Scaffolding Teachers' Construction of a Learning Trajectory for Mathematics Supported by ICT

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Abstract: In this paper, we present preliminary results related to a developmental project where we explore how technology can be used to improve teachers' teaching practices and their students' learning of mathematics. We have adopted a teacher-centered, collaborative approach that challenges the participating researcher to develop strategies for finding different forms of scaffolding to support the teachers' participation in this project. So far, we have used the software GeoGebra to provide competence development for the teachers in terms of mathematical representations. This software will also have a central role when investigating how the teachers perceive and make use of different affordances provided by technologies when addressing a specific learning goal related to the teaching of algebra at a lower secondary school.

Keywords: Affordances, co-design, GeoGebra, Hypothetical Learning Trajectory (HLT)

Introduction

We are currently involved in a developmental project with two mathematics teachers at a lower secondary school in Sweden where we explore how to make use of ICT to improve the teachers' mathematical teaching practices and their students' learning of mathematics. Drawing inspiration from the methodology of co-design [8] our objective is to develop technology-enhanced mathematical learning activities. Co-design provides a user-centered, collaborative approach to design research, suitable when requiring expertise from different areas, in our case mathematics, pedagogy and technology.

Involving different stakeholders and especially teachers in the design process is a key factor for an innovative learning activity to find its way to teachers' everyday practice [10]. The teachers' role in the co-design process should be regarded as dynamic and their influence should be increased in latter design iterations [10] with particular focus on refining the interplay among various forms of scaffolding required when implementing the activity in order to provide favorable conditions for its integration in the regular curriculum [11]. This methodological approach puts increasing demands on the teachers as the design process progresses and challenges the researchers to identify and provide appropriate theoretical and methodological underpinnings to support the teachers in the continued design process.

1. The Hypothetical Learning Trajectory

Within the iterative process of design research different phases of preliminary design, teaching experiment and retrospective analysis can be identified [2]. The preliminary design includes the development of a Hypothetical Learning Trajectory (HLT) which can be described as "the consideration of the learning goal, the learning activities, and the thinking and learning in which students might engage" [9, p. 133]. Although the notion of HTL has a constructivist origin [9] it has been used from other perspectives. Gravemeijer, Bowers, and Stephan have for example used the instructional theory Realistic Mathematics Education (RME) to guide the formulation of a HLT to develop classrooms activities that reflect the view of mathematics as a human activity and learning as a process within a social context [4]. For our objectives, this property makes the notion of HLT a highly adaptable and flexible tool suitable when discussing different aspects of teaching and learning within the co-design group.

The effects of integrating technologies in educational settings, are not necessary transparent. Even if technologies can be used to address some problems related to teaching, there is a possibility that they introduce new unexpected problems [5] [6]. Lindwall and Ivarsson claim that a common problem is that students tend to "exclusively focus on the operational aspects of the task without actually approaching the subject matter content" [6, p. 376]. This suggests that even if the intention of a learning activity is to afford students to work towards a predetermined learning goal, students might still focus on other aspects and other affordances. Another challenge for the designer of a HLT is that, each tool, although seemingly similar in many aspects, can have divergent effects on how students interact in an activity [6]. The result could be unexpected forms of interaction in conflict with the intentions of the HLT.

Formulating a HLT that integrates technologies requires knowledge about the affordances provided by the tool and also knowledge of how these affordances are perceived and used by teachers and students in a learning activity [1] [6]. Addressing these kinds of issues with a collaborative approach challenges the researcher to create conditions for the teachers to discuss and consider different affordances provided by technology as well as create awareness among the teachers on how digital tools might affect their educational settings [1] [5].

2. Key Idea

Our goal is to develop a HTL that is grounded in theory and attuned to the various prerequisites imposed by different stakeholders, including the possibilities and constraints that may rise when using technologies in mathematics instruction. The collaborative approach challenges the researcher to create appropriate conditions for the teachers to be able to make informed decisions. Therefore, we also seek to explore how to develop, provide and evaluate different forms of scaffold for teachers' participation in the design process. Our strategy is to monitor and qualitatively analyze the outcomes from the discussions held with the teachers and, based on the analyses, provide support for enhancing the teachers' knowledge base. So far, our efforts include two meetings with the teachers.

3. Preliminary Results

Two teachers from the current school volunteered to participate and the purpose of the first meeting was to discuss the goals of the project. At this meeting, the teachers brought forward that they were interested in addressing some of the difficulties their students had concerning algebra, e.g. the students' inability to make sense of the distributive law. The

analysis of the discussions rendered the following research question: “What mathematical knowledge requirements does a teacher need in order to participate in the co-design of a mathematical learning activity supported by ICT?”

