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Interactive Environment for Learning by Problem-Posing of Arithmetic Word Problems Solved by One-step Multiplication

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Abstract: In this paper, an interactive environment for learning by problem-posing targeting arithmetic word problems that are solved by one-step multiplication is described. Its practical use in an elementary school is also reported. We had already developed an interactive environment for learning by posing arithmetic word problems that can be solved by one-step addition or subtraction. Then, a practical use by the first grade students of an elementary school had been performed. The results suggested that the learning by using the environment was effective to improve the student's problem solving performance. As the next step of this previous research, we have developed another learning environment where a learner poses the multiplication word problems. In order to design the environment, we categorize word problems that can be solved by one-step multiplication based on the sorts of quantities consisting of the multiplication. Then, based on the categorization, we have built a task model of the problem-posing. The learning environment is used by a class of the second grade students for 9 class times, and we have confirmed that the scores of problem-posing and problem-solving were improved in the group of students who obtained low scores in the pre-test.

Keywords: Problem-posing, sentence-integration, interactive environment, multiplication, arithmetic word problem, problem structure

1. Introduction

Problem posing is well known as an effective way to promote learners to master the use of solution methods (NF Ellerton, 1986; Polya, 1945; Silver & Cai, 1996). Moreover, several researchers have already investigated that it is important for solving arithmetic word problems to understand the problem structure. They also have suggested that poor problem solvers often fail to elicit problem structures from the problems (Brown & VanLehn, 1980; Kintsch & Greeno, 1985; Mayer, 1982).

In light of these considerations, we started to make a model of problem structure of arithmetical word problems, and developed an interactive environment for learning by problem-posing based on the model (Nakano, et al, 1999). Moreover, we have already developed an interactive environment for learning by posing arithmetic word problems for one-step addition or subtraction (Hirashima, et al, 2007; Hirashima, et al, 2011). In the environment, the learner can pose an arithmetic word problem by selecting three cards from a set of given sentence cards and arranging them in a proper order. We call this problem posing as "problem posing as sentence integration" and the implemented learning environment as "MONSAKUN". These cards and the proper order of them have been designed on the basis of problem structure and a task model of problem posing (Kurayama & Hirashima, 2010). The learning environment diagnoses the posed problem on the basis of the task model of problem posing and gives the learner a feedback based on the results of the diagnosis.

In a previous research, we have implemented MONSAKUN on media tablet (This environment is called "MONSAKUN Touch"). Because of this implementation, it becomes possible for a teacher to conduct learning by problem-posing with MONSAKUN Touch in a usual classroom. Being able to carry out learning by problem-posing in a usual classroom, there are advantage not only to omit the time to move to a computer room or to avoid inconvenience to have a class in unfamiliar room, but also promote a teacher to fuse the learning in his/her usual teaching. From the results of an experimental use, it is understood that the first grade students in an elementally school were able to pose the problems properly, and their problem posing skills and their ability of problem solving were improved (Yamamoto, et al, 2012).

As the next step, we have designed and developed another interactive environment for learning by problem-posing for one-step multiplication or division arithmetic word problems. Because the problem structure and the task to pose the problems are important difference between addition/subtraction word problems and multiplication/division word problems, we defined a model of problem structure and tasks of problem posing for one-step multiplication or division arithmetic word problems. This paper also describes the design of above learning environment and its practical use targeting at the second grade students in an elementary school who were taught the way to solve multiplication arithmetic word problems just before this practical use.

A model of problem structure and a task model of problem posing are explained in the next section. Subsequently, we have described a developed learning environment by problem posing and a function for real-time analyzing the students learning data on MONSAKUN Touch. Procedure of practical use of MONSAKUN Touch and the analysis of the results are also described.

