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Experimental Study on Failures in Composing Solution Structures in Mathematical Problem Problems

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Abstract: Problem posing is identified as an important activity in mathematics education and a critical skill to be acquired. Several studies implemented support systems for learning of problem posing which aid novice learners in successfully posing appropriate problems. However, such learners may not necessarily success in posing appropriate problems without the support. Toward further support for novice learners in acquiring problem posing as a mathematical skill, we have to understand failures occurring in problem posing by novice learners. This study experimentally investigated problem posing by novices and empirically described their failures. In our investigation, participants were engaged in a learning task to study an example by reproducing it and a novel generation task to pose their own problems, with the results indicating that some participants composed problems whose texts and solutions were inconsistent in the learning task.

Keywords: Mathematical learning, problem posing, learning from examples

Introduction

Problem posing is identified as an important activity in mathematics education, as well as problem solving is [12, 13]. Although problem posing is rarely adopted in general education due to certain constraints in practical classrooms, it is as critical a skill as problem solving. One of the reasons why problem posing is unadopted may be that problem posing is extremely difficult for novice learners. Because problem posing is a production task that requires idea generation and synthesis of structures, it imposes heavy cognitive load on learners.

Several studies have addressed support for problem posing by learners. For example, some e-learning systems adopt problem posing as a learning task and aid it through the peer-assessment of learner problems [1, 4, 14, 16]. Hirashima and his colleagues implemented several systems that can evaluate problems posed by learners [3, 15]. Their environments offer computer-supported learning exercises to generate problems solved by specified solutions and to alter instance problems into new ones. These studies have also reported that learning with the systems improved learner understanding of domain knowledge or solution methods embedded in problems. Our previous studies proposed a support system that facilitates diverse problem posing through learning from examples [5, 6], and experimentally confirmed that our system could improve problem posing by learners to some extent.

Although problem posing is difficult, learners can successfully pose problems with support by the systems mentioned above. However, they may not necessarily success in posing appropriate problems without the support. Toward further support for novice learners in

acquiring problem posing as a mathematical skill, we have to understand failures occurring in problem posing by novice learners. Leung and Silver [10] studied problem posing by prospective elementary school teachers and empirically obtained a certain number of non-mathematical or unsolvable problems, even though, their focus was not on analysis of such inappropriate problems.
This study experimentally investigated problem posing by novices and empirically described their failures. In the investigation, we used a learning task of problem posing proposed in the previous study [6].

1. Experimental Method

In our investigation, participants were engaged in two problem posing tasks. One of them was a learning task to reproduce a problem given as an example, and the other was a task to generate novel problems.

1.1 Tasks

In each of the experimental tasks, participants were required to generate one or more problems in the domain of a problem initially given as a base. In the first learning task, they were provided with a base and an example problem as a good response in the task, which was generated by altering the base. They were then asked to reproduce the example. When reproducing, the example itself was hidden and information indicating how to generate the example from the base was shown. The generation process information of the example was automatically generated by our support system implemented in the previous studies, which included sufficient information to reproduce the example. This activity in the task had been designed to provide novice learners with ideas feasible in composing novel problems through *imitation* of varied examples. We empirically confirmed that learners could successfully transfer what they learned from an example through imitation with the system into novel problem posing by the learners. Figure 1 indicates the basic framework of the learning task (For more detail on the support system, see [6, 9]).

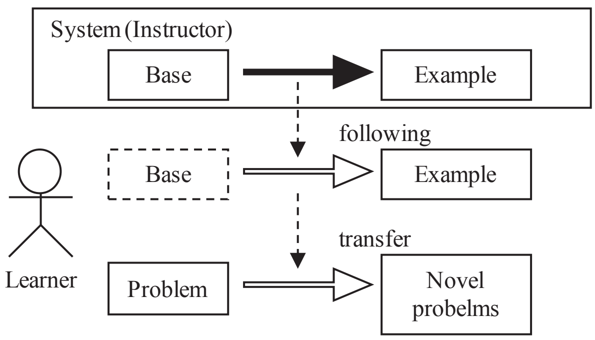

Figure 1. Basic framework of learning task

The learning task was followed by the novel generation task where the participants were asked to pose problems as many, varied and unique as possible from another base problem. In this task, participants' problems were evaluated based on four categories shown in Figure 2. These categories indicate similarities in *situations* and *solutions* between their problems and the base. Situations of problems denote surface features of contextual settings in problem texts (e.g., purchase of goods or transfer by vehicle), and solutions mathematical structures of the problems. Therefore, Category I / I indicates problems almost the same as the base, D / I indicates those generated by altering a situation of the base, I / D indicates those generated by altering a solution, and D / D indicates those generated by combining both alterations. It is desirable for mathematical learners to pose diverse problems across these categories controlling features of situations and solutions. However, previous studies revealed that leaner problems tend to lack diversity [2, 11] and have simple or inappropriate structures in their solutions [8].

