Exploring Student Difficulties in Divide and Conquer Skill with a Mapping Tool

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Abstract: Divide and conquer is an essential thinking skill for engineering undergraduates to have in order to solve engineering estimation problems. However, there exist no teaching-learning tools and strategies for divide and conquer skill for engineering estimation. In this paper, we report on the design of a mapping tool to improve students' divide and conquer skill while solving engineering estimation problems. We evaluated the tool to identify student difficulties in doing divide and conquer while using the tool. We identified several categories of difficulties which will be used to design appropriate scaffolds to support improvement of students' divide and conquer skill for engineering estimation

Keywords: divide and conquer, engineering estimation, mapping tool, external representations

1. Introduction

Consider this problem: "A toy manufacturer has designed a laptop for kids which helps them spell and read and is touch sensitive. Estimate how many AA batteries will be needed to run it for 5 hours at a stretch." Engineers must regularly make estimates like these in the workplace for purposes such as sanity check of results, to establish the feasibility of a design and to eliminate candidate design solutions (Shakerin, 2006; Adolphy et al, 2009). Thus estimation is an important skill for engineering undergraduates to have (Linder, 1999).

One of the key and initial skills practiced by professional engineers in solving problems such as estimation is breaking down large problems into smaller problems in order to make them tractable (Paritosh & Forbus, 2004). Thus breaking down large problems into small problems or divide and conquer is an essential thinking skill of engineering estimation. Divide and Conquer helps in approaching physical quantities that initially seem hard to estimate (Mahajan, 2014). For instance, it is difficult to estimate the energy consumption of a household in a month directly; however breaking down the energy consumption as the sum of energy consumed by all the appliances in a household makes the problem tractable. Thus divide and conquer is needed for many types of estimation problems and is applied repeatedly until one reaches quantities that can be directly estimated.

However the skill of divide and conquer for estimation is not taught in engineering classrooms (Adolphy et al, 2009). Therefore, it is important to develop teaching-learning tools and strategies that explicitly address the development of this skill among engineering undergraduates. In order to help students learn this skill, we propose a mapping tool that helps solvers create tree representations of divide and conquer (Mahajan, 2014). The tool has the provision of creating different kinds of maps such as concept maps and argument maps by adding different types of nodes and links, in addition to divide and conquer trees. We conjecture that using the mapping tool to create trees and maps while doing divide and conquer of estimation problems, will facilitate the doing and learning of divide and conquer skill for engineering estimation. The purpose of the study reported in this paper is to identify the particular difficulties which students face while doing divide and conquer of estimation problems using the mapping tool, so that we can design appropriate scaffolds in the mapping tool to overcome the difficulties.

We followed design based research (Reeves, 2006) to design and evaluate the teaching-learning tool. In the first iteration, we adapted an open-source mapping software called Compendium to create our mapping tool and evaluated it to identify student difficulties in doing divide and conquer of estimation problems while using our tool. Our research question was, “What difficulties do solvers encounter in divide and conquer skill for estimation while using the mapping tool?”

We conducted a lab study wherein engineering undergraduates worked with our tool to do divide and conquer for three estimation problems. We recorded all student questions and researcher responses, screen captures of students’ interactions with the tool and solver created artefacts. Using content analysis of the transcript of student-researcher interaction we identified categories and degrees of difficulties students faced. Our next step will be to translate these difficulties into suitable scaffolds to support student learning of divide and conquer skill.

2. Related Work

The design of the mapping tool is based on the theories of distributed and embodied cognition (Hollan et al, 2000) which argue that cognition emerges from an ongoing interaction between internal resources such as attention, memory and imagination and external resources such as the objects and artefacts in the surrounding environment. External representations facilitate this interaction as they allow processing which is difficult and often impossible in the mind (Kirsh, 2010).

The skill of divide and conquer for estimation includes the following sub-skills (Mahajan, 2014),

1. Identifying information about problem context, conceptual relations among quantities, structural knowledge about objects in the problem and the working of the systems in the problem.
2. Integrating all the above and selecting information and knowledge relevant for estimation.
3. Decomposing the quantity to be estimated as a sum or product of other sub-quantities.
4. Evaluating whether these sub-quantities are simpler to estimate.
5. Choosing a particular breakdown among many possible ones which makes the estimation process easier and more reliable.