2. Problem structure and task model of problem posing

2.1 Problem structure of arithmetic word problem solved by one-step multiplication or division

In this section, we described that a problem structure of 1-step multiplication or division arithmetic word problem is different from a one-step addition or subtraction one. Multiplication is generally expressed by "multiplicand multiplied by multiplier is product". The value arising from repeating the multiplicand as many times as there are units in the multiplier is called the product (Davies, 1841; Greer, 1992; Vergnaud, 1983). Example of multiplication word problem are shown Example (b) and (c) in Figure 1. In Example (b), the number of apple in "five apples in each box" is the multiplicand, the number of box is the multiplier, and the number of apple in total express the product. That is, the number that was repeated the "five apples in each box" as many times as the number of box is "apple in total". In this example, the properties of "the number of apples in each box", "the number of box" and "the number of apple in total" are clearly different but both "the number of apples in each box" and "the number of apple in total" expresses the number of apple. Therefore, in Example (b), in order to derive the calculation "5×3=15", distinguishing the properties of the given three quantities and relations between them. So, because the multiplication is considered as a relation of quantity, we call multiplicand is "base quantity", multiplier is "proportion quantity", and product is "compared quantity". Then, there are the two types of division from this different that is "base quantity" and "proportion quantity". Based on the above consideration, the multiplication or division has three types of calculations. (1) Compared quantity divided by base quantity is proportion quantity, (2) Base quantity multiplied by proportion quantity is compared quantity, (3) Compered quantity divided by proportion quantity is base quantity.

In Figure 1, Example (b) and (c) are solved by "5×3". However, "five apples" in Example (b) and "five apples" in Example (c) have different properties. That is, the property of quantity has not only the property of calculation like "base quantity", but also the property of itself. This means that "five apples" in Example (c) shows "apple in total" but "five apples" in Example (b) shows "apple in each box". Figure 2 shows this property. Firstly, the quantity is classified by either one quantity or relation of two quantities. Secondly, a relation of two quantities has two kinds of relations that is the same quantity's relation or other quantity's relation. Lastly, the relation of other two quantities is consisting of degree and quotient. Quotient has unit quantity and non-unit quantity. Because of page limitation, the details of this classification are omitted in this paper. The stories of multiplication or division arithmetic word problems are consist by applying these quantities to base quantity, proportion quantity and comparison quantity. This applied model is shown in Figure 3.

On the other hand, in the cases of addition or subtraction problems, the calculation operation is decided by a story of a problem which is classified in one of following four story types: (1) combine, (2) increase, (3) decrease and (4) compare (Riley & Greeno, 1988). In Example (a), the calculation operation "3+5=8" is derived by the combine story which is expressed by combining the number of apples with oranges because the properties of three given quantities are the same ones in addition and subtraction. In Example (b), in order to derive the calculation "5×3=15", it is important to distinguish the properties of the given three quantities and relations between them. Because of this difference, we arranged the task model of problem posing that can be solved by one-step multiplication and division word problems originally.

(a) Example of Addition word problem
There are three apples. There are five oranges. How many apples and oranges are there in total?

(b) Example of Multiplication word problem 1
There are five apples in each box. There are three boxes. How many apples in total?

(c) Example of Multiplication word problem 2
There are five apples. The oranges are three times of the apples. How many oranges are there?

Figure 1. Example of Each Operation's Problems.

Classification					Example
Extensive quantity	Single quantity				Head-count and so on
Multidimensional extensive quantity	Quantity to appear by multiplication between the quantity of the same unit				Square measure and so on
Intensive quantity	Quantity to appear by multiplication or division of two quantity	Degree		Quantity to appear by the multiplication and division between the quantity of the different unit	Velocity and so on
		Quotient	Unit	Quantity to appear by the multiplication and division between the quantity of the same unit	Cardinality and so on
			No unit	Quantity to express size of the other quantity to base quantity	Double

Figure 2. Classification of Quantity.