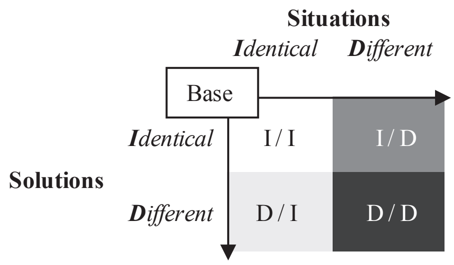

Figure 2. Categories for evaluating the variety of problems

1.2 Procedures

Undergraduates were participated in the investigation conducted in a lecture class of cognitive science. They were fist engaged in the learning task without the support system. The participants were told that the aim of the task was to learn what is problems posing and how it is done through an example before their own problem posing task. Prior to start of the learning task, the following problem was presented as a base.

> *Base*) I bought some 60-yen oranges and 120-yen apples for 1020 yen. The total number of oranges and apples was 12. How many oranges and apples did I buy?
> Solution.
> Let x denote the number of oranges and y denote the number of apples.
> $x+y=12$
> $60x+120y=1020$
> According to the equations above, $x=7$, $y=5$.

The base was printed in sheets of paper provided the participants. The participants were also presented the following problem as an example on a big screen of the classroom.

> *Example*) Last year I bought some 40-yen pencils and 110-yen pens. The total number was 13. This year I bought 2 times as many pencils as last year, the same number of pens as last year, and a 300-yen pen case for 1430 yen. How many pencils and pens did I buy last year?
> Solution.
> Let x denote the number of pencils and y denote the number of pens.
> $x+y=13$
> $40*2x+110y=1430-300$
> According to the equations above, $x=10$, $y=3$.

The example has the setting of purchase of goods identical to the base, and a solution formed by adding a third object other than x and y objects and an operation to calculate a coefficient of x in the lower equation to the base. Thus, it belongs to Category I / D in Figure 2.

When starting the task, the example was removed from the screen. The participants were asked to reproduce the example based on generation process information printed in the sheets. The generation process information contained the situation, numeric parameters appearing in the text, a basic structure of the solution, mathematical operations added in the solution, and keywords in the text of the example. It explicitly indicated that the situation of the example was identical to the base and the solution was altered. The participants were also instructed that they didn't have to completely literally reproduce words in the text of the

example as long as the contextual setting and equations in the solution were appropriately reproduced.
The participants were then engaged in the novel generation task. In this task, the following problem solved with a unitary equation was presented as a base.

> *Base*) I want to buy some boxes of cookies. If I buy some 110-yen boxes of cookies, then I have 50 yen left. If I buy some 120-yen boxes of chocolate cookies, then I need 20 yen more. How many boxes do I want?
> Solution.
> Let x denote the number of boxes.
> $110x + 50 = 120x - 20$
> According to the above equation, $x = 7$

They were told that their problems had to be necessarily solved with unitary equations and any problems in other domains were unacceptable.

1.3 Data Analysis

Participants were classified into groups based on problems they reproduced in the learning task. The groups were as follows.

Reproduced Appropriately (R-A): succeeded in composing a problem whose contextual setting and solution were identical to the example
Reproduced Sufficiently (R-S): almost succeeded in composing a problem identical to the example but partially changed its contextual setting (actually, *2 times* in the problem was not used as the number of pencils, but as the price of a pencil)
Reproduced but Modified (R-M): succeeded in composing the same solution structure but partially changed its surface parameters (numerals and their objects)
Altered solutions (A): composed a problem whose solution was different from the example
Lacked parameters (L): didn't succeed due to absence of numeric parameters, such as 300 (the price of a pencil box) or 13 (the total number).
Inconsistently composed (I): didn't succeed due to inconsistency between a text and a solution of a problem composed, although the solution was identical to the example

Problems newly composed by the participants in the novel generation task were categorized into the four categories in Figure 2. We also analyzed problems posed by altering solutions, from the aspect of structural complexity. However, this paper doesn't present more detail on the results in the novel generation task due to limitations of space. They will be reported precisely in another paper.

2. Results

One hundred and thirty-two undergraduates participated in the investigation. In the results below, eight undergraduates who didn't complete the learning task were excluded.