Thus, divide and conquer for estimation requires imagination of the problem context, structures of objects, behaviors of systems and qualitative relations among the quantities involved. Research has shown that epistemic actions (Kirsh & Maglio, 1994) performed on external representations during task performance make this imagination more reliable and memory & time efficient. Therefore external representations are required for performing divide and conquer for estimation.

Knowledge representation such as schematic diagrams have been shown to improve performance in problem solving (Hegarty and Kozhevnikov, 1999; Martin and Schwartz, 2009). From a learning point of view, research in scientific inquiry shows that knowledge representations such as models, explanations and argument maps support students’ inquiry and their learning of the skill of scientific inquiry (Quintana et al, 2004; Toth et al, 2002). Similarly, in ill-structured problem solving, the use of concept mapping (Stoyanov & Kommers, 2006; Hwang et al, 2014) and dual mapping (Wang et al, 2013) have been shown to improve problem solving learning and performance. In all these interventions students construct representations, such as argument maps, of the knowledge required for the task and are scaffolded in this process. Research has also shown that hierarchical knowledge structures, such as sub-goals, support problem solving performance and learning (Catrambone, 1998). This has been exploited to improve learning of problem solving in computer-based tutors (Koedinger, 2006) by including features to make the sub-goal structure explicit.

For divide and conquer of engineering estimation a representation showing the breakdown of the physical quantity to be estimated into sub-quantities is required. A tree is an appropriate representation of this breakdown as it depicts the hierarchy inherent in breaking down a problem into sub-problems. Further, the tree diagram serves as an external representation that can be used for restructuring the problem, which would otherwise have to be done in imagination (Kirsh & Maglio, 1994). Mahajan (2014) also recommends creating divide and conquer trees for the physical quantity to be estimated as it is a way of capturing the analysis with a single diagram. However, the strategies described in Mahajan (2014) to breakdown the physical quantity to be estimated are at a broad level and assume learner facility with concepts, which may not be true. So learners will need conceptual and estimation specific epistemic scaffolds (Quintana et al, 2004) to do the breakdown.

In this work, we flesh out the recommendations of Mahajan (2014) for creating divide and conquer trees with theoretical inputs from the cognitive and learning sciences to design a mapping tool that facilitates the doing and learning of divide and conquer skill for engineering estimation.

3. Design of the Mapping tool

We have chosen design-based research (Reeves, 2006) as our approach towards design of the mapping tool as it will require cycles of evaluation and solution refinement followed by producing design principles. Figure 1 describes how we applied design-based research in iteration 1. The last stage of mapping student difficulties to scaffolds will be done based on the results of this study.

The broad conjecture guiding the design of our tool is that creating external representations like the divide and conquer tree (Figure 2) will improve students' performance in the skill of divide and conquer. Therefore, the basic feature required in the tool is the ability to create a tree with a central node denoting the quantity to be estimated (say, mass of air) and nodes branching out from it, each node representing a quantity (say, volume and density) such that mass of air = volume x density. Similarly, there are nodes branching out from the volume node such that volume = length x breadth x height. However, divide and conquer skill for estimation includes several sub-skills (defined in section 2) which require additional actions and external representations from the learner. For instance, the sub-skills 1 & 2 require representations such as equations, graphs and schematics. Therefore a complete list of features required in the tool that will support performing divide and conquer are listed in Table 1.