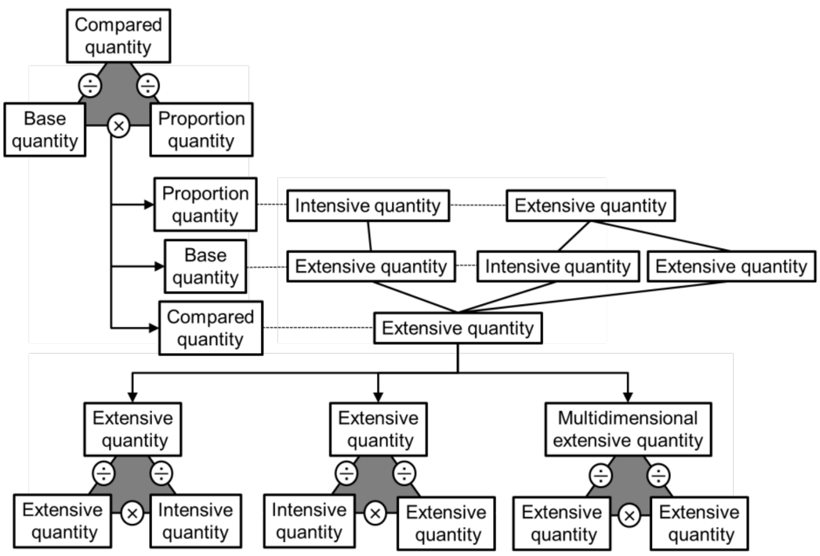

Figure 3. Story of One-step Multiplication Arithmetic Word Problem.

2.2 Task Model of Problem Posing for Multiplication and Division

In this section, we have described a task model of problem posing in the basis of previous multiplication and division's structure. Task model of problem posing consists of restrictions that should be satisfied in order to pose a problem. It is important to understand multiplication and division arithmetic word problems to distinguish each quantity, which are base quantity, proportion quantity and compared quantity. Therefore, we designed the task model of problem posing with a focus on these quantities and its relation.

The task model of problem posing for multiplication and division are shown in Figure 4. Deciding calculation operation structure is to decide the calculation for requiring answer. This is multiplication or division. In deciding quantity relation structure in calculation operation, the relation between each quantities to which the calculation operation structure correspond is decided. The problem operation structure is derived operation from timeline of problem's story. So, the calculation operation structure not necessarily corresponds to problem operation structure, because calculation operation is changed by given required value in problem. Next, story operation structure by quantity means the relation of the quantity that is not given required value. Of course, there is the story for deciding the story operation structure by quantity. Then, by referring to 2.1, the division story consists of partitive division for required base quantity and quotative division for required proportion quantity. Multiplication and division stories are also required each proper constitution of quantity, object and value. Exercises of problem posing and diagnosis function of posed problems are designed based on this task model.

Here, we explained the difficulty of the problem by a difference between calculation operation and problem operation by using example in Figure 5. Problem operation by quantity is expressed as "base quantity ×proportion quantity = compared quantity". But required value is proportion quantity in this example. Therefore, calculation operation by quantity is expressed as "compared quantity ÷ base quantity = proportion quantity ". The problem which problem operation by quantity doesn't correspond to calculation operation by quantity, like this example, is called as a reverse thinking problem. In contrast, the problem which problem operation by quantity correspond calculation operation by quantity is called as a forward thinking problem. The reverse thinking problem is more difficult than forward thinking problem because of the difference between structures.

