The Study of Mathematical Problem Solving through Eye Tracking Technique

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Abstract: The main purpose of this study is to explore the cognitive process of mathematical problem solving. This study adopted eye tracking technique to investigate some correspondences between behavior of problem solving and cognitive process of mental computation. Many interesting and deeper comprehension on problem solving were found. With these results, we believed that this study had contributed to the insight of problem-solving strategies and the usability of eye tracking technique in future studies.

Keywords: Eye tracking technique, mathematical problem solving

Introduction

Many studies had adopted the eye-tracking technique to explore how students with different backgrounds looked at while solving the science problems (Anderson, Bothell, & Douglass, 2004; Chuang & Liu, 2012; Epelboim & Suppes, 2001; Hyönä, 2010; Tai, Loehr, & Brigham, 2006). They found that novices looked at the specific areas including more expertise, and then the fewer fixation counts in specific areas, and the fewer saccades between areas. However, fewer studies had utilized eye-tracking technique to investigate mathematics problem solving. For example, Verschaffel et al. (1992) used eye-tracking technique to examine word problem solving about arithmetic strategies, e.g. addition used while subtraction was the correct strategy, and indicated that students made more comprehension when the order of the terms in the relational statement is not consistent with the preferred order. Moreover, Hegarty, Mayer and Monk (1995) also had conducted several eye-tracking studies to explore the process and the strategies for solving mathematics problems. They found some important results, such as numbers and variable names were fixated longer while solving word problem. In addition, high accuracy students spent more time on more difficult problems and the most time was located in the integration and planning phases of problem solving. So, they suggested that high-accuracy students focused more on variable names, but low-accuracy students focused more on the numbers and relational words such as more or less. Hence, the above studies on problem solving in science or mathematics educations through eye-tracking technique were very important, because it provided deeper insight from biological viewpoint.

According to these studies in science and mathematics educations, they gradually adopted eye tracking technique to gain more insight. Just and Carpenter (1980) suggested that eye tracking technology has been typically adopted to examine visual attention based on the eye-mind hypotheses. So, many studies had used the eye-tracking method successfully to apply in mathematics education research fields, such as arithmetic problem solving (Verschaffel, De Corte, & Pauwels, 1992), reading (Rayner, Chace, Slattery, & Ashby, 2006) and human-computer interactions (Jacob & Karn, 2003). However, the eye tracking technique even if can reveal important insights into the ongoing learning process

(Hyönä, 2010). It is important to note limitations of the eye-tracking method. For example, the student gazed the task-relevant information for a long time, but it did not tell us anything about success or failure of understanding the relevant piece of information. We only knew that the student spend a lot of time viewing a sequence of stimulus, however, we can't exactly know that he/she maybe solve the problem successfully or unsuccessfully. Thus, we had to complement with other performance measure, such as pre-test questionnaire or post-test interview protocols (Kaakinen & Hyönä, 2005).

To sum up, we are especially interested in how students solve mathematical problems. In this study, we used eye tracking technique to monitor and record the process of problem solving while participant view the numerical sequence. Furthermore, we analyzed the eye movement data, compared with post-interview protocols, and further found some important corresponding of eye movement models. These evidences could let us gain deeper insight on problem solving.

1. Methodology

1.1 Participant

There are thirty eight university students voluntarily participated in this study (mean age = 20 ± 1). All participants had taken and passed a visual acuity test and calibration test on the computer screen. They had normal or corrected-to-normal vision. Their typical educational background was science major. They were seated approximately 65 cm away from the screen. Before the experiment, the participant was asked to practice the procedure of clicking the buttons, and we could confirm every one give the response successfully. The eye tracker would collect eye movement data and behavioral data during the experiment.

1.2 Experimental Design and Procedure

The materials were adapted from different rule-based mathematical problems about numerical pattern. It consisted of four types of sequence problems: addition, subtraction, multiplication and division (see the figure 1). For example, question "1,3,5,7,9,11,__" was presented on the screen. The participant had to explore the rule of the sequence problem between the successive numbers and calculate the seventh number according to the same rule. Then he/she clicked the button and talked the answer orally. Each type of problems included three questions and all questions presented on the screen randomly. All participants were required to solve twelve questions in this study. They were asked to explore which rule generated this question and then answered aloud as soon as possible. Their spoken answers were recorded, and some indicators of eye movements were detected by the eye-tracking system.

Figure 1. The first question was generated from addition rule; the second one was generated from subtraction rule; the third one was generated from multiplication rule; the fourth one was generated from division rule.