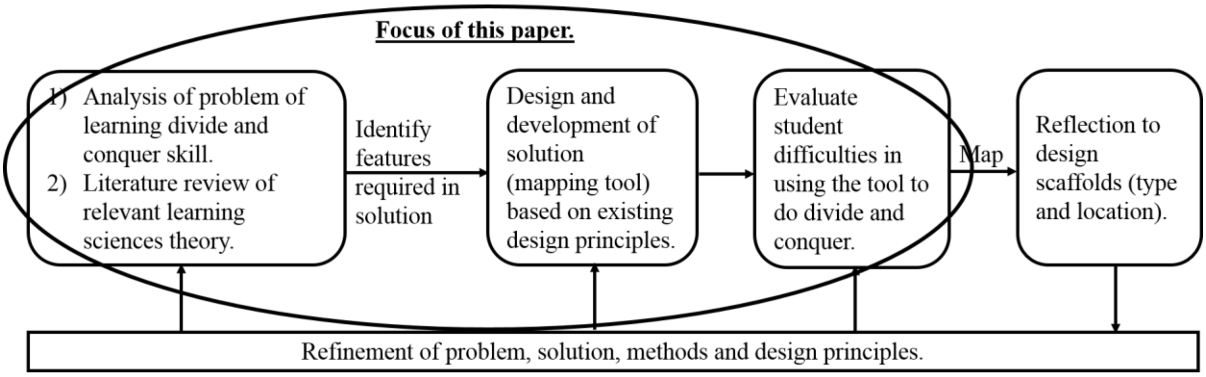

Figure 1. Iteration 1 of design-based research approach to design of mapping tool

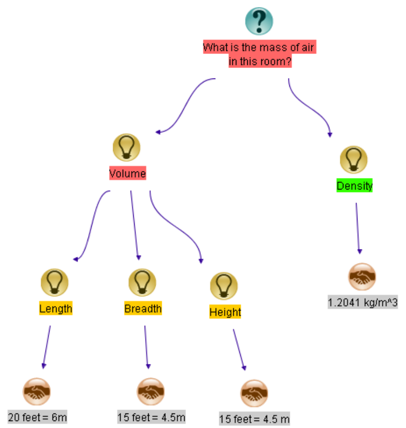

Figure 2. An example of a divide and conquer tree created in Compendium

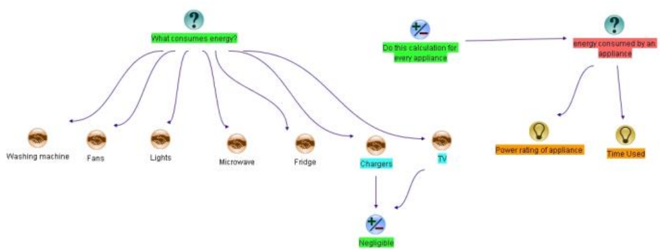

Figure 3. An example of a knowledge map created in Compendium

In the current iteration, we used an available open source knowledge mapping software called Compendium (http://compendiuminstitute.net/) to design the mapping tool. Compendium is a software that visually represents thoughts, ideas, issues and arguments (nodes), and the connections (links) between these. It has different types of nodes and links to represent different types of ideas and connections. Compendium was chosen among several available open source software like IHMC CMAP tools, yEd, FreeMind, etc. because it had the maximum number of features needed in our design.

The mapping tool was created by repurposing some of the available features in Compendium for divide and conquer of estimation problems as shown in Table 1.

Table 1: Features required in the mapping tool.

Feature Required	Available in Compendium?
Ability to create a divide and conquer tree.	Yes
Ability to construct knowledge maps for problem analysis (example in Figure 3).	Yes
Ability to zoom into a particular idea and explore it in depth by creating idea-specific argument and analysis maps (example in Figure 3).	Yes (using the map node)
Ability to move nodes around anywhere on the screen.	Yes
Ability to create different tree structures for sum and product breakdowns.	Yes (using different types of nodes for sums and products)
Ability to provide arguments for chosen breakdown at each level of the tree.	Yes (using argument node)
Ability to color nodes to indicate confidence level in quantities and arguments	Yes
Ability to link different types of knowledge such as information about problem context, conceptual relations among quantities, structural knowledge about objects and the working of the systems with appropriate representations; for example, equations for conceptual relations and diagrams for structural knowledge.	No (use pen and paper)
Scaffolds for doing and learning (Quintana et al, 2004; Ge & Land, 2004) at appropriate points	No (provided by researcher)

4. Evaluation

4.1 Experimental Procedure

To evaluate the first iteration of our mapping tool we performed a lab study with six engineering students (convenience sampling) from the freshman and sophomore years of engineering. These students had the prior knowledge required for the estimation problems we presented. The procedure involved the following steps:

1. Watching an introductory video about divide and conquer, divide and conquer trees and the mapping tool (6 minutes).
2. Watching a video detailing how to use the mapping tool for doing divide and conquer and an example of divide and conquer tree construction for an estimation problem (15 minutes).
3. Individual divide and conquer of three estimation problems using the mapping tool (open ended). An example problem is, “What is the output power of the human heart?”

