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Designing with mobile technologies for enacting the learning of geometry

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Abstract: Guided by the methodology of design research and the notion of seamless learning we develop a mobile learning activity for an outdoor context, where groups of 12 year old students are asked to coordinate themselves physically in terms of given distances with respect to both given and peer-defined points. Our learning activity consists of three connected tasks of successively increasing complexity, implemented at separate occasions over a period of 6 months. By participating in the activity, the students are offered opportunities to experience geometrical constructions in full-sized space. Specifically, they are stimulated to make use of their orientation ability, which differs cognitively from the visualization ability which is more commonly used to solve similar tasks in school. The outdoor explorations, the use of mobile technologies, and the distribution of the activity across time and locations, pose didactical as well as technological challenges which call for careful considerations regarding the design of the activity. In this paper, we account for the design process and its pedagogical grounding in ancient mathematics and modern psychology. Furthermore, we suggest to systematically combining the theory of instrumental genesis together with scenario-based design, within the methodological framework of design research, to guide the development of seamless mobile learning activities which provide a learning progression over time.

Keywords: Design research, geometry, instrumental genesis, seamless learning, scenario-based design, mobile technologies, technology-enhanced learning

1. Introduction

This paper reports on the design of a technology-enhanced learning (TEL) activity developed in collaboration between researchers in mathematics education and media technology. The activity emerged as an idea during a meeting where a selection of available mobile technologies and their didactical potential for the learning of mathematics were discussed. Several members of the current research team have previously collaborated in projects involving outdoor mathematics supported by mobile technologies and interactive visualization techniques [8,9,11]. These efforts have been inspired by Cobb, Confrey, diSessa, Lehrer, and Schauble [3], who formulate a mission for research in mathematics education as striving to develop, test and revise learning activities which are designed in order to support envisioned learning processes.

Research in mobile learning (m-learning) relates to a variety of subjects including school mathematics. An example of such an m-learning activity is MobileMath [13], designed as an outdoor activity for teams of students to compete against each other by constructing squares, rectangles, and parallelograms to cover as much area as possible while negotiating obstacles such as houses. The students define the vertices of the shapes by walking to chosen positions and clicking on a mobile device supporting GPS technology.

The device provides visual feedback to the students. As being a one-off activity, the game involves mathematics but it remains to exploit and research its potential for learning [13].

In this paper, we discuss the development and pedagogical grounding of a connected learning activity, whose three tasks have been initially tested with students. To achieve the specific learning objectives for the activity, we involve mobile technologies in an outdoor context. The development of learning activities for outdoor contexts is in itself a complex task, which becomes even more complex when attempting to support advanced mathematical learning objectives by making use of mobile technologies. In the next section, we account for the theoretical and methodological foundations that serve as a basis to support and guide our design efforts. In the last section, we suggest how to further strengthen the methodological approach in our future efforts. Moreover, the implemented tasks have to be negotiated with teachers and students. We address this latter issue of pragmatic roots in relation to dimensions of mobile-assisted seamless learning [14], and argue for using these dimensions as guiding principles for future work in educational design research.

2. Design research, scenario-based design, instrumental genesis, seamless learning

Two key aspects of design research are the central position of the design of learning activities and the cyclic character that allows their adjustment and improvement. One design cycle consists of three natural phases; the preliminary design phase, the teaching/learning experiment phase, and the phase of retrospective analysis [5]. The preliminary phase for design of a learning activity involves the negotiation between a) the design of a proposed activity, and b) a prospective analysis with focus on hypothetical learning trajectories [3], where the intended mathematical learning objectives guide the choice of trajectories. In our case, which concerns a learning activity with three connected tasks, the classical design cycle (Fig. 1, left pane) is naturally extended to involve three connected cycles (Fig. 1, right pane). This latter model involves transitions between designs (preparatory stage) as well as transitions between tasks (students in action).

