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Revealing Students' Thinking Process in Problem-Posing Exercises: Analysis of First Sentence Selection

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Abstract: We have developed a computer-based learning environment called MONSAKUN to realize learning by problem-posing, where students select and arrange several sentence cards to pose arithmetical word problems. We call this type of problem-posing "sentence integration". As the next step of MONSAKUN development, we have been analyzing the sentence selection process which is considered to reflect students' thinking process. In the first step of analysis, we focused on the first sentence card selected in the process of posing a problem. We found that the selection changed based on different type of approach, type of story and students' exercise experience. This result is an important step towards building elaborate process model of problem-posing and adaptive support of the process.

Keywords: Arithmetical word problems, learning analytics, problem posing, reverse thinking problem, sentence integration

1. Introduction

1.1 Background

Two activities that have been identified to be central themes in mathematics education are problem posing and problem solving. Problem solving practice, as the most popular way of teaching the solution method, has been long integrated into school mathematics (Stanic & Kilpatrick, 1988). Problem posing practice involves the generation of new problems in addition to solving pre-formulated problems (English, 1997; Silver & Cai, 1996). Although learning by problem posing has been suggested as an important way to promote learner's understanding (Ellerton, 1986; Polya, 1957), it was not until recently that the recommendations for the reform in mathematics education suggested the problem posing inclusion in students' activities (NCTM, 2000). Several investigations of various aspects of problem posing activities have been conducted as more educators and researchers realized its importance in mathematics education (English, 1998; English, 2003).

One of the most important issues in learning by problem posing is the way to assess and give feedback to posed problems. In traditional problem posing method, teachers and students were faced with difficulties to conduct the learning activities effectively. It is not easy for students to pose mathematically correct problems in a given time, especially students in lower grade of elementary school. Teachers were having problems to assess and give feedback to the wide variation of problems that students pose in a given time of class activity. The inefficiency of time and available method made problem posing activity less attractive for most mathematics educators.

In order to realize learning by problem-posing in a practical way, we have been investigating a computer-based learning environment to assess and give feedback to problems posed by students (Nakano, Hirashima & Takeuchi, 1999; Hirashima, Nakano & Takeuchi, 2000; Nakano, Hirashima & Takeuchi, 2002; Hirashima et al, 2007; Kurayama & Hirashima, 2010). The software, named MONSAKUN (means "Problem-posing Boy" in Japanese), provides an interactive support for learning arithmetical word problems solved by one operation of addition/subtraction.

The interface of an assignment in MONSAKUN is explained in Figure 1. A learner is provided with a set of sentence cards and a numerical expression, and then he/she is required to pose an arithmetical word problem using the numerical expression by selecting and arranging appropriate cards. Although learners do not create their own problem statements, they are required to interpret the provided sentences and integrate them into one problem, which is essentially the same as ordinary problem-posing activity. Hirashima & Kurayama (2011) call this style as "problem-posing as sentence-integration" and assert that this integration process is an essential activity in learning.