The students were allowed to watch the videos as many times as they wished, including while solving the problems. They were also given a set of instructions summarizing the two videos. Students used pen and paper to perform steps in the divide and conquer which the tool didn’t have provision for as described in Table 1. If students’ encountered difficulties while solving problems they asked the researcher who provided them scaffolds regarding how to proceed.

In pilot studies, we observed that even graduate students were unable to proceed in the absence of scaffolds. As a result, we could not get a complete picture of all student difficulties. Therefore scaffolds were provided as just-in-time-and-scope prompts to allow students to proceed. The initial scaffolds were reflective (Ge & Land, 2004), such as “To draw this tree I need to know...”. Subsequently, if students were still unable to proceed, the scaffolds became elaborative such as, “How are energy and power related?”

We audio recorded all questions asked by students and researcher responses, captured the on-screen interaction (using CamStudio) of students with the tool, saved the final maps produced in the tool and any rough paper students’ used.

4.2 Data Analysis

The audio recordings were transcribed and analyzed using content analysis with grounded codes. First the transcript of four students was coded; the initial codes were categorized into the "Category of difficulty" and "Degree of difficulty". Next these categories were used to code the transcript of the remaining two students. The codes and categories were revised by constant comparison until a final list of categories emerged. The final maps, screen captures and rough sheets were used while analyzing the audio recordings in order to identify the context of some of the questions that students asked.

5. Results

The categories and degrees of difficulties identified are shown in Table 2. The frequencies are not reported as we are interested in identifying all the possible difficulties that students encounter and providing scaffolds for those. There were three categories of difficulties, those emerging because of the nature of the mapping tool, those related to any of the five sub-skills of divide and conquer and those specifically arising in the process of solving estimation problems. The nature of the mapping tool led to usability issues such as underuse of map node to do problem analysis, using incorrect types of nodes and links and difficulties with the node colors. Related to divide and conquer skill students had difficulties in a) understanding and applying engineering concepts, principles & units and facts, structures & behaviors needed in the problem (sub-skill 1 & 2), b) breaking down the physical quantity into sub-quantities (sub-skill 3) and c) evaluating and choosing a breakdown (sub-skills 4 and 5). The estimation specific difficulties were making assumptions, quantity estimation, argumentation and assessing facts & numerical values. Finally the ill-structured nature of estimation makes it difficult for students to start the problem, proceed when stuck and identify problem requirements.

Table 2: Categories and Degrees of Difficulties.

Category of difficulty	**Sub-category of Difficulty**	**Degree of Difficulty**
Mapping tool	Map node; Color codes; Types of nodes; "-1" link; Argument node; General	Underused; Incorrectly used; Use not understood
Divide and Conquer Skill	Problem context-specific knowledge (sub-skills 1 & 2): Facts; Structures (Spatial); Behaviors	Unknown; Partially known; Incorrect; Unsure
	Prior engineering knowledge (sub-skills 1 & 2): Concepts and Principles	Misunderstood; Partially understood; Not understood; Unsure
	Prior engineering knowledge (sub-skills 1 & 2): Formulas	Inappropriate Application; Incorrect identification
	Prior engineering knowledge (sub-skills 1 & 2): Units	Incorrect
	Breakdown of physical quantity (sub-skill 3)	Incorrect breakdown; Incomplete breakdown; Tree structure not understood
	Evaluate and choose (divide and conquer sub-skills 4 & 5)	Inability to do
Estimation problem related	Assumptions	Inability to recognize; Partially justified; Unjustified; Inability to judge validity; Unable to make
	Quantity estimation	Inability to do
	Argumentation	Unable to write; Unable to judge
	Assessing facts & numerical values	Specificity (for look up); Reasonableness; Relative significance; Standardness; Relevance
	Ill-structuredness of problem	Inability to deal with low information; Inability to start solving; Inability to proceed when stuck; Inability to identify requirements; Inability to reason; Inability to relate; Incorrect identification of problem requirements
	Terminology	Causes misunderstanding; Unable to articulate

6. Conclusions and Future Work

From this study we identified usability issues in the tool that will need to be modified in iteration 2 of the design. Further we learned that the problem context-specific knowledge needs to be provided to students to enable them to begin divide and conquer. Finally we identified specific aspects of estimation problems and divide and conquer skill which students need scaffolds for such as making assumptions and breaking down physical quantities. We will design appropriate scaffolds to overcome these difficulties; for example "elaborative prompts" to get students to articulate their assumptions and providing a tree template to kick-start the breakdown process.