2.1 Problems Reproduced in Learning Task

Figure 3 indicates the proportions of participants in each group in the learning task. Although half of the problems were appropriately or sufficiently composed, the others were different from the example in some ways.

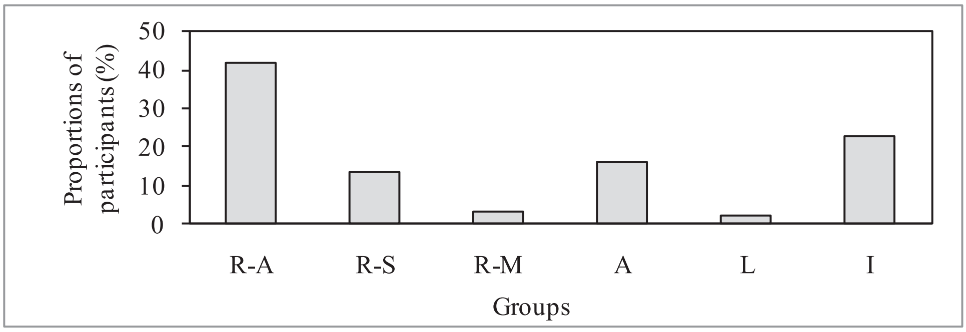

Figure 3. Proportions of reproduced problems in each group

2.2 Problems Posed in Novel Generation Task

In the following results, problems in domains different from the base and unsolvable problems were excluded. Figure 4 indicates the proportions of posed problems in each category in the novel generation task. “C” in the figure, denoting a *control* group, is the result of undergraduates who were engaged in the same novel generation task in the previous study [8] without learning of any example. This revealed that few problems in I / D were posed without supportive intervention. Although no significant differences between most of the groups and the control group were found due to the small numbers of participants, there was a significant difference between the R-A and control groups ($\chi^2(3)=15.29$, $p<.01$). Residual analysis revealed that the number of I / I problems was high in the control group and low in the R-A group ($p<.05$), and that of I / D was high in the R-A group and low in the control group ($p<.01$). Thus, appropriate reproduction of the example increased posed problems in I / D.

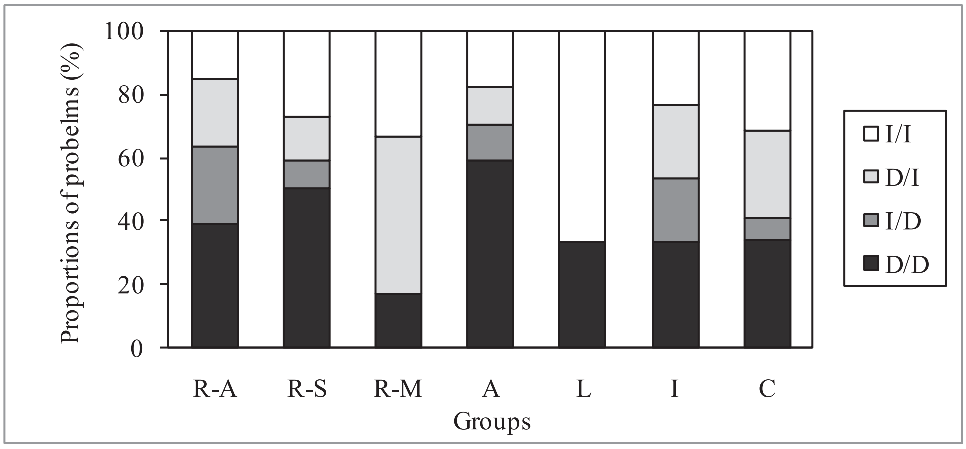

Figure 4. Proportions of posed problems in each category

Figure 5 indicates the proportions of problems posed by altering solutions whose operations in the solutions increased or decreased from the base. In the control group, half of the solution-altered problems were simpler than the base. The I group also posed many simple problems, whereas the R-A group posed many complex problems. There was also a significant differences between the R-A and control groups ($\chi^2(2)=11.36$, $p<.01$), and no difference between each of the other groups and the control group. Residual analysis revealed that the number of increase was high in the R-A group and low in the control group ($p<.05$), and that of decrease was high in the control group and low in the R-A group ($p<.01$). Thus, the appropriate reproduction also increased posed problems more complex than the base.