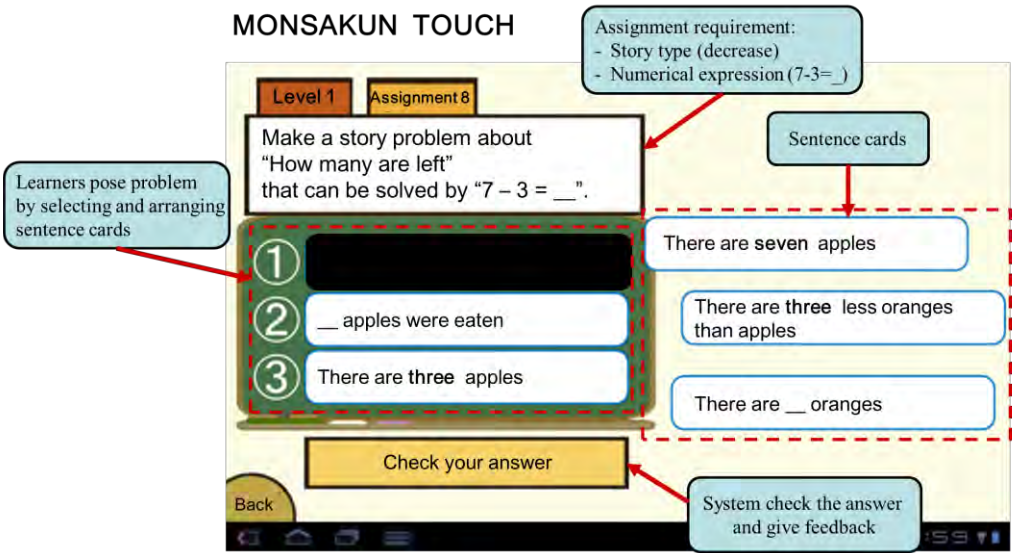

Figure 1. Interface of MONSAKUN

The practical use of MONSAKUN at several elementary schools has been reported in previous studies (Hirashima et al., 2008; Kurayama & Hirashima, 2010; Hirashima & Kurayama, 2011; Yamamoto et al., 2013). Through the analysis of pre-test and post-test of high-score group and low-score group of the students, effect of learning by problem-posing with MONSAKUN was investigated. It has been confirmed that the problem-posing exercise is effective to improve both problem-posing and problem-categorization abilities. Furthermore, after long term use of MONSAKUN in an elementary school, the result showed that both the students and teachers enjoyed using this system continuously and considered it useful for learning.

1.2 Purpose

One important direction in investigation of problem posing activities is to examine thinking processes related to problem posing (Brown & Walter, 1990). As the next step of MONSAKUN development, the purpose of this study is to examine learners' problem-posing process and to develop technologies for identifying learners' thinking process. By identifying learners' thinking process, we will be able to provide a better individualized feedback based on understanding of each learner.

Through previous practical use, we observed different ways of sentence selection in problem-posing process by the students. We assume that it is caused by the different way of thinking depending on the nature of problems and learner's understanding. Therefore, by examining the selection process of sentences, we aim to infer about a learner's thinking process in problem-posing.

While it is difficult to trace thinking process in a free problem posing activity, we can trace learners' card selection in MONSAKUN which can be considered to reflect their thinking process. Problem posing in MONSAKUN is defined as integration of provided sentences into one problem. Learner's assignment is to choose appropriate cards from several sentence cards provided by the system in order to fill the requirement of numerical expression and story type. This can be considered as search problem. Figure 2 illustrate a search space of an assignment in MONSAKUN which provides six sentence cards. The search space is a tree structure of combination of cards. Here, the root is the starting point and the numbers represent ID of cards, for example, the starting point is empty and the combination of cards 1, 2 and 3 indicates the correct answer. The nodes and arrows with bold line represents the paths of the learner's card selections during his problem posing activity. This learner committed mistakes twice and then got the correct answer. As shown in this figure, problem-posing in MONSAKUN is defined as a search problem in the structure of transition of card combinations. The rules for valid combination of the sentence cards are explained in the next section.

