Liu, C.-C. et al. (Eds.) (2014). Proceedings of the 22nd International Conference on Computers in Education. Japan: Asia-Pacific Society for Computers in Education

Practical Use of Interactive Environment for Learning by Problem-posing for One-step Multiplication and Division Word Problems

Sho YAMAMOTO[a*], Yuki AKAO[a], Mitsutaka MUROTSU[a], Takehiro KANBE[a], Yuta YOSHIDA[a], Kazushige MAEDA[b], Yusuke HAYASHI[a] & Tsukasa HIRASHIMA[a]
[a]*Graduate School of Engineering, Hiroshima University, Japan*
[b]*Elementary School Attached to Hiroshima University, Japan*
*sho@lel.hiroshima-u.ac.jp

Abstract: Problem-posing is known that effective method for promoting to master the use of solution methods. However, these methods have been rarely used because of the activity for posing problem and assessment the posed problem. So, we design and develop an interactive environment for learning by problem-posing continually. In previous research, we have already designed and developed the learning environment targeting one-step addition or subtraction word problems and one-step multiplication word problems. In this paper, we have designed and developed the learning environment targeting one-step multiplication or division word problem and its assignment newly. For realizing this system, the property of quantity and its relation is suggested as problem structure. And the diagnosis and feedback of posed problem are defined based on the property and relation. The level of assignment is defined too. Developed learning environment and its practical use in an elementary school are reported.

Keywords: Problem-posing, sentence-integration, multiplication word problems, division word problems, problem structure

1. Introduction

Learning by problem-posing is well known as effective method for promoting learners to master the use of solution methods (Polya, 1945; NF Ellerton, 1986; Silver, CAI, 1996). Moreover, it has been proposed that poor problem solvers often fail to elicit problem structures from problem (Brown, VanLehn, 1980; Mayer, 1982; Kintsch, Greeno, 1985). So, it is postulate that learning by problem-posing is effective for learner to promote to acquire the problem structure. However, this exercise also known that it is difficult to perform because of the cost of activity for posing problem and assessment the posed problem. Therefore, we design and develop the learning environment which learners acquiring the structure of arithmetic word problem by exercising the problem-posing continually (Nakano, et al, 1999; Hirashima, et al, 2007; Hirashima, et al, 2011). This learning environment is required a learner to pose a problem by selecting three cards from a set of given sentence cards and arranging them in proper order (Hirashima, et al, 2014). Also, the learning environment can generate feedback about posed problem. We call this learning environment as "MONSAKUN".

Until now, one-step addition or subtraction word problems and multiplication word problems are analyzed, and the structures of these problems are implemented on tablet PC for realizing the problem-posing exercise and an assessment of posed problem (Yamamoto, et al, 2012; Yamamoto, et al, 2013). MONSAKUN consists of MONSAKUN Touch and MONSKAUN Analyzer. By using this environment, a learner can exercise the problem-posing on MONSAKUN Touch, and a teacher can confirm the result of learner's problem-posing on MONSAKUN Analyzer via network. For this implementation, teacher can use our learning environment in their arithmetic class and lecture the arithmetic word problem by the problem-posing. Actually, in addition to the development of the learning environment, we have performed two experimental uses with elementary school teacher.

First experimental use is, which first grade students were used by MONSAKUN Touch for posing problem that can be solved by one-step addition or subtraction word problems because first grade students have just learned one-step addition or subtraction in arithmetic class. In second experimental use, MONSAKUN Touch 2 is used by the second grade students, which for posing problem that can be solved by one-step multiplication word problems because second grade students have just learned one-step multiplication in arithmetic class. The results of these experimental uses have proposed that not only the learner improve the problem solving performance, but also this learning environment was effective for the learner who can't judge the problem structure to acquire it.

Based on these researches, we have designed and developed a learning environment for posing problems that can be solved by one-step multiplication or division. Since third grade students learn not only one-step multiplication word problems but also one-step division word problems in contrast to second grade students, these scopes are targeted in our research as next step. In order to realize this, the design of a model of problem-posing and an assignment of problem-posing for the learning environment based on analysis of targeted problem should be performed. In this paper, a problem structure is explained in the following chapter. A design of developed learning environment based on this structure is described in section 3. A sequence of assignment is also expressed. Subsequently, a procedure of its practical use and an analysis of the results are reported.