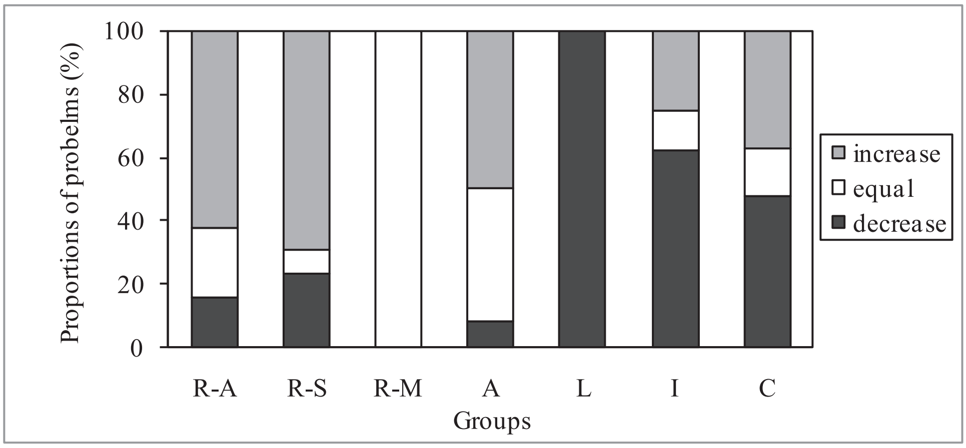

Figure 5. Proportions of solution-altered problems whose operations increased or decreased

3. Discussion

3.1 Failures in Reproduction

Despite sufficient information to reproduce the example was provided in the learning task, not more than half of the participants exactly reproduced it. The R-S and R-M groups didn't exactly do, though, their problems had solution structures identical to the example. Thus, it can be regarded that the two groups almost succeeded in the reproduction.

The A group didn't reproduce the example but composed problems different from the example. Therefore, the participants in the groups must have merely misunderstood the instruction in the learning task.

The participants in the I groups failed in reproducing the example. Although they described the same solution as the example, their problem texts were inconsistent with the solution. In the texts, some mathematical relationships were incorrectly described or inappropriate relationships were included[1] so that the solution was never formulated from the texts. Therefore, the participants didn't understand the inconsistency. Of course, none of the participants must fail in solving the example, which is a quite simple problem for undergraduates. We preliminary confirmed that undergraduates can successfully solve it [7].

The L group also failed in the reproduction. However, all of their problems could be completed by adding a description such as "the total number of the pencils and pens was 13". Thus, the participants must have carelessly forgotten to include some numerals into their problem texts.

As described in Section 1.1, the reproducing task adopted in this investigation is used in our support system [6]. No participants failed in the same reproduction task in an experimental evaluation of the system [9], although a few participants composed problems different from the example like the A group did. According to the facts, novice learners who successfully poses problem with supporting intervention can fail in appropriate problem posing without the intervention. Another important insight is that novice learners occasionally pose problems whose texts and solutions are inconsistent. To improve problem posing of novice learners, hence, further support is needed to endow the learners with a skill to appropriately compose problems.

[1] Some examples of problems in the learning task are presented in Appendix.

3.2 Novel Problem Posing after Learning

As described in Section 2.2, learning through appropriate reproduction of the example increased posed problems in I / D. It also increased problems whose solutions were more complex than the base, because the example allowed the participants to learn how to add operations. These results in the current study are consistent with experimental evaluation of the support system in the previous study [9]. On the other hand, sufficient learning effect wasn't gained through inappropriate reproduction.
According to the results in Figures 4 and 5, half of problems posed by the I group were in I / D or D / D, which fact indicates that many of the problems had solutions different from the base. The I group varied solutions in their problem posing to some extent. However, such problems in the I group were mostly simpler than the base which was quite simple and elementary. Therefore, the participants in this group didn't thoroughly learn the example, although they were examined it.

4. Conclusions

This study experimentally investigated problem posing by novices and empirically described their failures. In our investigation, participants were engaged in a learning task to reproduce an example and a novel generation task to pose their own problems, with the results indicating that some participants composed problems whose texts and solutions were inconsistent, in other words, they failed in reproduction. Our next task is, of course, to study and design a supporting method to prevent such failure in the learning activity.