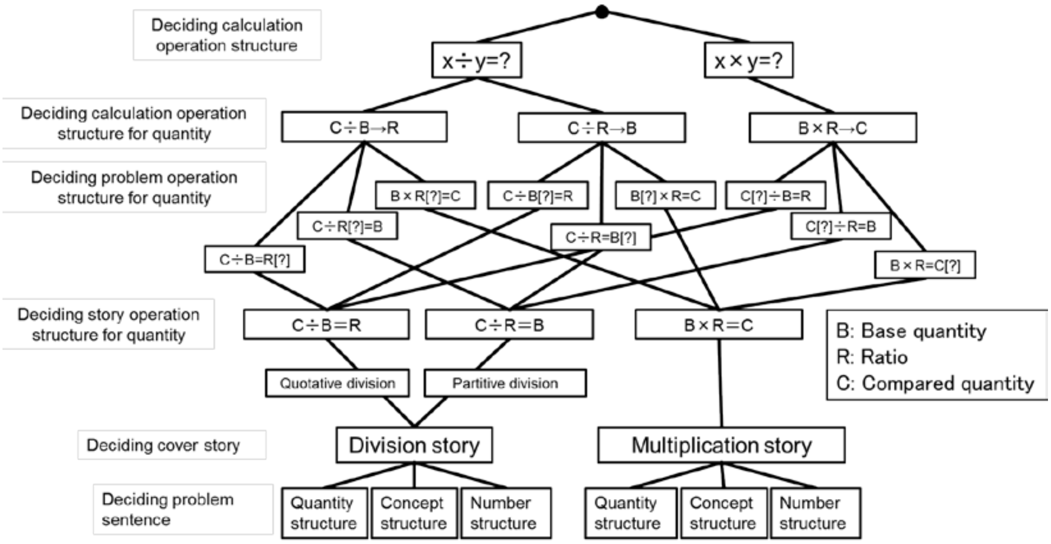

Figure 4. Task Model of Problem Posing.

There are three apples in each box. There are several boxes. *There are fifteen apples in total. How many boxes are there?*

Figure 5. Example of One-step Multiplication Word Problem.

3. MONSAKUN Touch 2 and MONSAKUN Analyzer

3.1 Framework of Learning Environment

MONSAKUN Touch 2 and MONSAKUN Analyzer are connected via Wi-Fi network and MONSAKUN Analyzer is able to visualize learning data sent from MONSAKUN Touch 2. This framework is shown in Figure 6. Each learner poses word problems by using MONSAKUN Touch 2 respectively. The posed problems are diagnosed by MOSAKUN Touch 2. The diagnosis judged not only whether it is correct or not, but also categorizes the incorrectness if the posed problem is inadequate one. As learning data, the posed problems and the results of the diagnosis are sent to MONSAKUN Analyzer via Wi-Fi network. MONSAKUN Analyzer visualizes the learning data with several types of graphs. Details are described in the next section.

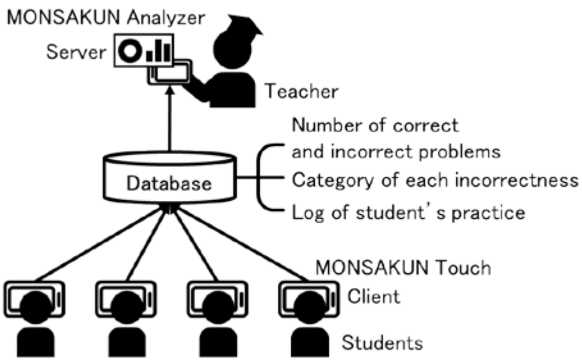

Figure 6. Framework of Learning Environment.

3.2 Learning environment for problem posing by one-step multiplication or division

This learning environment called MONSAKUN Touch 2 is shown in Figure 7. In this practical use, we have only implemented multiplication problem posing into MONSAKUN Touch 2 because our target is only the 2nd grade students in this time. In the learning by problem-posing with the system, a learner is given an assignment of problem posing and a set of sentence cards from the learning environment. We call a sentence card as a "simple sentence card" that consists of an object, an attribute of the object, and a quantity of the attribute. The environment can diagnose a posed problem based on the task model of problem posing and give the learner a feedback for the posed problem. The learner is required to select three simple sentence cards and arrange them in a proper order for posing the problem. The set of cards given to the learner includes several dummy cards that cannot be used to pose an adequate problem for the problem posing assignment. An assignment is expressed by a calculation operation or a problem operation like “3×2=?” or “3×?=6”. Figure 8 shows a procedure of diagnosis of the posed problem. The examination is carried out by checking out of several constraints that the posed problem is required to satisfy. The constraints are defined based on the task model. Then, MONSAKUN Touch 2 has two methods to give feedback to students. One method is that system feeds back to the students that a posed problem is "correct" or "incorrect", and the other method is that system feeds back an explanation of why a posed problem is incorrect as shown in Figure 8 (e.g. Check the objects that are appeared in the posed problem). Also, MONSAKUN Touch 2 diagnoses the order of base quantity and proportion quantity because of verifying the learner knows the meaning of quantity. If the learner cannot distinguish the types of each value, he/she will make a mistake for the order of the sentence cards.

