. Scaling the tape with measure if learner wants.
iv. Put the tape in answer area (below and pink background) if you think the tape is correct.

Learner can execute these steps as many times as needed. If fourth step has done, computer starts assessing. If the answer is correct, computer goes next question, else if error, computer shows feedback message and leaner tries again.

4. Experiments

We verified the validity of fraction block model by using the learning environment at attached elementary school of Hiroshima university. In this section, we explain the methodology and outcomes.

Methodology

Subjects are 76 students, they are all 6th grades and have been taught fraction enough in the usual way. Additionally, it is assumed that they have good conceptual understanding of fractions because their correct answer rate of the typical problem on the pre-test is 87.8%, is better than 40.6% which is the rate of academic ability survey all over Japan. At least, they can distinguish the meanings of fraction.

We carried out a pre-test a few days before students used our learning environment, and just after the use, we carried out a post-test and a questionnaire. We used the same test in the pre and post tests, that is, a typical problem: “How many meters is 1/3 of 2 meters?” by showing a picture instead of words and request them to select an answer from several choices (Figure 9). We adopted this type of problem because we needed to know their conceptual understanding of fractions. To understand a picture is assumed to be more intuitive task than reading words.

After using our learning environment, we also carried out a six-point scale questionnaire on which smaller is better, asked two questions, “Can you use this software enough?” and “Do you think fraction block is fraction?”, to confirm subjectivity of students.

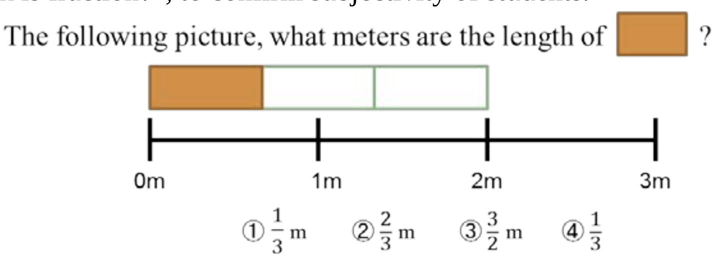

Figure 9. Paper Test

In the class, we delivered one tablet for one person and students could use our learning environment individually. First, they were given instructions of how to use our learning environment by the tablet computer. Second, they solved 13 practices of transforming fraction block to formula. Third, they solved 13 practices of transforming formula to fraction block. Total time was about 30 minutes. Several students who finished the exercise early, they were allowed to use fraction block to make a tape from another tape.

Hypothesis

If they can use our learning environment smoothly and recognize that fraction block is fraction, fraction block is assumed to be consistent with student's understanding. In order to confirm this validity, we analyzed three types of data, (1) system log, (2) questionnaire and (3) pre-post paper test. (1) A tablet computer sends log data to the server at any time, thus we can examine each learner's activity history that is, how many times a learner answered, how many tapes was made, how long learner solve a practice, and so on. The analysis of the log data indicates objective information. (2) We used questionnaire to confirm learners' subjectivity. We asked learners directly that they accept fraction block is fraction. (3) We also adopted pre-post paper test because it is useless if our learning environment have students confuse or make a mistake even if students can use and accept our model.

Results

Based on analysis of the log data, we confirmed that all students solved all practices of transforming fraction block to formula. Also, 74 in 76 students solved all practices of transforming formula to fraction block. Because most of them solved practices more than necessary, we judged that students can use our learning environment actively (Table 1).

In the questionnaire, there are two questions, "Can you use this software enough?" and "Do you think fraction block is fraction?" that were replied by a six-point scale on which smaller is better. Most of them answered 1 in 1^{st} question and 1 or 2 in 2^{nd} question (Table 2). These results suggested that the students felt they can use fraction block well and accepted fraction block as fraction of ratio.

Comparing the results of the pre-test and post-test, there were no students who got worse scores. There are 65 students who answered correct in the pre-test, and all of them answered correctly in the post-test. There are 9 students who answered incorrectly in the pre-test, 4 of them answered correctly in the post-test (Figure 10). This result suggests that using the learning environment might cause some learning effects to them but this result is not enough, thus we have to investigate that our model is truly helpful for learner who don't have conceptual understanding of fractions enough.

Table 1: System Log Data.

Practice	Answered	Correct	Percentage	Tape
Formula → Fraction Block	26.57	24.25	0.91	45.33
Fraction Block → Formula	40.58	30.37	0.75	---

Table 2: Results of Questionnaire.

question	1	2	3	4	5	6
1	71	0	1	1	2	0
2	57	13	1	0	3	1

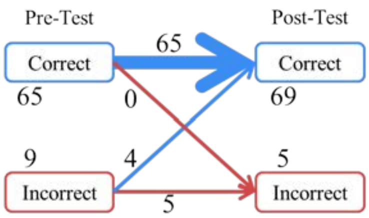

Figure 10. Transition of Paper Test

5. Conclusion

In this paper, we proposed fraction block model which is a kind of educational reifications of fraction for intuitive and constructive learning. For that, fraction block has three features, visible, direct-manipulatable and assessable. Learner can watch and directly manipulate fraction of ratio, and try to create a tape by ratio operations practically. Also, computer can trace learner's activities and give feedback to their operations.

We have already developed a learning environment based on fraction block model and verified the validity by using at an elementary school. As results, we found that students who already understood fraction conceptually could use fraction block and accept the fraction block as fraction of ratio. These results suggest that the fraction block is consistent with student's knowledge. However, our experiment is not enough to verify learning effect. Therefore, we will verify it by using our learning environment by students who have not studied fraction in elementary school.

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