2. Problem Structure of One-step Multiplication or Division Word Problem

In this section, the model of one-step multiplication or division arithmetic word problem is explained. One-step word problems can be expressed by three sentences in our research. Example is shown in Figure 1. Because there are three values in one-step arithmetic word problem, this problem can be expressed by three sentences. These sentences consist of two sentences mean existence and one sentence means relation between other two values. We call each sentence as existence sentence and relation sentence. In this example, "There are three boxes" and "There are several apples" are existence sentence. "There are four apples in each box" is relation sentence because this sentence shows the relation between the apple and box. These sentences consist of value, object and predicate.

In addition to the kind of sentence, multiplication and division word problem have a property of quantity (Yamamoto, et al, 2013). Generally, multiplication is expressed by “multiplicand multiplied by multiplier is product” (Greer, 1992; Vergnaud, 1983; Davies, 1841). In other words, each quantity has different property. In Figure 1, multiplicand is expressed as "There are five apples in each box", multiplier is "There are three boxes" and product is "There are several apples". That problem contains the story that the value of apples is expressed as the amount of apple when there are three boxes and the value of apples in each box is basis. Since, in Japanese Education, multiplicand is also called "base quantity", multiplier is "proportion" and product is "compared quantity". Then, the arithmetic word problems that can be solved by one-step multiplication or division has three types of story. This is, (1) Compared quantity divided by base quantity is proportion, (2) Base quantity multiplied by proportion is compared quantity, (3) Compered quantity divided by proportion is base quantity. The story of the problem in Figure 1 is (2).

All of these stories contain the relation that is “Base quantity multiplied by proportion is compared quantity”. One-step multiplication or division word problems are expressed by changing the one quantity to required value in each story. Therefore, it is important to extract the base quantity, proportion and compared quantity from problem and to make the relation between these quantities with “Base quantity multiplied by proportion is compared quantity”. Next section, the implementation of the problem structure mentioned above to tablet PC is described.

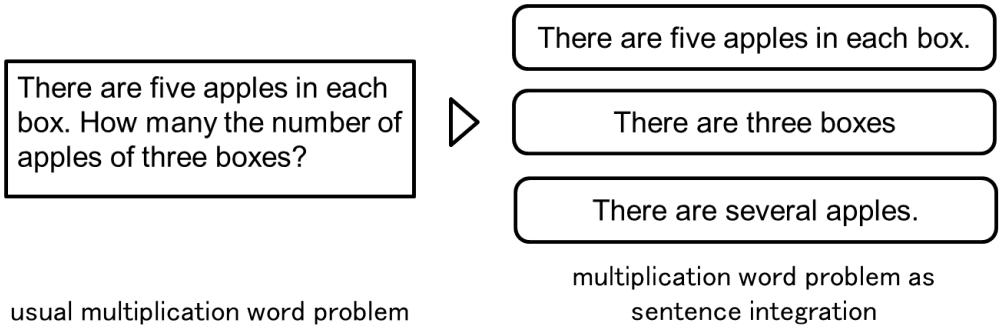

Figure 1. Example of Problem Expression as Sentence Integration.

3. Learning Environment for Problem-posing "MONSAKUN Touch3"

3.1 Framework of Learning Environment

This learning environment consists of MONSAKUN Touch 3 for learners and MONSAKUN Analyzer 3 for teachers. A result of the learner's learning by problem-posing on MONSAKUN Touch 3 is sent to database server via network. This framework is shown in Figure 2. MONSAKUN Touch 3 developed by using Android, MONSAKUN Analyzer 3 by using PHP and JavaScript. The each software can be run on Android Tablet. RDBMS is used MySQL. The teacher can confirm the graph of learner's learning by using MONSAKUN Analyzer 3 that receives a learning data from database server. The learning data are saved as three data: the number of correct problem, the number of incorrect problem, the number of the each incorrectness and the learner's log. Category of incorrectness is explained in next section. MONSAKUN Analyzer 3 generates some graphs by using these data and displays teacher it. Teacher can limit to an assignment that learners can exercise on MONSAKUN Touch 3 by using MONSAKUN Analyzer 3.

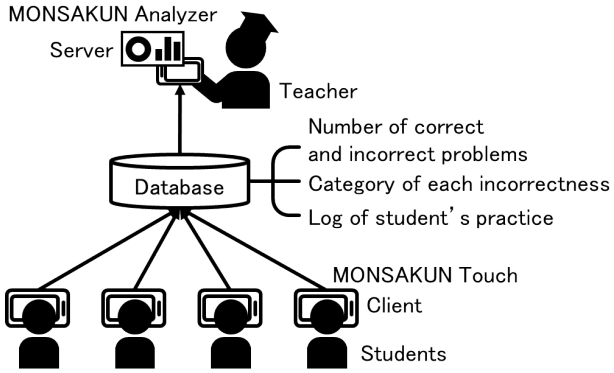

Figure 2. The Framework of Learning Environment.