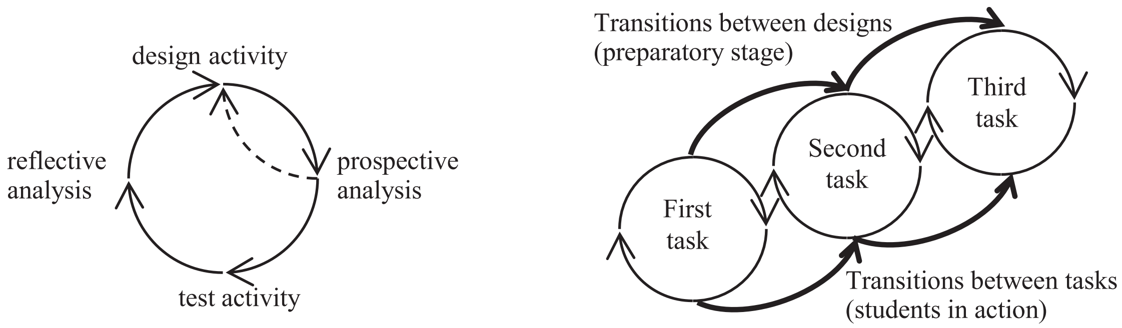


Figure 1: Design cycle (left pane) and connected design cycles (right pane)

In the current project, as well as in our previous efforts related to design research [8,9,11], the learning activities are highly self-regulated and stimulate collaboration in the sense that the students need to engage in a coordinated effort to solve tasks within the activity [4]. From a didactical point of view, we negotiate the design of a specific learning activity with prospective analysis in a local cycle and simultaneously negotiate technological requirements of the mobile applications with the technological domain experts. These discussions and negotiations are guided by scenario-based design (SBD) which enables "rapid communication about usage possibilities and concerns among many different stakeholders" [10] by providing narratives about the students' possible actions and interactions during a proposed learning activity. The scenarios make it possible to communicate about learning trajectories without having to engage deeply in specific subject matters.

The implementation of any learning activity, and particularly an activity that makes use of mobile technologies, has to include a phase where the students try out and learn to use the available tools. The process of turning a tool into an instrument (in our case, an instrument for learning) is summarized by the notion of instrumental genesis [12]. The theory of instrumental genesis makes a distinction between the tool as an object and the instrument, as a cognitive construct, emerging when the user interacts with the tool. A central feature within the current activity is a customized mobile application to be used in an outdoor context. The design considerations regarding the students' instrumental genesis could fortunately be limited to the first task, as the functionality of the application was changed just slightly from the first to the second and third tasks. As a consequence, the students' cognitive efforts during the latter tasks could be focused on the mathematical challenges.

Our three tasks being distributed across time and locations make them fulfill two of the ten dimensions (MSL3 and MSL4) characterizing mobile-assisted seamless learning (MSL), as recently suggested by [14]. The tasks were implemented at four different occasions beginning in December 2010 and ending in June 2011. Each task involved an indoor preparatory session combined with a self-directed group effort in an outdoor setting. We will discuss the dimensions of MSL in further detail in the last section.

Our general design approach can be characterized by the use of design research to develop learning activities which support specific learning trajectories. The prospective analysis is based on narratives, which account for the students' hypothetical action (and learning) trajectories. These narratives guide the pedagogical design as well as the technical implementations throughout our work. The reflective analysis in this contribution is limited to a concluding discussion in relation to the dimensions of MSL.

3. Grounding of the activity in ancient mathematics and modern psychology

Our activity offers the participating students enacted experiences of school geometry which are not commonly offered in school contexts. The activity, which will be described in the next section, was initiated during a team meeting where a selection of available mobile technologies and applications were introduced and implemented by researchers from media technology. The research team promptly agreed, influenced by the didactical opportunities offered by one of the mobile applications, to design an activity involving outdoor constructions of large-scale triangles. We now proceed to account for the theories and research findings which were identified and used to guide the design process in order to refine and support specific hypothetical learning trajectories within the activity.