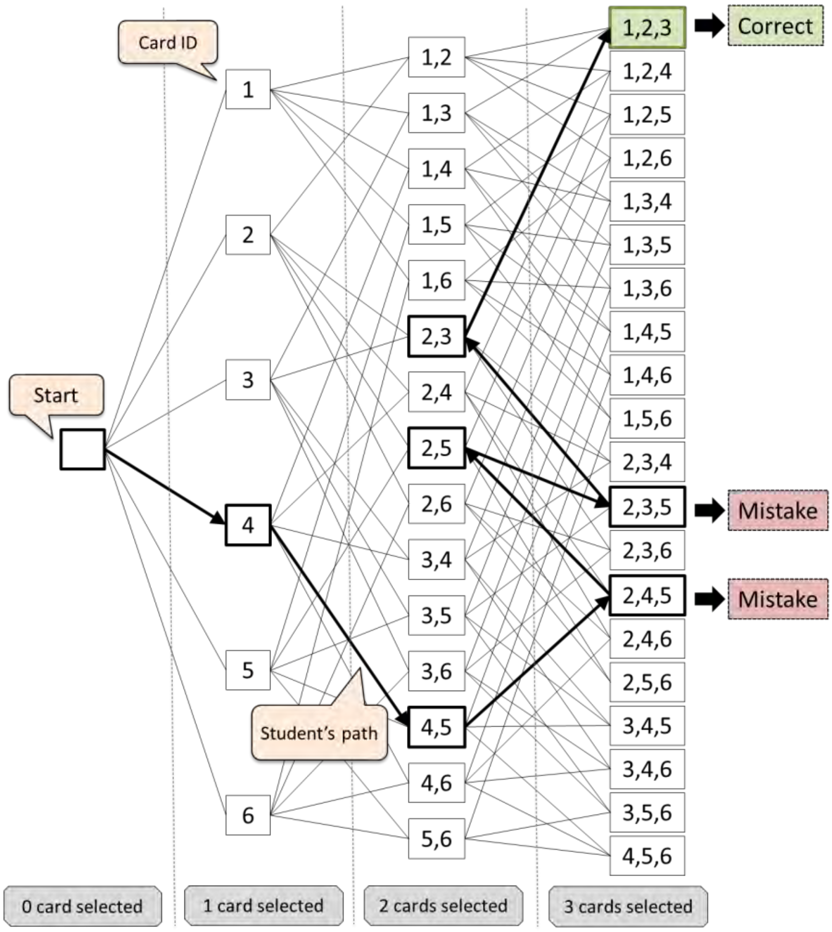

Figure 2. Example of learner's card selections shown in a graph

In this study, we examine how learners pose arithmetical word problems as sentence integration on MONSAKUN. Our assumption is learners do not choose sentence cards randomly - they arrange sentence cards based on some sort of thinking. In the analysis, as the first step toward analyzing problem-posing activity, we especially focus on what kind of sentence card was firstly selected by the learners.

The composition of this paper is as follows. The next section gives an overview of MONSAKUN and the definition of two types of problems: forward-thinking and reverse-thinking problem. Section 3 describes data about initial card selection by learners and discusses what happens in problem posing on MONSAKUN. Finally section 4 concludes this paper and shows some prospects for future study.

2. Problem Posing Activity in MONSAKUN

2.1 Categorization of Problem-posing Exercises

In arithmetical word problems, sentences are divided into two types: existence sentence and relational sentence. An *existence sentence* represents a number of single objects. A *relational sentence* contains keyword that represents a story type. An arithmetic word problem of binary operation is integration of two existence sentences and one relational sentence.

There are four types of story in arithmetic word problems of addition and subtraction: 1) combination, 2) increase, 3) decrease, and 4) comparison (Riley, Greeno and Heller, 1983). In MONSAKUN, the differences among them are defined as differences of integration of sentences. For example, a decrease story type problem is composed as follows:

a) There are seven apples *(existence sentence)*,
b) Several apples were eaten *(relational sentence that contains decrease story type)*, and
c) There are three apples *(existence sentence)*.

2.2 Forward-thinking and Reverse-thinking Problem

An arithmetical word problem includes two kinds of numerical relations: story operation structure and calculation operation structure. Story operation structure is the equation expressing the numerical relation according to the story, while calculation operation structure is the equation used to derive the required number in the assignment.

Based on this relation, there are two groups of problem in arithmetical word problems: forward-thinking problem and reverse-thinking problem. In forward-thinking problem, a story represented in the problem has the same structure with the calculation to derive the answer, while in reverse-thinking problem, the story and the calculation operation structures are different (Hirashima and Kurayama, 2011).

For example, in the following problem:

There are seven apples. Three apples were eaten. There are several apples.
How many apples are there?

Based on the sentence "Three apples were eaten", we understand that the story focuses on "decrease" number of an object. The story operation structure is "7-3=?", and the calculation structure is also "7-3(=?)", which can be found easily by reading the story in order from the first sentence. Since the two structures are the same, this type of problem can usually be solved easily by the learners.

Meanwhile, in the following problem:

There are seven apples. Several apples were eaten. There are three apples.
How many apples were eaten?

We can derive the story operation structure as "7 - _ = 3", and the calculation operation as "7 - 3 = _". Since the two structures are different, a learner is required not only to understand the story but also to derive the calculation operation structure from the story. This type of problem is called "reverse-thinking problem".