In MONSAKUN Touch 2, the learner logs in to the system at first. Then, the learner selects the level for learning. After that, the learner starts to pose problems that are fit for the given assignment. If the learner finishes all given assignments in a level, the learner is allowed to move to another level of problem-posing assignments.

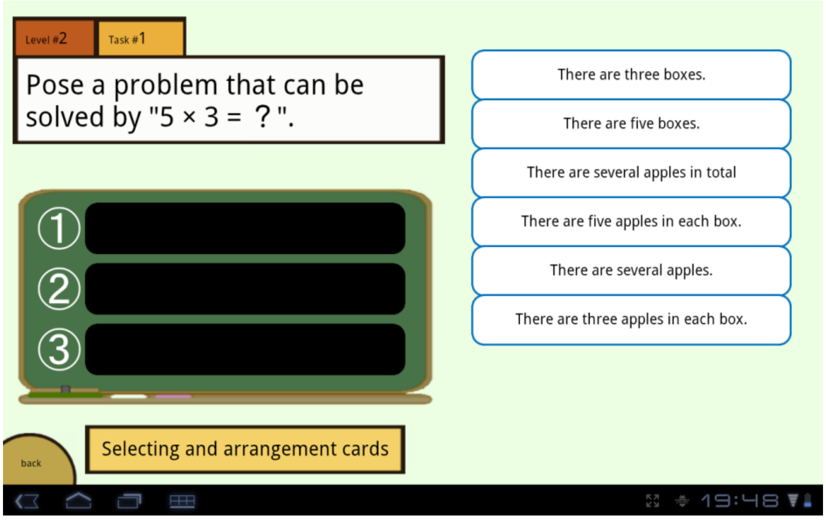

Figure 7. Interface of MONSAKUN Touch 2.

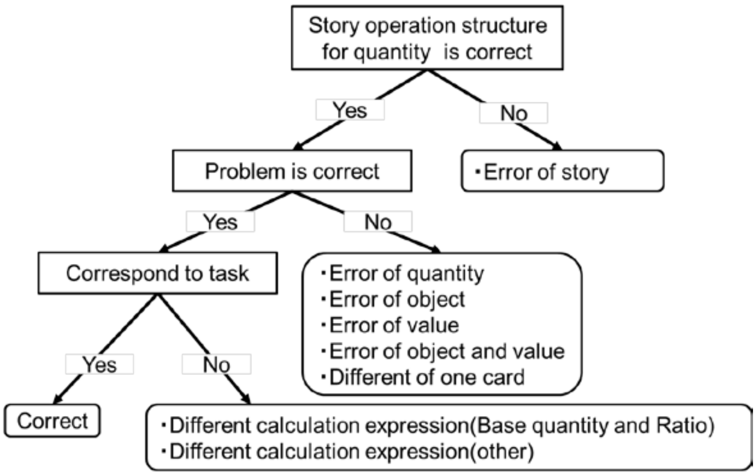

Figure 8. The Process for Diagnosing the Posed Problem.