From a mathematical perspective, the activity may be interpreted as the construction of a triangle with three given sides. A while after the task was proposed we felt confident that such a geometric construction must have been considered by Euclid (~300 BC). Indeed, we found such a proposition in Book 1 in Euclid's Elements [6]. The construction can be regarded as a traditional school task related to geometrical constructions, problem solving and visualization. Students may readily solve the task on a piece of paper by using a compass and a ruler. In that case, they make use of a spatial ability which is sometimes referred to as *object manipulation*. This ability includes abilities for spatial visualization and spatial relations and concerns manipulation of spatial forms from a fixed perspective, involving an object-to-object representational system [7]. Within the psychometric research tradition, spatial visualization and spatial relations are contrasted with a third spatial ability, namely spatial *orientation*, which involves "movement of the egocentric frame of reference" [7, p. 745] and a self-to-object representational system. The self-to-object system activates another part of the brain than does the object-to-object system [7], which implies that object manipulation and spatial orientation should be considered as separate spatial abilities.

Steering documents for compulsory school in Sweden have a one-sided focus on object manipulation and consider spatial orientation explicitly only in pre-school. This fact may be contrasted with the claim by Bishop [1, p. 260] that "insofar as we are concerned with spatial ideas in mathematics as opposed to just visual ideas, we must attend to large, full-sized space, as well as to space as it is represented in models, and in drawings on paper". Activities taking place in full-sized space may be related to Bruner's [2] enactive mode of action and corresponding mode of thinking, as one out of three modes – enactive, iconic, symbolic – characterizing an individual's interaction with the world. We find it reasonable to claim that these different modes, which Bruner considers as emphases (rather than stages) in a child's development [2, p. 28], may be fruitful to draw on during learning activities also for older children, especially with respect to learning subject matter of abstract nature, such as mathematics.

Our ambition has been to design a learning activity that stimulates students' enactive mode of action by putting special focus on spatial orientation while minimizing features related to spatial visualization. We argue that this singular activity may serve as a general frame of reference regarding students' future geometric constructions on paper, using compass and ruler, where the outdoor activity may provide a connection between iconic constructions on paper and constructions imagined to be enacted in an outdoor setting.

4. Design and implementation of our learning activity

The current learning activity draws on the use of GPS technology available in a mobile device. The research team has developed a mobile application, which allows a student to measure distances between her own device and mobile devices held by other students. Based on this feature, we have designed a learning activity providing opportunities for students to experience spatial self-orientation in full-sized space. The activity was tried out by twelve students in grade 6 (13-14 years old) at a school located in a rural area in southern Sweden. The students worked with three tasks within the activity during a number of sessions that took place during the period December 2010 - June 2011.

4.1. The first task of the learning activity

In the first task, implemented in December 2010 on a field covered in snow, the students worked in pairs. They were asked to use one mobile device to coordinate themselves with respect to two given distances measured against two fixed points, which were respectively marked on the field by a triangle and a square (Figures 2&4).

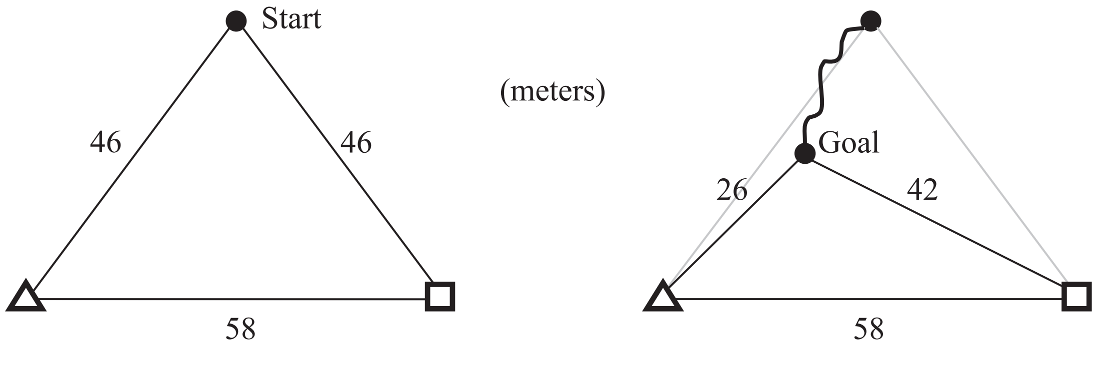
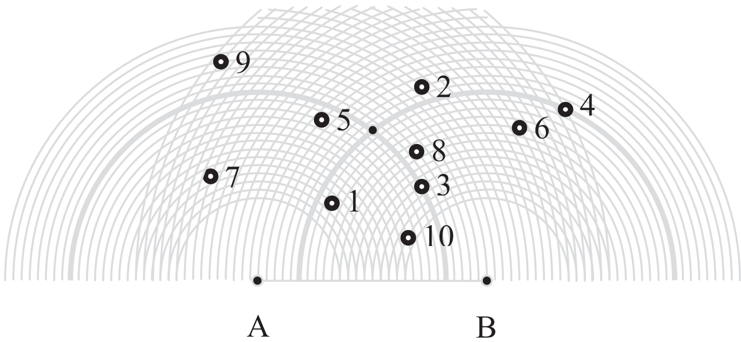


