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The Role of Epistemic Agency and Progressive Inquiry in the Transfer of Mathematical Thinking

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Abstract: Instructional design of inquiry-based classrooms need to regard learning as dynamically flexible and adaptive with opportunities for emergent teaching and learning strategies as well as assessments. Consequently, we argue that emergent instructional design and emergent learning systems should focus on the discovery of instructional principles, instructional strategies and technologies that promote the development of inquiry, with teacher beliefs as a key design factor. We scope our study to the inculcation of Mathematical thinking because Mathematical thinking focuses on the identification of similarities among instances, leading to the development of general principles. The ability to formulate principles grows alongside learners' search for abstract problem-solving methods and mental schemata. These serve as analogy-enhancing transfer between different task situations. Consequently, in this exploratory study, we aim to help learners to learn to inquire and to reason, to be able to identify patterns, describe patterns and apply patterns to solve problems. Furthermore, we aim to identify how teacher's beliefs influence the design of teaching-learning practices. Subsequently, we suggest implications to the design of creative inquiry-oriented-based curriculum, pedagogy and technologies for the learning of Mathematical thinking.

Keywords: Transfer, Mathematical thinking, Engineering education, epistemic agency, progressive inquiry, technological scaffolds

1. Introduction

The 21st century requires dynamic flexible adaptive thinking. Hence, expectations have shifted from merely problem-solving to the inculcation of creative problem-solving. An example of such change is reflected in ABET's Engineering Criteria (1995). They indicate that Engineering education needs to address a wide scope of concerns, including environmental, political, social, international, and legal and ethical ramifications of decisions. More recently, Duderstadt's (2008) report highlights that the economic, political, social, and environmental context of engineering practice needs to be explicitly taught, in order to situate core scientific and technical courses within real-world concerns.

We argue that the aim of scientific inquiry and experimentation is to discover appropriate instructional strategies which can be used as conditions of learning. These conditions of learning can and should subsequently be applied as design factors in different contexts, and be adopted, adapted and refined through further scientific inquiry and experimentations.

Hypothetically, these conditions of learning should be relevant across all disciplines, and especially so for science-based disciplines such as Engineering. However, since this is an exploratory study, we scope our study to investigate and discover suitable curriculum, instructional strategies and technologies for the teaching of Mathematical thinking. We focus on the development of Mathematical thinking because Mathematical thinking (Schoenfeld, 1992) can help learners to develop the competency to identify, formulate, and solve problems. These competencies are crucial in the formation of abstract problem-solving methods and mental schemata. According to Schoenfeld, the latter competencies can serve to enhance analogy transfer between different task situations. Consequently, design factors or conditions of learning which can contribute positively to such analogical transfer needs to be identified or discovered.

Subsequently, we argue that conditions of learning are not independent from the beliefs that teachers and learners bring to the classroom. We further argue that beliefs are part of the key design factors underlying Chan's (2007) Humanity-based epistemology, which espouses that teaching should be caring and learning joyful. Consequently, we hypothesize that the mapping between teacher and learner beliefs can be used as design factors to discover suitable instructional strategies and technologies for the teaching of Mathematical thinking.

We also argue that to help learners proactively identify similarities among instances leading to the development of principles, we need to help learners to inquire and to reason, to be able to identify patterns, describe patterns and apply patterns to solve problems. As highlighted by Schoenfeld (1992), these metacognitive skills would enable efficient analogy-based transfers between different task situations. For such meta-cognitive methods to develop, we need to first help learners to refine their inquiry skills. Subsequently, we regard cases as instances of a general principle or combinations of principles. These cases present huge potential for emergent learning.

The inquiry approach we have adopted is based on Hakkarainen's (2003) gradual approach to inquiry, i.e., "progressive inquiry." Aimed at young learners learning science, progressive inquiry continuously guides learners to systematically generate their own research questions, develop their own intuitive working theories, generate intuitive ideas, critically evaluate these ideas, search for new scientific information, engage in progressive generation of related questions, and construct new working theories throughout the inquiry process.

In summary, our main aim is to discover appropriate instructional strategies and technologies that can help learners to develop a fuller understanding of principles through inquiry-based incremental knowledge transformation and refinement. Our research question is how do specific teacher's beliefs towards the learning of a specific Mathematical concept, i.e., addition influence the design of teaching practices? Subsequently, we suggest implications to teacher professional development on the design of inquiry-oriented curriculum, pedagogy and technologies for the learning of Mathematical and creative thinking.

2. Related work

Bruner (1986) and Thorndike (1906) agree that transfer of learning occurs when learning source and learning target share common stimulus-response elements. The theory of "identical elements", currently the most prevalent notion about transfer, highlights that what is more crucial is the identification of similarities among instances, which lead to general principles. The formulation of principles indicates transfer of learning arising from understanding. On top on this, Brown (1987) posits that meta-cognitive awareness of task features, problem structures, and solution methods is a crucial skill to develop because it makes relations between different situations cognitively *salient.*

With regards to teacher beliefs and practices, studies such as by Crawford (2007), Sandoval and Daniszewski (2004), Weinberger and Fischer (2006), Tillema and Orland-Barak (2006) and Jacobson, So, Teo, Lee, Pathak, & Lossman (2010) demonstrate that teacher beliefs and teaching practices are intertwined. Hence, changes in teachers' beliefs shape their planning decisions as well as interpretations of the curriculum. Subsequently, research methods for examining teacher practices, are based on inquiry instructional principles, with the aim of capturing specific key elements in inquiry practices.