A synthesis of two complementary models was used suggest that “teachers’ mathematical knowledge requirement would be to consider and be able to judge and compare the didactical value of various mathematical representations” [7]. This requirement includes also the use of technologies in the sense that technologies provide affordances for multiple and multi-modal representations, simulations, manipulation of data, and conversions of representations [5]. Furthermore, the analysis of the discussion indicated that there was a need for competence development among the teachers regarding mathematical representations. For this purpose a second meeting was held where the teachers where shown some activities, implemented in GeoGebra, where numerical expressions are connected with their corresponding geometrical representations (Figure 1).

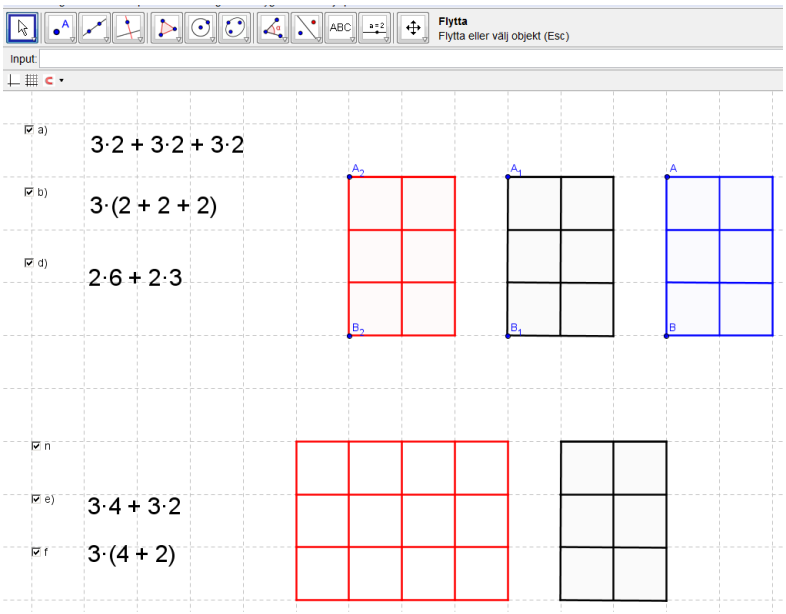

Figure 1: The activities implemented in GeoGebra

The dynamics of GeoGebra provides possibilities to move and rearrange geometrical figures with ease. This affordance is used in the activities to illustrate how algebraic treatments of a numerical expression can be interpreted and represented geometrically. The possibility to display and hide text and figures in Geogebra is used to create freedom of choice to display the two different mathematical representations either simultaneously or one at a time. Although figures and expressions are organized in a determined order in the activities, the teacher still needs to consider how the different representations should interact within the activities. The activities are designed to address students’ conceptions of the distributive law by affording transformations of representations which can be regarded as a characteristic of mathematical proficiency [3]. The activities are also designed to provide affordances for different forms of interaction between the teacher, the students and the activities.

4. Reflections on the Outcomes of the Meetings

The teachers had never used GeoGebra before the second meeting and they became interested in the features of this software and in the activities presented. Furthermore, they

wanted immediately access to the activities in order to use them with their students. During this meeting the teachers adopted the mathematical ideas presented in the activities and they came to realize the didactical potential in interpreting and representing algebraic laws geometrically. This was clearly a new insight for them and they recognized the limitations of alternative explanations that they normally used that where exclusively based on instructions on how to manipulate different variables.

The teachers were encouraged to use the activities with their students but they were not provided any details on how to create conditions for using the activities in instruction. Instead they were invited to modify and use the activities in GeoGebra in any preferable way and thereby challenging the teachers to create their own HLT. This last meeting has not yet been followed up and the outcome of these efforts has therefore not been analyzed yet.

5. Future Work

The next step is to discuss with the teachers how they have used the activities and also attend lessons where the teachers have implemented the activities as part of their daily work. The analysis of these sessions has two objectives. First we are interested in understanding and discussing how the teachers coordinate the different parts of the hypothetical learning trajectory when using GeoGebra. Secondly, we are interested in evaluating our efforts of providing competence development in terms of mathematical representation. We want to see if and how the teachers perceive and make use of affordances for representation and communication provided in the specific activities implemented in GeoGebra.

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