3.2 MONSAKUN Touch 3

3.2.1 Flow by Using MONSAKUN Touch 3

In this section, the flow of problem-posing by using MONSAKUN Touch 3 is described. This is same as the MONSAKUN Touch 1 and 2 but an interface of problem-posing is a little changed. First, a learner logs in this system by selecting his/her grade, class and number. After that, the learner selects a level of assignment on an interface for selecting level. The learner is also able to select particular assignment and switch feedback on or off. The level of assignment is elaborated in 3.2.3. After selecting the level, he/she sees an interface of problem-posing is shown in Figure 3. This interface presents the assignment for posing problem, the set of given sentence card and three blank for arranging given sentence cards. Sentence cards are written sentence like "There are three apples".

These sentence cards were mentioned in previous section. Hence the learner can pose the problem by selecting three sentence cards from given cards and arranging them in proper order. Given sentence cards are consists of correct card set and dummy card set for leading to errors. In the MONSAKUN Touch 3, the text means the property of quantity is shown in the left side of each blank because the environment lets the learner consider the property of each sentence card when they learn by using this environment. If three blank is filled with three sentence cards, diagnosis button will be active. Then, the learner can tap this button and the system diagnoses and feed back his/her posed problem. When the learner finishes answering all assignment in selected level correctly, the interface for posing problem backs to the interface for selecting level.

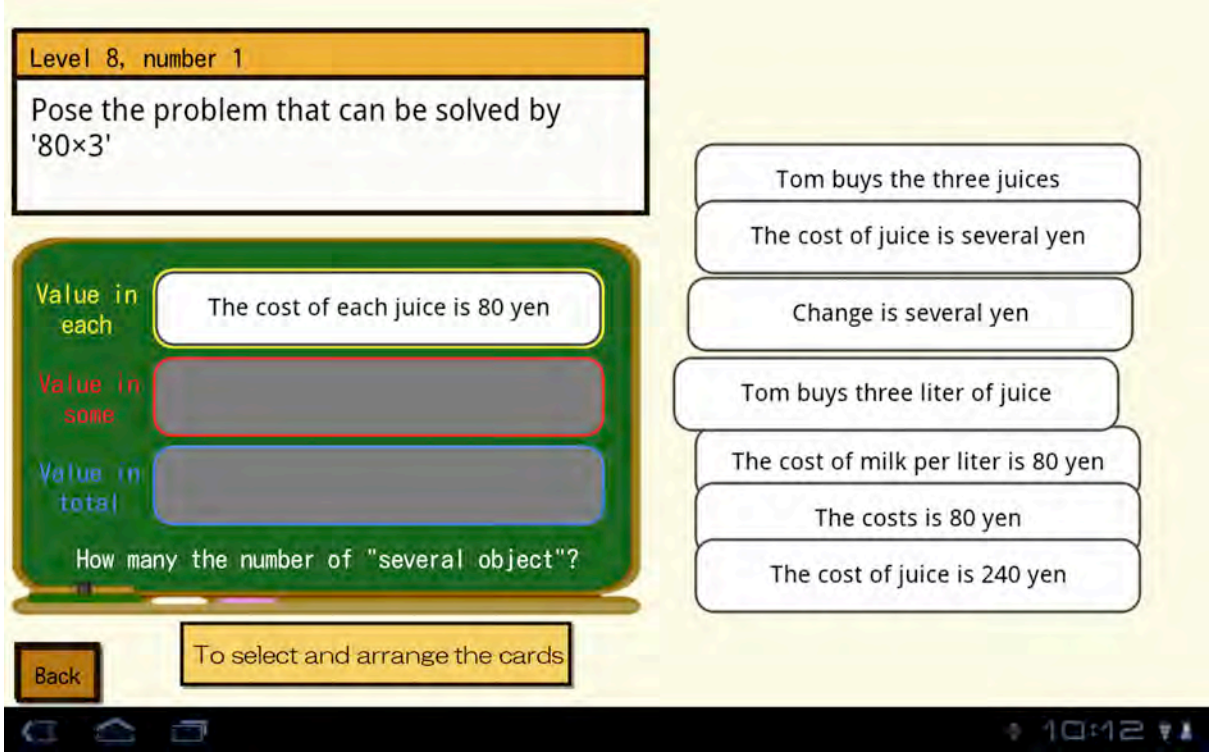

Figure 3. Main Interface of MONSAKUN Touch 3.