2.3 Task Model of Problem-Posing

Based on the consideration of problem types, we have proposed a task model of problem posing as sentence-integration shown in Figure 3 (Kurayama & Hirashima, 2010). There are four main tasks in problem posing activity: (1) deciding calculation operation structure, (2) deciding story operation structure, (3) deciding story structure, and (4) deciding problem sentences. A learner should complete these tasks to pose a correct problem, although the execution procedure of the tasks is not decided in the model.

In the first step of MONSAKUN, subtraction or addition is selected as a *calculation operation structure*. In the second step, a story operation structure is decided. For example, for subtraction, four *story operation structures* can be selected. Among them, only one story operation structure is the same with the calculation operation structure (subtraction), and two of them have completely different story operation, that is, addition. Because this is an abstract transformation, it is often very difficult for learners to decide.

The next task of deciding *story structure* involves selection from four types of story: combination, increase, decrease, or comparison problem. Each type of story has its own structure, as explained in the sections above.

In the last task of deciding *problem sentences*, sentences are put into the story structure following the story operation structure. This task is divided into three more tasks: deciding sentence structure, deciding concept structure and deciding number structure. Deciding *sentence structure* is to select and order sentences following the story structure. For example, if the story structure is decrease problem, learner should make a sentence structure composed of an existence sentence, a decrease type of relational sentence, then followed by another existence sentence.

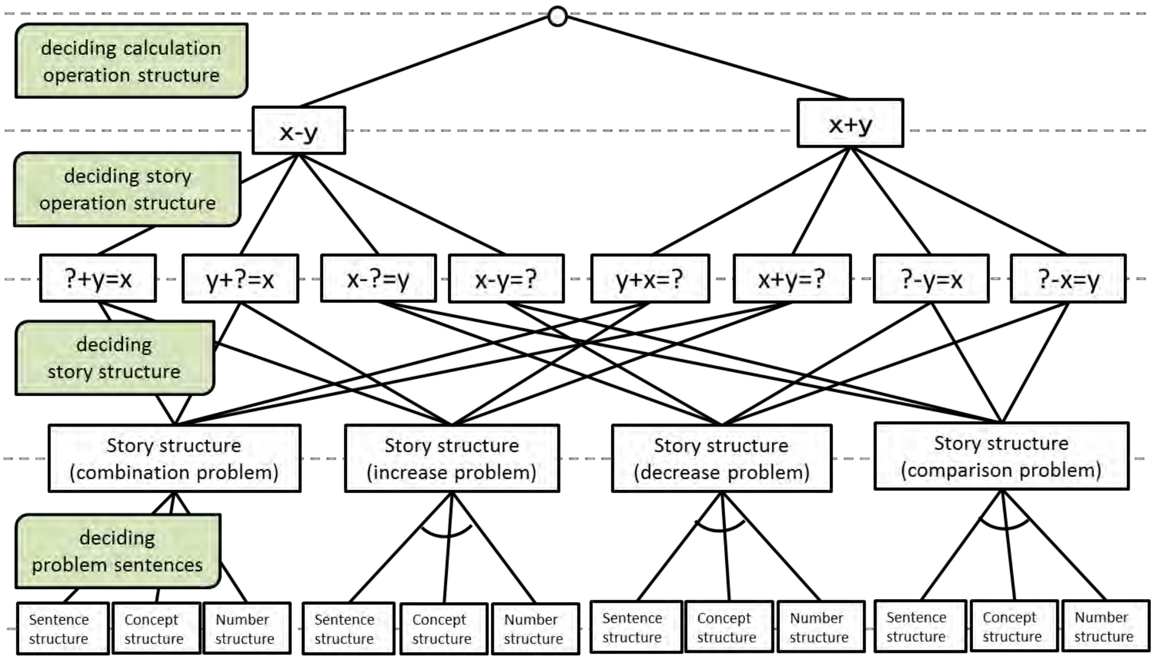

Figure 3. Task model of problem-posing as sentence integration

For the decision of *concept structure*, concepts dealt with the problem are decided. For example, if the problem requested learner to answer about the total number of apples and oranges, then the sentences should be dealt with apples and oranges as the concept. For the decision of *number structure*, the numbers dealt with the problem is decided. In arithmetic word problems, a negative number should not be used.