In this study, the eye-tracking experiment was designed to explore the procedure of mathematical problem solving and their thinking. Four types of problems in this study were displayed on the monitor randomly. The eye movement data was recorded through eye-tracking system during each participant solved problems. To confirm the thinking or strategies of problem solving, post-test interview was conducted. Hence, the procedure of this study involved four phases. The first phase was calibration test to insure the reliability of the eye-movement data. Each participant had to undergo a visual acuity test to calibrate the position of the eye. In the second phase, we explained the purpose of this study and taught them how to respond the problems by clicking the button. Each participant practiced how to press the button until they felt comfortable with the procedure. In the third phase, each participant solved 12 questions that were randomly generated on the monitor. They solved the problems displayed on the monitor while their eye-movements were being recorded. In the fourth phase, the post-test interview was conducted to ascertain the thinking processes of solving problem after the experiment. During the interview, the participant explained why he/she made a particular move and what he/she was thinking at that time. The scan paths of eye-movement data were display again in the interview, the purpose was to help the participant recall what he/she were thinking, what they did, and what they felt as they were watching a particular part of the screen. Participants were also requested to complete a short questionnaire about the strategies they used in solving the problems. For example, "which rule was generated from this question?" and "what kind of strategies was used to solve the problem?"

2. Data Collecting and Analysis

Eye-movement data included eye fixation paths, a video of gaze overlay, and the data of the gaze location with statistical calculations. During the experiment, the participant's eyes stayed in one location for 200 ms or more, we defined it as ''a fixation''. Fixation is considered as an indicator of perceived points of interest, and the duration of fixation indicates the cognitive complexity of information being acquired (Henderson & Hollingsworth, 1998). Slykhuis et al. (2005) suggested that the total numbers and the duration of fixation within a region can be considered an indicator of perceived importance accompanied by a high probability for long-term memory encoding. Therefore, the duration or counts of fixation may actually be a better indicator of the student's focus areas (Jacob & Karn, 2003; Liu & Shen, 2011). This study proposed that students take a strategic approach when solving sequential problems according to their acquired visual information. They gazed at the regions of the problem where they could extract the most relevant information for solving the problem. In addition, an interview was conducted immediately after completing the experiment. Whereas eye tracking provided one possible insight on problem solving, interviews were found to be a more interpretive source of information on the usefulness and quality of the problem solving.

3. Result and Discussion

To investigate whether participants used different strategies to solve four types of problems, this study conducted a repeated measure ANOVA with some indicators of eye movement on four types of problems as the dependent measure. According to the statistical results, we inferred that some indicators of eye movement should be strongly related to the cognitive process. First, a significant difference was found in total time of problem solving among four types of problems ($F=51.81$, $p< .05$, $\eta 2= .583$), and the total time of solving multiplication and division problems was longer than addition and subtraction problems. It showed that participants spent more time to solve multiplication/division problems. However, this result didn't revealed why participant spend more time to solve multiplication/division problems or how did they process the information. Therefore, we further analyzed these indicators of eye movement, such as Fixation Count (FC), Fixation Duration (FD) and Frequency of Fixation (FF).

To distinguish some certain difference in FC, FD and FF among four types of problems, we compared the eye movement data with interview protocols. The significant difference was found in FC among each type of problems, except between multiplication and division problems ($F=31.65$, $p< .05$, $\eta2= .461$). FC showed the more counts of fixation while solving multiplication/division problems than addition/subtraction problems. However, we thought that there was more FC while solving multiplication/division problem maybe because of the long time of problem solving. Therefore, we compared with interview protocol. We found that most of participants viewed the numerical sequence from left to right but they viewed the sequences of subtraction or division problems from right to left. The result revealed that participants are used to check the rule from the least number. In addition, the significant difference in FD was found and FD of solving multiplication/division problems was longer than addition/subtraction problems ($F=37.12$, $p< .05$, $\eta2= .501$). At the same time, the significant difference in FF was also found ($F=5.98$, $p< .05$, $\eta2= .139$). These results showed that when participants solved multiplication or division problems, they spent more time to recall the facts, such as 2 times 16 equal 32 ($2 \times 16 = 32$). In addition, when they solved addition or subtraction problems, they directly viewed two numbers to calculate the difference between the next numbers. This inference was justified from eye-tracking data and interview protocols.

Hence, this study combined the eye-tracking data and interview protocols. We gained more insight about the cognitive process of problem solving. With these results, we believed that this study has contributed to the insight of problem-solving strategies and the usability of eye tracking technique in future studies.

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