3.2.2 Diagnosis and Dummy Card

Figure 4 is the procedure for diagnosing of posed problem. This procedure is processed based on the problem structure is described in chapter 2. The posed problem is required to satisfy these several constraint. First, MONSAKUN Touch 3 assesses the composition of sentence. This is, the combination of existence sentences and relation sentence, and the setting cards and its property of quantities. If these are not correct, MONSAKUN Touch 3 gives a learner feedback it. If these are correct, next, MONSAKUN Touch 3 assesses whether a composition of story is correct or not. This means that the system checks the relation of each object, value, unit in answered three sentence cards. In addition to this, the learning environment assesses not only the relation between two objects of base quantity and object of other sentence cards but also the base value of base quantity like “apple per 2 boxes”. After that, MONSAKUN Touch 3 assesses whether the calculation expression of posed problem and given calculation by assignment is same or not. If learner causes the error, the system feedbacks its reason based on diagnosis in Figure 4. When posed problem satisfies the above constraint, this problem is diagnosed as correct.

Dummy card is included in given sentence card in order to let the learner cause the error that is show in Figure 4. These cards make by changing object, number or predicate of correct card. Because of dummy cards, the learner needs to be vividly aware of problem structure when they posed problem on MONSAKUN Touch 3.

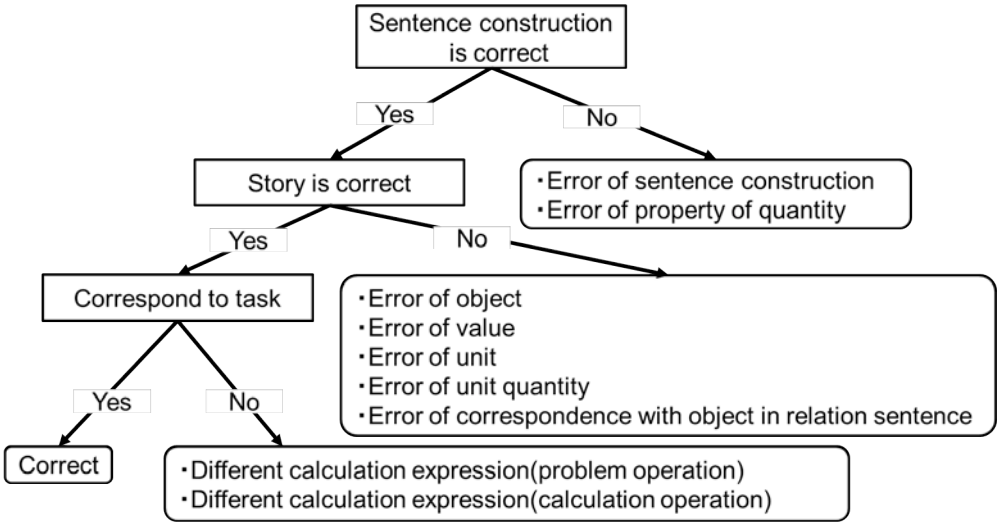

Figure 4. The Procedure for Diagnosing the Posed Problem.

3.2.3 Designing the Level of Assignment

In this research, we have designed the level of assignment gradually so that the learner acquires the problem structure seamlessly. Table 1 shows the all level of assignment by dividing into the number of level, assignment, required activity, contents of assignment and number of assignment. Then, each level designed on the basis of "Base quantity multiplied by proportion is compared quantity" for third grade students on elementary school. The learner is required to pose the story from level 2 to 7, to pose the problem from level 8 to 9.

In level 1, the learner is given the story of one-step multiplication and four calculation expression which are expressed by "Base quantity multiplied by proportion is compared quantity", "Proportion multiplied by base quantity is compared quantity" and the cumulation of same number like “4+4+4=12” and “3+3+3+3=12”. This assignment is the confirmation of the relation between multiplication and addition. Then, the learner is required to select the correct calculation expression. The purpose of this level is which let the learner comprehend the relation of multiplication story and addition calculation. The learning environment gives the story and several sentence cards to the learner in level 2. The given story as sentence integration consists of two fixed sentence cards and one blank. The learner is required to fill this blank by considering the property of quantity. In this assignment, they learn the property of quantity that is contained each given sentence card. Given sentence cards in level 3 are included two sentence cards that have different text representation and same property. For example, "There are two boxes." and "The number of box is two.". In this level, let the learner learn that the sentence cards include the same property of quantity have various text representation. MONSAKUN Touch 3 presents the three blank for putting the sentence cards and several sentence cards in level 4. Then, the learner is required to pose the story by selecting three sentence cards and by arranging them in proper order based on the relation of "Base quantity multiplied by proportion is compared quantity". The assignment of level 5 requires the learner to pose the two stories by using one common sentence card. For example, "There are three apples in each box. There are six boxes. There are eighteen apples." and "There are two boxes in each shelf. There are three shelves. There are six boxes" are used same card that is "There are six boxes.". Through this exercise, the learner comprehends that existence sentence card is able to have two property of quantity. In other words, both proportion and compared quantity are expressed by existence sentence. After that, in level 6, the learner learns that the story has three kinds of calculations expression that are mentioned in section 2. This purpose is that the learner notices the multiplication story contains the calculation (a) and (c). Thus, the learner is given the multiplication and division calculation expression as assignment for posing story. In order to let the learner confirm three properties of quantity and its relation again, assignments of level 7 includes improper assignment which cannot solve because of lack of one proper sentence card. Then, the learner is given a specific sentence card for posing the story in this level, which is labeled "proper sentence card is not given" instead of lacking sentence card. Because the assignments in level 7 are composed of usual assignment and