Figure 2: Visual representation of the first subtask.

The standing markers were placed 58 meters apart and could be readily identified from a large distance (Fig. 4). The starting point was marked with a pole and located 46 meters from each of the markers, as indicated in Fig. 2 (left pane). We chose to design ten subtasks, based on a diagram of level curves used to secure variation between longer and shorter distances (Fig. 4). The goal point for the first subtask, involving the students coordinating themselves with respect to the distances 26 m and 42 m, is indicated in Fig. 2 (right pane).



Point	A	B
1	26	42
2	62	50
3	46	28
4	86	46
5	42	56
6	74	38
7	28	72
8	50	36
9	54	84
10	38	22

Figure 3: Distribution of points for the tasks (unit: meter).

The rather large distances, 22-86 meters, were chosen for two reasons. Firstly, we wanted the students to move substantially (and reasonably far) within the chosen field. Secondly, inaccuracy of the GPS values resulted in an error for the computed distances. The research team tested both these aspects outdoors and found that a tolerance of two meters was enough to compensate for the inherent inaccuracies of the GPS technology. At the final stage of implementation (December 2010), the students were randomly organized into six groups. They worked simultaneously with the activity on the same field, which was covered with 20 cm snow. To avoid having the groups follow each other (in order to complete their ten tasks) six variations of the initial sequence of points were constructed based on symmetry (interchanging distances for A and B) and taking the ten points (Fig. 3) in different order (1-10, reverse order 10-1, and 3-10 followed by 1-2). A reference point was marked on the field with the two starting distances 46, 46 (meters). Between the points A and B, we provided distance markers for 5 and 10 meters which the students could use as references either before or during the activity. In order to put focus on the spatial orientation ability, we decided not to provide visual references on the mobile device although this was technically possible (such as maps with marked attempts). They were instructed in the classroom about the activity and the functionality of the mobile application. To promote students' reflections during the activity, their new distances were shown on the display of the mobile device only when so prompted by the students (Fig. 4, right pane).

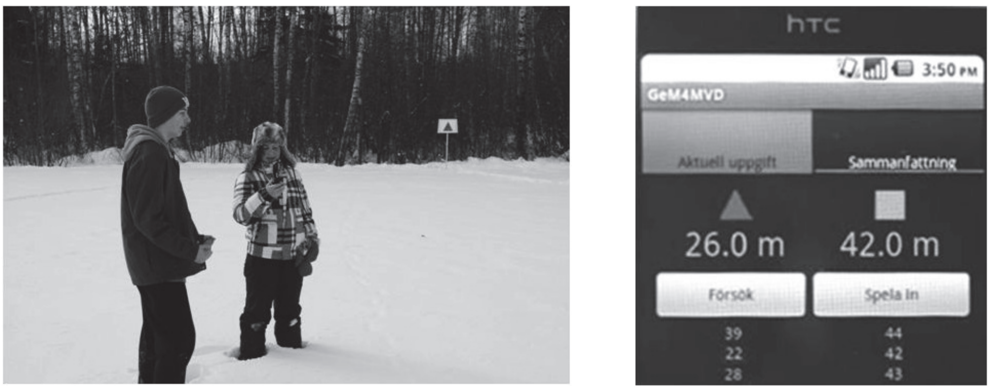


Figure 4: Picture from the outdoor activity. The display of the mobile device.