We are adopting Song and Looi (2011)'s five principles of inquiry, i.e. working on authentic problems, encouraging diverse ideas, making progressive inquiry, providing collaborative opportunities and doing embedded assessment to assess teacher beliefs and practices. In their study, findings confirm that the teachers' enactment of different beliefs led to the design of different practices and the design of students' progressive inquiry and learning. They find that the teacher who had "innovation-oriented" beliefs is inclined to enact the lesson in patterns of inquiry-principle-based practices as well as enhance learning experiences with the use of technology. These patterns consequently positively influenced student inquiry processes and the effective use of technology affordances. We hope to discover more such innovation-oriented beliefs and practices in our current and future study as learning is dynamically flexible and adaptive with opportunities for emergent teaching and learning strategies as well as assessments.

3. Methodology

The design of our mechanism for transfer of learning is based on on Perkins and Salomon's (1992) conditions for transfer, These are:
Comprehensive and diverse practice through extensive practice of the performance in a variety of contexts.
Creating mindfulness through relating new material to material learnt earlier and through relating the material to the learner's surroundings.
Active self-monitoring. Metacognitive reflection on one's thinking processes promotes transfer of skills, highlighting thinking processes.
Explicit abstraction. Abstraction emphasizes the structure of the situation. Explicit abstractions of principles from a situation foster transfer.

We interviewed an elementary school teacher who had four years of experience teaching in the school and another four years in tuition classes to better understand his beliefs about the teaching of Mathematics. The design of the interview questions was adapted from teacher perception categories in Song and Looi's (2011) study. To assess and validate our design, we asked him to try out our learning system, aimed at helping learners in grades 2-3 identify similarities across instances, leading to the development of general principles. Two types of exercises were designed and developed based on Perkins and Salomon's mechanism: pattern recognition exercises and question posing exercises formulated by student peers. Each type of exercise comprise of five levels of difficulty.

4. System design

We argue that challenges should be regarded as embedded assessment as assessment has often successfully shaped students' learning goals, strategies and motivation to be challenged and to learn. We designed and developed two types of exercises: pattern recognition exercises and question posing exercises formulated by student peers. Each type of exercise comprise of five levels of difficulty. Every successful answer will be awarded Ten Points. A brief explanation of each type of exercise is elaborated on below:

Pattern Recognition

The system will randomly generate questions where students need to add several numbers presented in a matrix format. The number of rows and columns in the matrix increase incrementally, building up complexity as the students progress. The simplest is a 2 x 2 matrix which builds up to a 6 x 11 matrix. The main objective of this exercise is to allow students to discover the patterns/principles behind addition through repeated practice. We hope that once the students are able to recognize these patterns/principles, they would be able to transfer these patterns/principles to answer all questions regardless of the level of complexity.

Question Posing

The aim of this type of problem is to help students with the most important processes in inquiry: question-driven process of understanding, formulation of theories in the search for new scientific information and generation of one's own explanations, hypotheses or conjectures and ultimately, deepening of the students' understanding of the concept.

5. Findings

5.1 Teaching beliefs

The instructor opined that what is most important in the design of curriculum is to help students to make associations among various concepts.

5.2 Teaching strategies

Instructors and students often face the following difficulties in learning Mathematics:

- Many students lacked interest in learning Mathematics. There were two possible reasons: either the students do not like Mathematics or they are too smart and thus find the curriculum boring.

However, they might face difficulty when the teacher teaches the advanced curriculum. One possible reason when faced with more advanced curriculum is that since the students are very young (grades 2-3), they often face difficulty expressing themselves and understanding expressions in Mathematical problems.

- Most students will encounter difficulties in learning the concept of addition. He noted that it would be beneficial if the instructor takes advantage of the way that of students communicate and learn because then, students are more able to accept and understand

His common practice in addressing these difficulties is first to teach based on the school curriculum framework. If the students do not understand, then he will use pictures to illustrate the method or demonstrate the steps and explain why to increase students' understanding and create a deeper impression on the why rather than the how. To him, it is very important for students to understand both why and how because if students could not understand, they would eventually lose the motivation to learn.

5.3 Working on authentic problems

He gave real-life examples in relating curriculum to real-life when students could not understand the concepts of Mathematics. For example, using changes to teach the concepts of positive and negative numbers can make students relate to real-life.

5.4 Encouraging diverse ideas

He agreed that different kinds of questions would need different problem-solving skills. As such, the design of the kinds of questions must consider the students' problem-solving and thinking processes. Furthermore, he believes that the expected type of answer should be determined by the degree of difficulty.

5.5 Making progressive inquiry

He regards challenging students with open-ended questions scaffolded by technology as beneficial. This is because these questions, especially those of the application type, can attract students' attention; crucial to motivating them to learn. He also thinks that teachers should not limit the number of times students can be allowed to answer in order to increase students' willingness to be challenged, and to increase the likelihood of obtaining a sense of accomplishment.