assignment which mentioned above, the learner is required to consider each property of quantity and its relation again. As the next step, the learner is required to pose problem in level 8 because the learner learn to pose the story through level 2 to 7. Finally, in level 9, the learner is required to pose the two problems by using one common sentence card. This assignment is same as assignment of level 5. Through the exercise from level 1 to 9, the learner can acquire the problem structure gradually.

Table 1: The Assignment Level on MONSAKUN Touch 3.

Level	Required activity	Contents of assignment	Number of assignment
1	Select calculation expression	Select calculation express given story	12
2	Pose story	Pose story that is expressed by given calculation (one-step multiplication) Required story has already given two sentence cards	12
3	Pose story	Same as assignment of level 2 Include same property and different text representation	12
4	Pose story	Pose story that is expressed by given calculation (one-step multiplication) Select three sentence cards and arrange them	10
5	Pose story	Pose two stories by using same sentence card	10
6	Pose story	Pose story that is expressed by given calculation But given calculation expression is one-step multiplication or division	12
7	Pose story	Same as assignment of level 6 But one proper sentence card is not given	12
8	Pose problem	Pose problem that is expressed by given calculation Select three sentence cards and arrange them	12
9	Pose problem	Pose two problems by using same sentence card	12

3.3 MONSAKUN Analyzer 3

Here, MONSAKUN Analyzer 3 for visualizing the learner's learning data on MONSAKUN Touch 3 is explained in line with the function of MONSAKUN Analyzer 3. After the teacher loged in the learning environment by inputting id and password, MONSAKUN Analyzer 3 changes the interface that is shown in Figure 5. In this interface, the learning environment displays the average of student's learning data in each lesson that are received some learning data from database server. This interface generates and shows a three bar charts and a doughnut chart. Three bar charts consist of the average number of posed problem in total, the average number of correct problem and the average number of incorrect problem. A doughnut chart shows the rate of each error that category is classified in 3.2.2. In addition to this information, MONSAKUN Analyzer 3 indicates the average progress of the level and assignment number in class. These displayed learning data can be filtered out based on the each level and assignment in each class. MONSAKUN Analyzer 3 displays these graphs not only in each lesson but also in each student by clicking the link "see the each student's data". The interface element is same as Figure 4 but each graph are visualized in each student's learning data in total. This interface also only displays the learning data in each level or each assignment. Then, the teacher can see the posed problem of each student by clicking "see the each student's log". These data are updated in real-time. The teacher is able to arrange his/her lesson on the basis of these visualized data. For example, by seeing the learning data in each student, the teacher can know the students who failure in the exercise of MONSAKUN Touch 3 and support their exercise.