They were instructed to try to minimize the number of prompts/tries for each task and were also asked to record audio messages (on the same device) to be used for reflection in the classroom. The activity took less than one hour to complete by the six groups, despite the snow, cold weather, and humidity in the students' clothes.

4.2 The second task of the learning activity

The first part of the activity was followed by a more complex activity, implemented in early February 2011 involving a task where the students had to handle repeated coordination of distances. The new requirement for the mobile application was that distances had to be measured with respect to moving targets as the initial reference points A and B (triangle and square) had to be replaced by the other students as new reference points for the measuring of distances. Hence, each group needed three GPS enabled mobile phones which were running a customized application, so that relative distances between the students were measured. The activity was supposed to be tried out by three groups, one with four students and the two remaining groups with three students each (due to two students being absent on the day of the activity). The students who were chosen had all participated in the first part of the activity and were familiar with the functionality of the mobile application.

The groups prepared for the outdoor activity in the classroom. They were presented with maps (size A4) of a construction presented on a neutral white background and with marked distances on each edge (Fig. 5, left pane). They were asked to find the goal point, indicated with a circle in Fig. 5. Before attempting the constructions outdoors, they were asked to discuss possible strategies for approximately 15 minutes and to decide on a strategy for reaching the goal point before engaging in the outdoor activity.

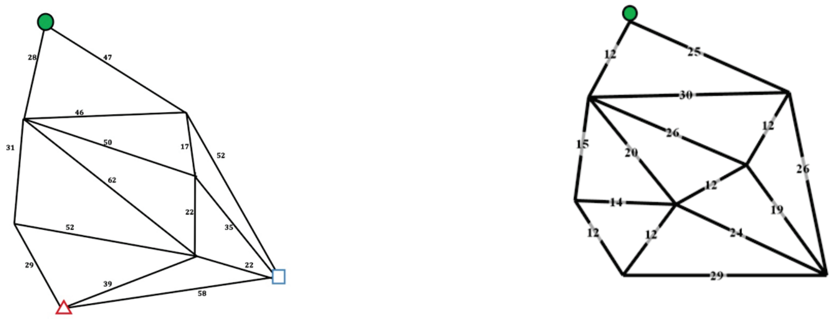


Figure 5: Maps used in February 2011 (left pane) and in April 2011 (right pane).

During the first implementation of the task, in February 2011, some incidents appeared that made us implement a re-designed version of this construction (Fig. 5, right pane) before the third and final task was implemented. These incidents concerned the students' preparation of strategies and the scaffolding of their outdoor communication. First, the students did not engage sufficiently in their indoor preparations of strategies for solving the task. Their insufficient strategies made them insecure and caused confusion when they tried to solve the task outdoors. Their confusion was amplified by communication problems due to the large distances in the construction and the strong winds on the day of implementation. For these reasons, the second task was redesigned with a technologically supported indoor session for preparing strategies. Furthermore, the distances in the construction were halved with the starting distance shortened from 58 m to 29 m in order to facilitate communication. This second iteration of the task worked out well when implemented in April 2011.

4.3 The third task of the learning activity

The last task of the activity, implemented in early June 2011, involved each of the three groups making a similar construction from different starting points (Fig. 6, left pane) by using the map above (Fig. 5, right pane). When all three groups had identified their different goal points (Fig. 6, left pane) they were instructed to collaborate in a jig-saw construction to construct the center of mass for the triangle (Fig. 6, right pane). The instructions did not mention "center of mass" which was an unknown and possibly distracting concept for the students. Instead, the goal points were named A, B, C, a form of notation they were already familiar with. The midpoints on the line segments AB, AC, BC were named D, E, F, respectively. The students were instructed to find the (final goal) point where the line segments AF, BE, CD intersect each other (Fig. 6, right pane).