3. Analysis of MONSAKUN Log Data

In this section, the analysis of MONSAKUN log data from an experiment of MONSAKUN used by eleven undergraduate students from Faculty of Education is reported. Although MONSAKUN is intended for elementary school students, the subjects of this experiment are undergraduate students. The reason is that undergraduate students are supposed to be able to solve both forward-thinking and reverse-thinking problems rather easily, because they have already understood the structure of simple arithmetic word problems. They are only expected to learn how to make problems through the use of MONSAKUN.

On the other hand, elementary school students firstly learn about the problem structure through the use of MONSAKUN before they become able to pose problems, which takes several times of class schedule. Because the undergraduate students do not need to learn but only to recognize the problem structure, they are expected to show clearer changes in thinking process towards different problem types than elementary school students. For this reason, our study analyzed data from the experimental use of MONSAKUN by university students as the subjects. In the experiment, the subjects are firstly given explanation about the software, and then posed problems in a given time.

Our aim in this study is to examine learners' way of thinking from selection of sentence, especially the first selected sentence in each assignment. We analyzed the subjects' log data in assignments at Level 1 and Level 5 which require the subjects to pose forward-thinking problems and reverse-thinking problems, respectively. Both levels consist of 12 assignments that include four types of stories: combination, increase, decrease, and comparison. Each type of story has three assignments. Subjects carried out the assignments in order, and they can only move on to the next assignment when the current assignment has been answered correctly.

3.1 Difference in First Selected Card between Level 1 and Level 5

Figure 4 shows log data in Level 5 Assignment 1 mapped on a graph. Black nodes and links represent the ones selected by the subjects, while gray ones represent the ones not selected. In this experiment, not all paths were observed in subjects' selection. Subjects only took some particular paths, which show that the card selections are not random. In addition, focusing on the card firstly selected by subjects, most of them chose Card 4 (8 subjects out of 11). In this assignment, there is a decided tendency based on some sort of thinking. If we can clarify the tendency for learners to choose a specific first sentence card, it will be useful to diagnose learners' understanding. Therefore, as the initial step of analysis of students' thinking process, this study aims at revealing the characteristics of first selected sentence card.

The sentence cards in MONSAKUN contain different number according to the numerical expression in the given assignment. For example, in an assignment "Make a story problem about 'how many are the difference' that can be solved by 7 – 3.", the required calculation expression is "7 – 3 = _". In MONSAKUN, several sentence cards with numbers are provided to the users. The cards are distinguished by the order of numbers in the required calculation expression. If a card contains the first number in the required expression, for example, 7 in the example above, it is called *"first number card"*. Similarly, if it contains the second number or the third number, it is called *"second number card"* or *"third number card"*, respectively. One of the numbers in every assignment is an unknown number, which is represented by the blank mark.