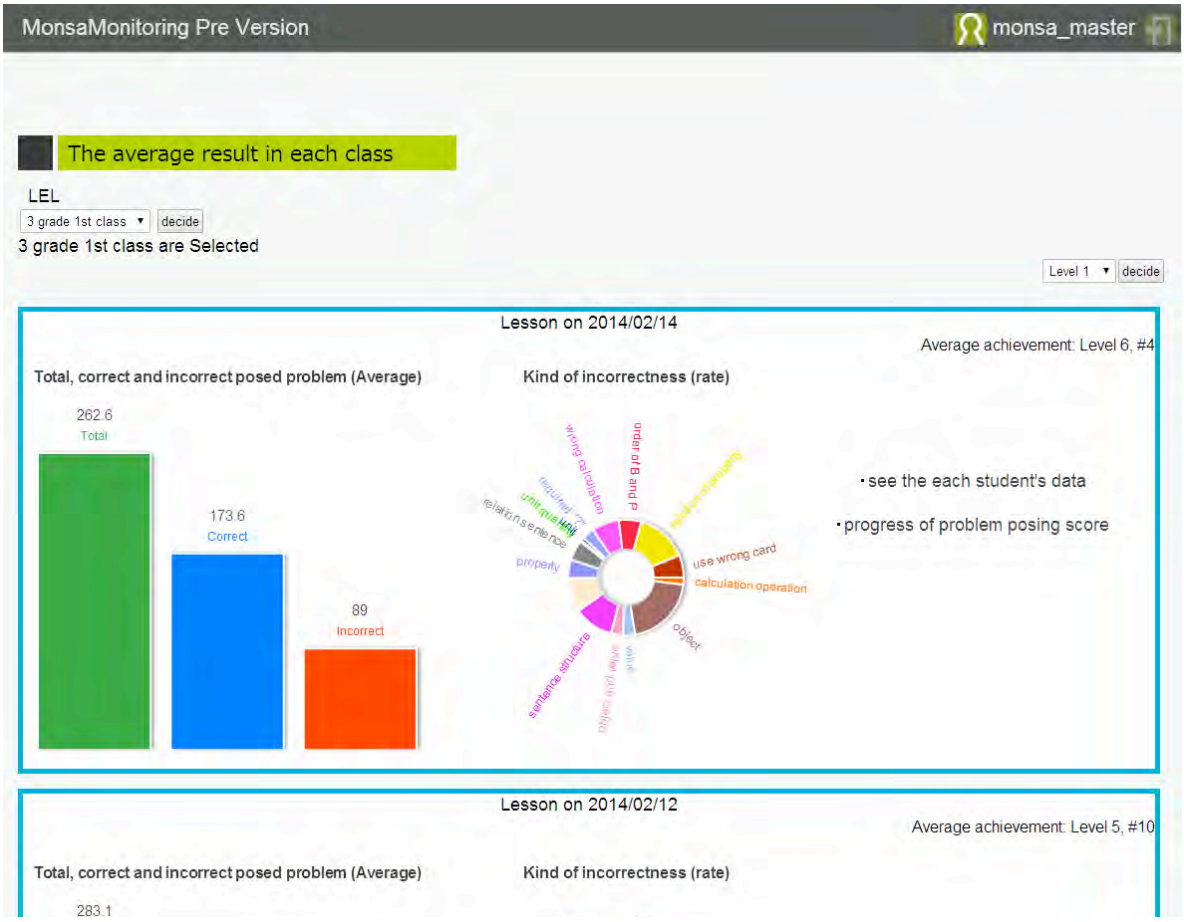

Figure 5. The Main Interface of MONSAKUN Analyzer 3.

4. Experimental Use of MONSAKUN Touch 3

4.1 Procedure of Experimental Use

Subjects were thirty-nine students in the third grade of an elementary school. They were divided into subjects who experienced MONSAKUN and who did not experience it in previous experimental use of our research (Yamamoto, et al, 2012; Yamamoto, et al, 2013). Inexperienced group of MONSAKUN has learned one-step addition, subtraction or multiplication word problem by usual lesson only. Moreover, they had just learned to solve arithmetic word problems that can be solved by one-step multiplication or division. This experimental use has been performed during thirteen lessons that consist of pretest in one lesson, eleven lessons by using MONSAKUN and posttest in one lesson (45 minutes per lesson, in 5 weeks). A lesson by using MONSAKUN has composed of teaching about problem-posing by a teacher and problem-posing exercise by using MONSAKUN Touch 3. The teacher decided the time of using MONSAKUN Touch 3 based on the progress of each lesson. If the subjects have finished twice the current level when they exercise the problem-posing after teaching, they were allowed to work on the previous level. The purpose of this experimental use is to examine the effects of MONSAKUN Touch 3 by using a usual problem solving test, an extraneous problem solving test and a problem-posing test, and the effects of experience MONSAKUN is also examined.

We used these three tests: the problem solving test, the extraneous problem solving test and the problem-posing test. The problem solving test is the usual problem solving test can be solved by one-step multiplication or division that is expressed by three sentences. This test is included five stories that are made as the permutation of base quantity, proportion and compared quantity without "Proportion multiplied by base quantity is compared quantity" because of commutative law. Therefore, the usual problem-posing test has fifteen questions because each quantity can be the required value in these five stories. Extraneous problem solving test includes extraneous information

that is not necessary to solve the problem. It is more difficult for learner to solve the extraneous problem than to solve the usual problem solving (Muth, 1992). The subjects are required to judge the relevance of each sentence and find the sentence including as the extraneous information for solving the problem. Therefore, the extraneous problem solving test is useful to assess leaner's comprehension of the problem structure. These problems consists of twelve problems that including the two kinds of extraneous information that change sentence cards except sentence contains required value in each six stories. The problem-posing test examines the problem-posing performance to let the subject pose the problem as he/she can within the time limit. The subject pose problem from scratch. The time limit is ten minutes in each test. The difference between pretest and posttest is order of each problem.