5.6 Providing collaborative opportunities

The teacher should first assess whether students' oral communication skills could enable them to express the concepts and their strategies adequately. Hence, the design of the kinds of questions must consider not only the students' problem-solving processes but also communication skills. For more difficult questions, the teacher can provide a longer period of time to allow students to think and then provide the answers.

6. Conclusion and implications

Prior research has indicated that changes in teachers' beliefs change their planning decisions as well as interpretations of the curriculum. The research methods employed for examining teacher practices were based on inquiry instructional principles as these help capture specific key elements in inquiry practices. Song and Looi (2011) have investigated how specific teacher beliefs impacted specific teacher practices in Mathematics and how these practices influenced the design of student inquiry learning in specific domain in a CSCL environment. Their findings highlight that the teacher who exhibited "innovation-oriented" beliefs was inclined to enact the lesson in patterns of inquiry-principle-based practices as well as enhance learning experiences with the use of technology. These patterns in turn refined student inquiry processes and the effective use of technology affordances. We are concerned with teachers' and learners' knowledge building processes and outcomes refined through their beliefs. Hence, we regard the classroom as emergent learning systems where the beliefs function as design factors argued that to help learners proactively identify similarities among instances leading to the development of principles, we need to help learners to inquire and to reason, to be able to identify patterns, describe patterns and apply patterns to solve problems. Subsequently, we investigated how specific teacher beliefs impacted the design of student inquiry learning in specific domain for a CSCL environment. Our objective was to help learners to identify similarities among instances which would lead to the development and testing of general principles in order to improve transfer of learning, arising from understanding.

Our system is designed to help learners search for abstract problem-solving principles and methods by forming analogies between different task situations. We provided ***comprehensive and diverse practice.*** Two different categories of exercises, i.e. pattern recognition and question-posing exercises (fill in the blanks (self) and peer-teaching) would be randomly generated. Each contains five levels of complexity. Pattern recognition aims at affording ***explicit abstraction***, while question-posing to create ***mindfulness*** and ***active self-monitoring***. Pattern recognition is sequenced before pattern description as students need to recognize patterns in order to generalize principles. There are two levels to the question-posing exercises, i.e., fill in the blanks (self) and peer-teaching. Students need to achieve of level of mastery of 80% right for the current level before they can proceed to the next level. We hoped that students would be engaged in investigations not only in learning content but also discipline-specific reasoning skills and practices collaboratively, by constructing and testing their own mathematical models. Ultimately, we hope to make learning real - to enable learners to experience what it means to contribute to social-technological development through Mathematics and Mathematical thinking.

Our interview data show that our instructor practices principle-based inquiry. He stresses on teaching based on systems thinking in order to help learners to associate concepts and methodologies/processes. Furthermore, he emphasizes that Mathematics learning is not only about the computations but more of Mathematical thinking, i.e., teaching the *why* underlying the *what* and *how* by using different cases and strategies.

The epistemology behind our system design is congruent with his beliefs. Therefore, he accepts the system and can identify the benefits that we hoped for, i.e. to inculcate Mathematical thinking and motivation to learn further. He could also identify that we are attempting to highlight cognitive *salience* of similarities among instances be they task features, problem structures, solution methods or relations between different contexts of increasing complexity. He was thus positive regarding the potential learning outcomes if the system were to be implemented in the actual classroom.

We conclude with implications arising from this study and our future work. To foster development of big ideas and principles, a possible means that can be explored is Lee's (2012b) suggestion that teachers be trained to teach based on pedagogical schema and ontological connections. Focusing training on these two factors is likely to help teachers determine identify topical and pedagogical similarity. This may result in more effective ontological mapping and experimentation in the adoption, adaptation and scaling of teaching-learning strategies.

To inculcate creative teaching-learning approaches, Lee (2012a) suggests that we should provide for variations in teaching-learning strategies among behaviorist-cognitivist-constructivist approaches to enhance learning experiences. The choice of approach depends on the learning needs and the instructor's beliefs. She suggests regarding learning approaches as testbeds for variable experimentation along three dimensions, i.e., creative types (exploratory creativity, combinatory creativity and adaptive creativity), creative processes and learning approach. Lee (2012b) further suggests that teachers be trained to teach based on pedagogical schema and ontological connections. She hypothesizes that the ability to identify topical and pedagogical similarity may result in more effective ontological mapping, foster development of big ideas and principles, and experimentation in the adoption, adaptation and scaling of teaching-learning strategies.

Furthermore, consistent with inquiry-based epistemology, experiments are regarded as the essence of meaningful and fun teaching and learning. Hence, we suggest regarding learning approaches as testbeds for variable experimentation along three dimensions, i.e., creative types (exploratory creativity, combinatory creativity and adaptive creativity), creative processes and learning approach. Beginning with the design of technology-mediated learning activities and content along these three dimensions will create a viable testbed. From this study, we add that open-ended challenges should be regarded as embedded assessment. These preliminary findings will help us will design and experiment with different instructional strategies in our future work.

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