3.3 MONSAKUN Analyzer

MONSAKUN Analyzer is the web-based system which is developed by using PHP and MySQL for visualizing a student's learning data on MONSAKUN Touch 2. The interface of MONSAKUN Analyzer is shown in Figure 9. The learning data of all students in a class are gathered and displayed with an interface that consists of a bar chart shows the number of posed problems and a doughnut chart shows the rate of each error. In the left side, three bar charts indicate the teacher the average number of posed problem in total, the average number of correct problem, and the average number of wrong problem. The doughnut chart displays the teacher the sorts of incorrectness which are in accordance with diagnosing of MONSAKUN Touch 2 in Figure 8. In addition to the visualization of the number of posed problem and the rate of each error, MONSAKUN analyzer displays the average progress with a level and number of task. This system can display these data not only in each lesson but also in each student by clicking the link "see the each student's data". This interface shows the learning data which is the same type of the data are shown in each lesson. When MONSAKUN Analyzer shows the learning data of each student, the teacher can see the posed problem of each student by clicking "see the each student's log". These data are updated in real-time. By using this system, the teacher can confirm the process of student's problem posing performance by each lesson and the student's learning data in performed lesson.

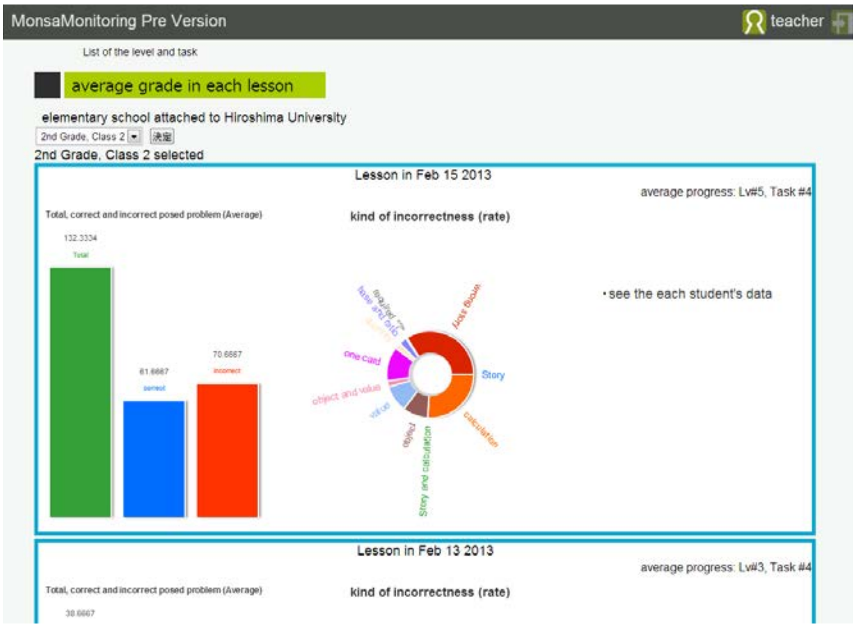

Figure 9. MONSAKUN Analyzer.

4. Practical Use of MONSAKUN Touch 2

4.1 Procedure of Practical Use

The subjects were 39 students in the second grade of an elementary school. The subjects had just learned the way to solve arithmetic word problems that could be solved by one-step multiplication. This practice used nine lesson times (45 minutes per lesson, in 3 weeks). One lesson consists of teaching of problem-posing by a teacher and problem-posing exercise of MONSAKUN Touch 2. The teaching is designed based on the task model of problem posing for one-step multiplication or division in 2.2. At first, the learner poses the problem by using the MONSAKUN Touch 2 for introduction of a new level of problem posing. Second, the teacher teaches a method of problem posing by using the task of problem posing and several sentence cards each of them written on a sheet of paper. In this teaching, the task and cards are displayed on blackboard for teaching the problem structure following our model of problem posing as sentence integration. Then, the students answer a teacher's question or discuss with other students about given problem posing task. This teaching consists of selecting of necessary sentence cards and explaining its reasons, and describing the pictorial expression of the posed problem. The learner is able to acquire the problem structure through learning in this lesson of problem posing on the basis of the task model described in Section 2.2. Then, they used the MONSAKUN Touch 2 as exercise of the teaching. The teacher monitored students learning data for adjusting progress of lesson during each exercise by using MONSAKUN Touch 2.