4.2 Analysis of Pretest and Posttest

An analysis of pretest and posttest are reported in this section. And the level by using lecture is described. The teacher has performed the lecture based on the level on MONSAKUN Touch 3 and treated one level in one lecture. However, the subjects can not relate between multiplication calculation expression and text representation contain "cut" because "cut" is associated with division calculation expression. Thus, the teacher has to spend three lessons for resolving this difficulty. The lessons have been performed from sixth lesson for level 4 continually and the subjects have worked on level 9 in eleventh lesson.

The result of average score and SD in usual problem solving, extraneous problem solving and problem-posing test are shown in Table 2. These scores are divided into experienced and inexperienced group of MONSAKUN that the subjects learn by problem-posing in the scope of one-step addition, subtraction or multiplication word problem. In addition to this result, the results of ANOVA in each test are shown in Table 3. There was an interaction in the score of usual problem-posing test between experience of MONSAKUN and pre-posttest (p=.03). So, we analyzed simple effect. There was a significant difference in the score of posttest between experienced group and inexperienced group (F(1, 36)=3.193, p=.008). This result suggested that it is effective for the subjects to experience the learning by using MONSAKUN for improving their usual problem solving performance. Next, there was a significant difference in the score of extraneous problem solving test between experienced group and inexperienced group (p=.04), and effect size is medium ($|\eta^2|$=.10). Also, there was a significant difference in the score between pretest and posttest (p=.02), and effect size is small ($|\eta^2|$=.02). In addition to this analysis in the score of extraneous problem solving test, we analyzed the correlation between the pretest score and the difference posttest and posttest score. In this result, there are a negative correlation between them (Spearman's rank-correlation coefficient, $|rs|$=.59, p=2.5E-06). These results suggested that the lesson by using our learning environment promote the subjects to improve their problem structure, in particular, more effective to the subjects who the score of extraneous problem solving test is lower. MONSAKUN is more effective for the subjects who have experienced MONSAKUN to comprehend the problem structure particularly. This result same as the result of experimental use by MONSAKUN Touch 2 (Yamamoto, et al, 2013) so this effect is that the main effect of learning by problem-posing on MONSAKUN Touch. Last, there was no significant difference in the number of posed problem between experienced and inexperienced group. But, there was a significant difference between pretest and posttest (p=.005), and effect size is medium ($|\eta^2|$=.07). These results suggested that MONSAKUN is useful for the subjects to improve their problem-posing performance regardless of whether the subjects have experienced MONSAKUN.

Table 2: Result of Each Pretest and Posttest in experienced group (N=18) and inexperienced (N=20).

Test	Experience of MONSAKUN	Pretest		Posttest	
		M	SD	M	SD
Problem-posing	experienced	2.50	1.57	3.56	1.42
	inexperienced	2.10	1.45	2.7	1.52
Usual Problem solving	experienced	13.61	1.34	14.28	0.80
	inexperienced	13.30	1.52	12.75	2.05
Extraneous Problem Solving	experienced	10.83	2.14	11.38	0.76
	inexperienced	8.80	3.54	9.75	3.40

Table 3: Two factor ANOVA of Each Pretest and Posttest.

(a) Result of the score of usual problem solving test

factor	SS	df	MS	F	
experienced × inexperienced group	16.02	1	16.01	4.56	*
pre × post-test	0.06	1	0.06	0.05	n.s.
interaction	7.01	1	7.01	5.32	*
total variation	196.88	75			

(b) Result of the score of extraneous problem solving test

factor	SS	df	MS	F	
experienced × inexperienced group	63.88	1	63.88	4.54	*
pre × post-test	10.74	1	10.74	5.63	*
interaction	0.74	1	0.74	0.39	n.s.
total variation	651.41	75			

(c) Result of the score of problem-posing test

factor	SS	df	MS	F	
experienced × inexperienced group	7.47	1	7.47	2.28	n.s.
pre × post-test	12.98	1	12.98	9.19	**
interaction	0.98	1	0.98	0.70	n.s.
total variation	190.04	75			