Acknowledgment

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References

[1] Barak, M., & Rafaeli, S. (2004). On-line Question-Posing and Peer-Assessment as Means for Web-Based Knowledge Sharing in Learning. International Journal of Human-Computer Studies, 61, 84-103.
[2] English, L. D. (1998). Children's Problem Posing within Formal and Informal Contexts. Journal for Research in Mathematics Education, 29, 83-106.
[3] Hirashima, T., Yokoyama, T., Okamoto, M., & Takeuchi, A (2007). Learning by Problem-Posing as Sentence-Integration and Experimental Use. In R. Luckin, K. R. Koedinger & J. Greer (Eds.) Proceedings of the 13th International Conference on Artificial Intelligence in Education (pp. 254-261). Amsterdam, Netherlands: IOS Press.
[4] Hirai, Y., Hazeyama, A., & Inoue, T. (2009). Assessment of Learning in Concerto III: A Collaborative Learning Support System Based on Question-posing, In V. Uskov (Ed.) IASTED International Conference on Computers and Advanced Technology in Education (pp. 36-43). Calgary, Canada: ACTA Press.
[5] Kojima, K., & Miwa, K. (2008). A System that Facilitates Diverse Thinking in Problem Posing. International Journal of Artificial Intelligence in Education, 18, 209-236.
[6] Kojima, K., Miwa, K., & Matsui, T. (2009). Study on Support of Learning from Examples in Problem Posing as a Production Task. In S. C. Kong et al. (Eds) Proceedings of 17th International Conference on Computers in Education (pp. 75-82). Jhongli, Taiwan: Asia-Pacific Society for Computers in Education.
[7] Kojima, K., Miwa, K., & Matsui, T. (2010). Experimental Study on Methods of Learning from Examples and their Effects in Problem Posing. 62th SIG on Advanced Learning Science and Technology in Japanese Society for Artificial Intelligence, SIG-ALST-B001,13-18. (In Japanese)

[8] Kojima, K., Miwa, K., & Matsui, T. (2010). An Experimental Study on Support for Leaning of Problem Posing as a Production Task. Transactions of Japanese Society for Information and Systems in Education, 27, 302-315. (In Japanese)
[9] Kojima, K., Miwa, K., & Matsui, T. (*in press*). Study on the Effects of Learning Examples through Production in Problem Posing. 19th International Conference on Computers in Education.
[10] Leung, S. S., & Silver, E. A. (1997b). The Role of Task Format, Mathematics Knowledge, and Creative Thinking on the Arithmetic Problem Posing of Prospective Elementary School Teachers. Mathematics Education Research Journal, 9, 5-24.
[11] Mestre, J. P. (2002). Probing Adults' Conceptual Understanding and Transfer of Learning via Problem Posing. Journal of Applied Developmental Psychology, 23, 9-50.
[12] Polya, G. (1945). How to Solve it. Princeton, NJ: Princeton University Press.
[13] Silver, E. A. (1994). On Mathematical Problem Posing. For the Learning of Mathematics, 14, 19-28.
[14] Takagi, M., & Teshigawara, Y. (2006). A WBT System Enabling to Create New or Similar Quizzes Collaboratively by Students. In J. R. Parker (Ed.) IASTED International Conference on Education and Technology (pp. 263-268). Calgary, Canada: ACTA Press.
[15] Yamamoto, S., Waki, H., & Hirashima, T. (2010). An Interactive Environment for Learning by Problem-Changing. In S. L. Wong, S. C. Kong & F. Yu (Eds) Proceedings of the 18th International Conference on Computers in Education (pp. 1-8). Malaysia: Universiti Putra Malaysia.
[16] Yu, F., Liu, Y., & Chan. T. (2005). A Web-based Learning System for Question-Posing and Peer Assessment. Innovations in Education and Teaching International, 42, 337-348.

Appendix: Examples of problems in Learning Task

A group

I bought some 100-yen apples and 30-yen bananas. The total number was 13. I gave a 1000-yen bill and received 190 yen as the change. How many apples and bananas did I buy?

Solution.

Let x denote the number of apples and y denote the number of bananas.

$x+y=13$

$100x+30y=1000-190$

According to the equations above, $x=6$, $y=7$.

I group

Last year I bought some 40-yen pencils and 110-yen pens. The total number was 13. <u>This year I also bought 13 pencils and pens</u>. The number of pencils this year was 2 times as many as last year. In addition to pencils and pens, I bought a 300-yen pencil box. The payment was 1430 yen. How many pencils and pens did she buy?

Solution.

Let x denote the number of pencils and y denote the number of pens.

$x+y=13$

$40*2x+110y=1430-300$

According to the equations above, $x=10$, $y=3$.

(The total number this year is wrong)

A girl bought pencils and pens. The total number was 13. <u>The number of pencils was 2 times as many as pens</u>. A pencil was 40 yen and a pen was 110 yen. She found a 300-yen lovely pencil box near the cash desk, and took it with pencils and pens. The payment was 1430 yen. How many pencils and pens did she buy?

Solution.

Let x denote the number of pencils and y denote the number of pens.

$x+y=13$

$40*2x+110y=1430-300$

According to the equations above, $x=10$, $y=3$.

(Parameters associated with the relationship “2 times” are wrong)