The students used MONSAKUN Touch 2 before and after the teaching for 5 - 10 minutes respectively. The teacher taught the method of problem posing for 25 - 35 minutes. Problem posing exercises on MONSAKUN Touch 2 and the teaching were divided into 7 levels that are shown in Table 1. The contents of each level were classified by three scales that are the calculation operation, the problem operation, and the number of problems that should be posed in an assignment. The students are only given multiplication problem because they have not learned the division yet. So, they are given problem operation in Lv.5 and calculation operation in other levels. In Lv.1, the students are required to pose the story. Lv.4 and 5 are composed of the reverse thinking problems. Lv.6 and 7 requires a learner to use the distributive law. These contents of each level were decided by the consultation of the teacher and us on the basis of the task model. The teacher has performed one level by one lesson in order from level 1 to 7. In the sequence of this learning activity, the learner is able to sophisticate the problem structure that he/she acquired by posing problems on MONSAKUN Touch 2 after he/she is taught the method of problem posing based on the task model by the teacher.

Figure 10 shows the scene that the subjects were using MONSAKUN Touch 2. The subjects took a pretest in one lesson time before the period, and a posttest and a questionnaire in one lesson time after the period. In the exercise of MONSAKUN Touch 2, if the subjects finished problem posing at all tasks, they repeat previous level.

In this practical use, MONSAKUN Touch 2 only gave feedback that the posed problem is correct or not because we have aimed that the subjects pose the problem by using the taught problem structure. Three teaching assistants have supported setting of the wireless LAN and operation of the MONSAKUN Touch 2, but they didn't help the learners to pose problems.

As the pretest and posttest, we used the same usual problem solving test, extraneous problem solving test and problem posing test. The usual problem solving test used to assess the subjects usual problem solving performance. The extraneous problem test includes extraneous information that is not necessary to solve the word problem. It is more difficult for learner to solve the extraneous problem than to solve the usual problem solving (Muth, 1992). Because the learner has to judge the relevance of each sentence and find the sentence including as the extraneous information in order to solve the extraneous problem. In this process, the problem structure plays a crucial role. Therefore, the extraneous problem solving is useful to assess leaner's comprehension of the problem structure. In problem-posing test, the students are required to pose four problems by composing several sentence cards provided beforehand. This test is used to examine the student's problem-posing performance.

Table 1: Levels of MONSAKUN Touch 2.

Level	Calculation operation	Problem operation	Number of posing problem per level	Number of posing problem
1	x×y	x×y=z (story operation)	1	10
2	x×y	x×y=?	1	10
3	x×y	x×y=?	2	10
4	x×y	?÷x=y ?÷y=x	1	10
5	x÷y	?×y=x y×?=x	1	10
6	x×y	(a+b)×y=? (a-b)×y=?	1	10
7	x×y	x×(b+c)=? x×(a-b)=?	1	10

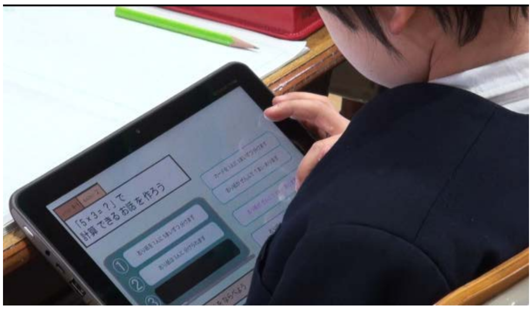

Figure 10. Scene of Using MONSAKUN Touch 2.