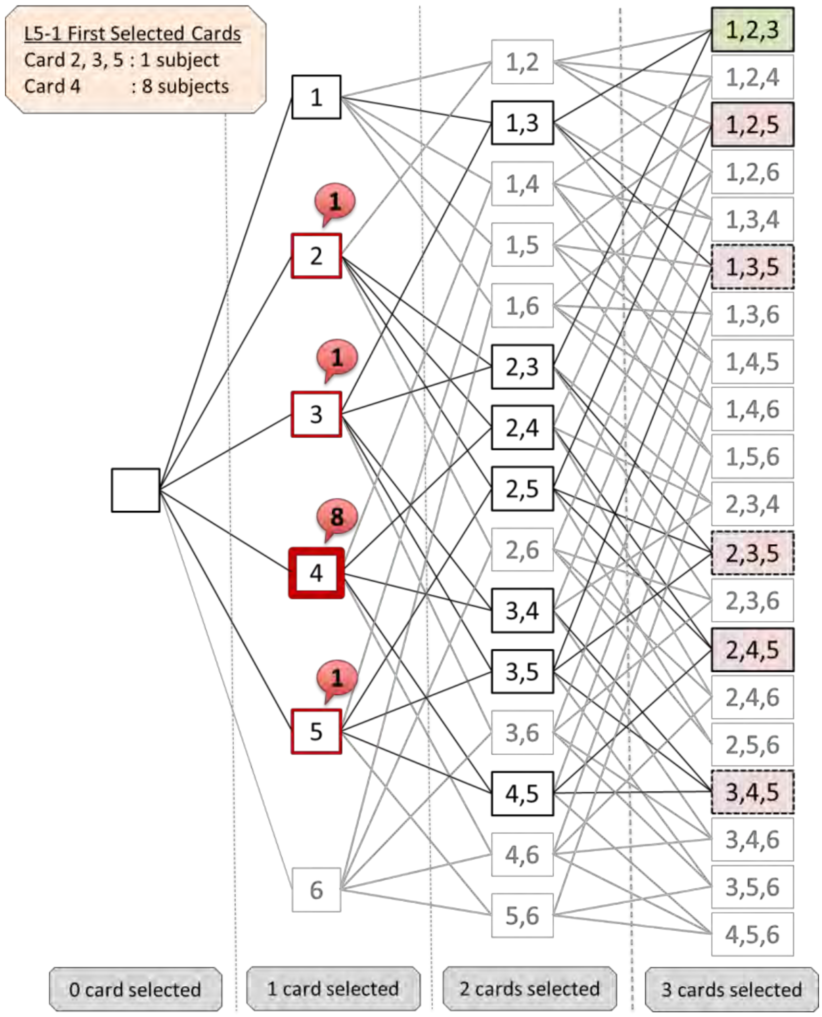

Figure 4. University students' card selection in Level 5 Assignment 1 shown in graph

From the analysis of all subjects' first card selection in Level 1 and Level 5 assignments, we found that the proportion of each sentence card to be selected firstly is entirely not even. Table 1 shows the proportion of first card selected by subjects in Level 1 and 5. In every assignment, only one or two cards are significantly chosen by them. We found that there is a bias against first selected card. This finding proves our assumption that subjects did not choose a card randomly, but with some sort of approach.

Table 1 Percentage of first selected card by the subjects

Type of first selected card	Level 1 (%)	Level 5 (%)
First number card	91.8	58.7
Second number card	3.3	16.5
Third number card (blank mark)	4.9	24.8

Furthermore, we found different trends of first card selection between Level 1 and 5. We presume that this difference appeared because subjects had different approach to pose either forward-thinking or reverse-thinking problems. In forward-thinking problem, the approach to order cards following the order of numbers in the numerical expression can be applied easily. However, in reverse-thinking problem they cannot pose problem with the same approach. This type of problem requires learners to think about the numerical relation in the given problem and reflect it to the choice of cards.

3.2 Change of Approach through the Exercise

In the previous section, we have presumed that subjects had different approach to solve forward-thinking and reverse-thinking problems. In this section, we would like to explain further how the subjects change their way of thinking during problem posing exercise by looking at the type of story, order of assignment, type of first selected card, as well as the type of sentence. We especially analyzed subjects' selection in Level 5 assignments, where they posed challenging reverse-thinking problems.

Table 2 shows the characteristics of first selected sentence card from each assignment at Level 5 that has marginal or significant difference in number of selection from the average. These results were analyzed with binomial test to the amount of each card being firstly chosen or not in each assignment. Binomial test is an exact test of the statistical significance of deviations from a theoretically expected distribution of observations into two categories. Based on our assumption that students posed problems by selecting cards through a thinking process, we expect the distribution of firstly chosen cards to have a significant difference in comparison with other cards.

Table 2. Result of binomial test of first selected card in Level 5 assignments

No	Type of story	Order of assignment	Type of first selected card	Type of sentence	p-value	
1	**Combination**	1st	*First number card*	*Existence*	$7.05*10^{-5}$	**
2		2nd	First number card	Relational	$1.88*10^{-7}$	**
3		3rd	First number card	Relational	$1.97*10^{-3}$	**
4	**Increase**	1st	*First number card*	*Existence*	$1.89*10^{-5}$	**
5		2nd	Second number card	Existence	0.0504	+
6		3rd	*First number card*	*Existence*	0.0504	+
7	**Decrease**	1st	*First number card*	*Existence*	$2.35*10^{-4}$	**
8		2nd	Second number card	Existence	$2.35*10^{-4}$	**
9		3rd	Second number card	Existence	$2.35*10^{-4}$	**
10	**Comparison**	1st	-	-	-	
11		2nd	Third number card	Relational	0.0266	*
12		3rd	Third number card	Relational	0.0266	*