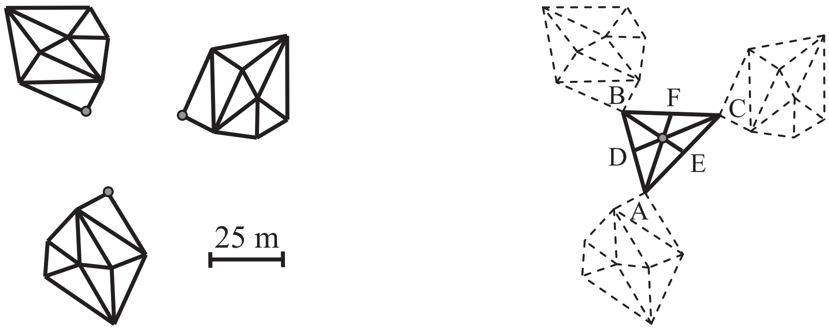


Figure 6: First phase (left pane) and the second phase (right pane) of the third task.

Our observations indicate that the students have been deeply engaged and motivated in solving the tasks, particularly when enacting the constructions in the outdoor contexts.

5. Lessons learned and future efforts

Based on the experiences from the first iteration, we plan to more systematically collect data during a second implementation of the activity with new groups of students. We plan to evaluate the outcomes based on a comparison of two groups of students, one which will do the tasks and one that will make similar constructions by solving standard tasks in textbooks. The students' ability to make geometrical constructions will be tested by letting them solve tasks with varying mathematical representations and modes of action.

As already mentioned, our activity obviously involves the two dimensions MSL3 and MSL4 (distribution across time and location) out of the ten dimensions for mobile-assisted seamless learning [14]. The tasks also encompass personalized and particularly social learning, (MSL2), they involve physical and digital worlds, (MSL6), and combined use of multiple device types – digital as well as traditional, (MSL7). Furthermore, the activity involves seamless switching between multiple learning tasks (MSL8) as individual students' roles in particularly the second and third tasks were changed frequently as they solved the tasks. Finally, managing the tasks in the connected activity requires a combination of prior and new knowledge (MSL9) as each new task builds on the previous tasks and each task involves both strategic planning and application of problem solving skills.

Although the participating researchers have been collaborating for several years in similar projects, there remains a need for a deeper understanding of both the didactical intentions of the activity and the functionality of the supporting technological applications in order to improve the design and realize the learning objectives for the students. In the current research effort, the ten-dimensional model for mobile-assisted seamless learning [14] has been used mainly for assessment purposes and has not been explicitly used in the design process, although some of the researchers are familiar with the model. Similarly, the methodology of scenario-based design has been used only implicitly. In our future efforts, we will attempt to make more systematic use of the dimensions of MSL as guiding design principles and apply the SBD methodology explicitly to further enhance the communication within the research team and to improve the quality of the design process.

Although our activity is designed for and implemented in a school context, we argue that it contains several features with strong impact for broadening students' learning experiences beyond both the activity itself and also beyond the formal school context. In a formal school context, the flow of learning is controlled and supported by the teacher, while the learner herself becomes primarily responsible for her learning in informal contexts. As noted by Wong and Looi [14], the "learners need to be engaged in an enculturation process to transform their existing epistemological beliefs, attitudes, and methods of learning". In our activity, we contribute to some extent to the learners' enculturation but even more to the enculturation of the teachers who are involved in the design process. By targeting teachers and transforming their attitudes and methods in the direction of seamless learning, we indirectly target their students and other teachers (and their students). The acceptance of our approach is underpinned by current steering documents for Swedish compulsory school, that highlight the development of general abilities (problem solving, communication, reasoning, representation, choosing and evaluating methods) which naturally encompass both formal and informal contexts, before specific content knowledge (arithmetic, algebra, functions, etc) which is more closely associated with the formal school context. By offering activities that are highly self-regulated and involve collaboration and communication with peers, we contribute to preparing the students for a future which requires them to take initiatives, be creative, take informed decisions, and puts high demand on their social skills.

A didactically relevant challenge for the future design improvements of our seamless learning activity would be to continue optimizing the hypothetical learning trajectories, aiming at specific mathematical learning objectives, while simultaneously attempting to incorporate additional dimensions of MSL in the activity.

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