4.2 Analysis of the Pretest and Posttest

Analysis of the pretest and posttest are reported in this section. The results of these tests are shown in Table 2. In the problem posing test, the subject are required to pose four problems by selecting and arranging twenty eight sentence cards. There was a significant difference in the number of correct problem between pretest and posttest (paired t-test, p=.011), and effect size is medium (|r|=.50). These results suggest that the subjects improved their performance of problem posing through the learning by problem-posing. The usual problem solving test composed of three problems in Lv.1-3, two problems in Lv.4, four problems in Lv.5, one problem in Lv.6. Total number of problems are ten. The extraneous problem solving test also has the same composition, too. There was no significant difference in the scores between the pretest and posttest (paired t-test, p=.22) in the usual problem solving test. In the extraneous problem solving test, there was also no significant difference in the scores between the pretest and posttest (paired t-test, p=.12).

Because the SDs in two usual problem solving tests were much bigger than the ones of problem-posing test, we analyzed the results of problem solving and extraneous problem solving test by dividing into high group and low group based on average of the score of extraneous test in pretest. Table.3 shows the results. There are no significant differences between the score of each test in high group. In contrast, there are a marginally significant between the low group's score of pretest and posttest in usual problem solving test (paired t-test, p=.09), and the effect size is small (d=.49). Also, there are a significant difference in the scores of extraneous problem solving test between pretest and posttest (paired t-test, p=.004), and the effect size is large (d=1.19). These results suggested that not only the subject improve the problem solving performance, but also this learning environment was effective for the learner who can’t judge the problem structure to acquire the problem structure. Based on these result of analysis, we have considered that suggested problem structure and its task model of problem posing are reasonable, and this learning environment by posing problem was effective for understanding the problem structure of one-step multiplication word problem. However, this result needs more investigation and we should analyze the student's log and show a teacher's teaching method.

From the results of MONSAKUN Analyzer, the each subject has posed an average forty four problems in each lesson. Among them, twenty seven posed problems were correct problems and seventeen were wrong problems. This indicates that the subjects learned how to posed problem with MONSAKUN Touch 2 because the learner posed the problems with several errors.

Table 2: Score of Pretest and Posttest in total (N=39).

Test	Pretest		Posttest		
	M	SD	M	SD	
Problem posing	2.54	0.96	2.94	0.99	**
Usual Problem solving	8.18	1.68	8.51	1.38	n.s.
Extraneous Problem Solving	7.72	1.74	8.23	1.75	n.s.

** p<.01

Table 3: Score of Pretest and Posttest in high group (N=25) and low group (N=14).

Test	Group	Pretest		Posttest		
		M	SD	M	SD	
Usual Problem solving	High group	8.80	1.30	8.92	1.32	n.s.
	Low group	7.07	1.71	7.79	1.15	+
Extraneous Problem Solving	High group	8.84	0.83	8.60	1.45	n.s.
	Low group	5.71	0.96	7.57	1.99	**

** p<.01, + p<.10

5. Concluding Remarks

In this paper, we have described a model of problem posing in one-step multiplication or division arithmetic word problem, the development of an interactive environment based on the model, and the results of its practical use. In order to design and develop the learning environment, at the first step, we have indicated that the multiplication word problems are defined by three quantities and its relation. These three quantities are base quantity, proportion quantity and compared quantity. Secondly, the task model of problem posing in multiplication or division word problems is built on the basis of this problem structure. After that, we have developed learning environment for problem posing implemented the task model of problem posing. This environment consists of MONSAKUN Touch 2 and MONSAKUN Analyzer. MONSAKUN Touch 2 provides the environment for learning by problem posing as sentence integration to the learner. MONSAKUN Analyzer provides the visualized student's learning data on MONSAKUN Touch 2 to the teacher. By using this environment, it is realized that the lesson for acquiring the problem structure through our task model of problem posing has been realized. In addition to above, we have reported its practical use and the results of brief analysis. These results suggested that the second grade students were able to improve their problem solving performance. We have judged that the proposed task model of problem posing and the problem structure were reasonable.

As our future works, we need to analyze the student's log in this practical use to understand student's learning performance. Furthermore, we should perform the practical use for problem posing in division to third grade students of an elementary school continuously. Sophistication of the task model of problem posing and evaluation of learning effect of the teaching method with MONSAKUN Touch 2 is also an important future work.

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