* $p<.05$, ** $p<.01$

5. Conclusion

In this paper, we have described the model of problem and problem-posing in one-step multiplication or division arithmetic word problem, the development of the interactive environment for problem-posing based on the model, and the results of its practical use. We analyze the problem structure as processing information and realize the development of interactive environment for learning by problem-posing based on its problem structure continually. This learning environment called MONSAKUN Touch. Until now, MONSAKUN Touch is introduced by elementary school teacher into first and second grade arithmetic class on an elementary school for practical use. MONSAKUN Touch is developed for learning by problem-posing in the scope of one-step addition, subtraction or multiplication arithmetic word problem. Therefore, as the next step, we designed and developed the learning environment by posing problem that can be solved by one-step multiplication or division word problem. In order to realize this system, firstly, we have mentioned that three quantities and its relation define one-step multiplication or division word problem. These three quantities are called base quantity, proportion and compared quantity. Its relations are "Base quantity multiplied by proportion is compared quantity". At the second step, the problem-posing based on this problem structure and the diagnosis and feedback of posed problem are defined. The levels of assignment are designed by this problem structure as the learner can acquire the problem structure gradually. After that, we have developed learning environment for problem-posing as sentence integration. This environment consists of MONSAKUN Touch 3 and MONSAKUN Analyzer 3. The learner can exercise the problem-posing on MONSAKUN Touch 3 and MONSAKUN Analyzer 3 provides the visualized student's learning data on MONSAKUN Touch 3 to the teacher. Lastly, an eleven lesson experimental use is reported. The results of brief analysis suggested that the third grade students who have learned by using MONSAKUN in the past time are improved their problem solving performance and sophisticated their acquired problem structure. In addition to this result, the third grade students who didn't acquire the problem structure well are improved their problem-posing

and sophisticated their acquired problem structure. This result same as the result of experimental use by MONSAKUN Touch 2 so it is suggested that this result is main effect of learning by problem-posing on MONSAKUN Touch.

As our future works, we need to verify the quantity of the effect to high group by using MONSAKUN Touch 3. Furthermore, we should perform the practical use for problem-posing in one-step addition, subtraction, multiplication or division to fourth grade students of an elementary school continuously.

References

Brown, J.S., VanLehn, K. (1980). Repair theory: A generative theory of bugs in procedural skills. Cognitive Science, 4, 379-426.

Davies, C. (1841). Arithmetic: Designed for Academies and Schools. A.S. Barners.

E.A. Silver, J. CAI. (1996). An analysis of arithmetic problem posing by middle school students. Journal for Research in Mathematics Education, 27(5), 521-539.

Greer, B. (1992). Multiplication and Division as Models of Situations. Handbook of Research on Mathematics Teaching and Learning, 276-295.

Hirashima, T., Kurayama, M. (2011). Learning by Problem-Posing for Reverse-Thinking Problems, Proc. of AIED 2011, 123-130.

Hirashima, T., Yamamoto, S., & Hayashi, Y. (2014). Triplet Structure Model of Arithmetical Word Problems for Learning by Problem-posing. In Human Interface and the Management of Information. Information and Knowledge in Applications and Services (pp. 42-50). Springer International Publishing.

Hirashima, T., Yokoyama, T. Okamoto, M. ,Takeuchi, A. (2007). Learning by Problem-posing as Sentence-Integration and Experimental Use. Proc. of AIED2007, 254-261.

Kintsch, W., Greeno, J.G. (1985). Understanding and Solving Word Arithmetic Problem. Psychological Review, 92(1), 109-129.

Mayer, R. E. (1982). Memory for algebra story problem. Journal of Educational Psychology, 74, 199-216.

Muth K.D. (1992). Extraneous information and extra steps in arithmetic word problems, Contemporary educational psychology. Vol. 17, pp.278-285.

Nakano, A., Hirashima, T., Takeuchi, A. (1999). Problem-Making Practice to Master Solution-Methods in Intelligent Learning Environment. Proc. of ICCE'99, 891-898.

NF Ellerton. (1986). Children's made-up mathematics problems: a new perspective on talented mathematicians. Educational Studies in Mathematics, Vol.17, 261-271.

Polya, G. (1945). How to Solve It. Princeton University Press.

Vergnaud, G. (1983). Multiplicative Structures. Acquisition of mathematics concepts and processes, 127-174.

Yamamoto, S., Kanbe, T., Yoshida, Y., Maeda, K., Hirashima, T. (2012). A Case Study of Learning by Problem-Posing in Introductory Phase of Arithmetic Word Problems. Proc. of ICCE2012, Main Conference E-Book, 25-32.

Yamamoto, S., Takuya H., Kanbe, T., Yoshida, Y., Maeda, K., Hirashima, T. (2013). Interactive Environment for Learning by Problem-Posing of Arithmetic Word Problems Solved by One-step Multiplication, Proc. of ICCE2013, Main Conference E-Book, 51-60.