**: significant difference (p<.01), *: significant difference (p<.05), +:, marginal difference (p<.1)

When firstly used MONSAKUN, subjects are given simple forward-thinking problems to pose at Level 1. From the analysis mentioned in Section 3.1, we found that they first simply chose a card with the first number in the required numerical expression (*"first number card"*), and then proceeded to choose other appropriate cards. This approach worked well for assignments in Level 1, where all of the assignments are forward-thinking problems.

When subjects arrived at the first assignment of Level 5, they initially approached the assignment with the same way of thinking in choosing the first sentence card. However, this did not work well, and they tend to make more mistakes than in the previous levels. We presumed that the subjects were aware that the previous approach did not work for reverse-thinking problems, because in the second assignment of Level 5 they tend to choose another type of card.

In a similar way, subjects changed their approach from the first assignment in a story type to the second and third assignment in the same story type. As shown in Table 2, in the first assignment in each type of problem, they generally took the simple approach to firstly select a *"first number card"* containing an *existence sentence*. Only in the case of comparison story there was no significant difference in cards selected by subjects in the first assignment. On the other hand, in the second and third assignments of the same type of story, they did not choose it as the first card. For example, in combination stories, most of them firstly did not select *existence sentence*, but "*first number card"* containing *relational sentence*. This is also the same as in the decrease story type.

Meanwhile, in the case of increase stories, we did not found any evident change between the assignments. At the second assignment they tend to select *"second number card"* containing *existence sentence* (shown by a marginal p-value), while at the third assignment they took the simple approach just like in previous levels of forward-thinking problems.

Furthermore, in comparison stories, there is no trend in first card selection at the first assignment. However, at the second and third assignment, there is a trend to select *"third number card"*, that is a blank mark, containing a *relational sentence*. Consequently, we observed that there is a change of approach in comparison story compared to the previous story types.

This leads to two findings about changes in subjects' way of thinking through the exercises. The first one is that subjects change their approach to pose problems after they had experienced posing the same type of story. As shown in Table 2, trends of first card selection are different between the first assignment and the rest in the same story type. The next finding is that the change of approach depends on the type of story, as we can see that subjects made different first card selection in different story type.

Table 3. Average of steps and mistakes in Level 5 assignments

Story Type	Average No. of Steps		Average No. of Mistakes	
	Assignment		Assignment	
	1st	2nd & 3rd	1st	2nd & 3rd
Combination	11.60	4.20	1.5	0.3
Increase	45.50	16.50	8.4	1.4
Decrease	24.90	16.30	3.3	1.9
Comparison	10.00	9.80	1	0.5

These changes of approach seem to bring a good effect to subjects' thinking process in posing reverse-thinking problems. Our analysis of the average of steps and mistakes in Level 5 problems showed that in comparison to the first assignment of each story type, the average of steps and mistakes in the second and third assignments of the same story type are mostly decreased, as shown in Table 3.

4. Concluding Remarks

In this research, we have conducted analysis of MONSAKUN log data of university students' problem posing activity to investigate their way of thinking in posing different types of arithmetical word problems. From the analysis, we found that the first sentence selected in each assignment were different in several ways. In forward-thinking problems, subjects generally used a simple approach to select *"first number card"*. However, in reverse-thinking problems, they changed the approach to select *"second number card"* or *"third number card"*. Depending on the type of story and subjects' exercise experience, they applied different approach of first card selection. These finding proves our assumption that learners who used MONSAKUN did not chose cards randomly, but with some sort of thinking process. Furthermore, we infer that learners who used MONSAKUN were able to recognize the differences in structure of problems depending on types of story, as they changed their approach to pose problems for different story types. The recognition of the difference is important for learners to understand the nature of arithmetic word problems.

For the next step of this research, we plan to perform the analysis to a larger data of MONSAKUN used by elementary school students to infer their thinking process of the same problem posing activity. The results will be used to make an elaborate process model of the problem-posing and adaptive support of the process.

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