An interesting finding was that even though we had ensured that students had learned the concepts and principles necessary to solve the problems, students have difficulties in understanding and applying prior conceptual knowledge. There are two ways to manage this difficulty; either we can target our tool to advanced engineering students (juniors and seniors) or we can incorporate conceptual knowledge as a scaffold in our learning tool. For our next iteration, we propose to do the former as we do not want the emphasis of this tool to shift from learning divide and conquer skill to learning engineering principles. In future iterations we will incorporate conceptual knowledge in the tool and evaluate the difference between beginning and advanced engineering students in learning divide and conquer skill. We will also try to identify whether students difficulties with the tool are related to their difficulties with divide and conquer skill for estimation problems, i.e., whether they are unable to or underuse certain features of the tool because of their difficulty with divide and conquer skill and/or their inability to think and reason about estimation problems.

References

Adolphy, S., Gericke, K., & Blessing, L. (2009). Estimation and its role in Engineering Design - An introduction. In *DS 58-9: Proceedings of ICED 09, the 17th International Conference on Engineering Design, Vol. 9, Human Behavior in Design* (pp. 255–266). Palo Alto, CA, USA.

Catrambone, R. (1998). The subgoal learning model: Creating better examples so that students can solve novel problems. *Journal of Experimental Psychology: General*, 127(4), 355–376.

Ge, X., & Land, S. M. (2004). A Conceptual Framework for Scaffolding Ill-Structured Problem-Solving Processes Using Question Prompts and Peer Interactions. *Edu. Tech. Research and Devt*, 52(2), 5–22.

Hollan, J., Hutchins, E., & Kirsh, D. (2000). Distributed Cognition : Toward a New Foundation for Human-Computer Interaction Research. *ACM Trans. on Computer-Human Interaction*, 7(2), 174–196.

Hwang, G., Kuo, F., Chen, N., & Ho, H. (2014). Effects of an integrated concept mapping and web-based problem-solving approach on students' learning achievements, perceptions and cognitive loads. *Computers & Education*, 71, 77–86.

Linder, B. M. (1999). Understanding estimation and its relation to engineering education (Doctoral dissertation, Massachusetts Institute of Technology).

Kirsh, D. (2010). Thinking with external representations. *AI and Society*, 25, 441–454.

Kirsh, D., & Maglio, P. (1994). On distinguishing epistemic from pragmatic action. *Cog Science*, 18(4), 513–549.

Koedinger, K. R., & Corbett, A. (2006).*Cognitive Tutors: Technology Bringing Learning Science to the classroom*.

Mahajan, S. (2014). The Art of Insight in Science and Engineering: Mastering Complexity. The MIT Press.

Martin, L., & Schwartz, D. L. (2009). Prospective Adaptation in the Use of External Representations. *Cognition and Instruction*, 27(4), 370–400.

Paritosh, P. K., & Forbus, K. D. (2004). Using Strategies and / OR Decomposition for Back of the Envelope Reasoning. In *Proceedings of the 18th International Workshop on Qualitative Reasoning*. Chicago.

Quintana, C., Reiser, B. J., & Davis, E. A. (2004). A scaffolding design framework for software to support science inquiry. *The Journal of Learning Sciences*, 13(3), 337–386.

Reeves, T. C. (2006). Design research from a technology perspective. *Educational design research,* 1(3), 52-66.

Shakerin, S. (2006). The Art of Estimation *. *International Journal of Engineering Education,* 22(2), 273–278.

Stoyanov, S., & Kommers, P. (2006). WWW-intensive concept mapping for metacognition in solving ill-structured problems. *Intl. J. Continuing Engineering Education and Life-Long Learning*, 16(3/4), 297.

Toth, E. E., Suthers, D. D., & Lesgold, A. M. (2002). "Mapping to Know": The Effects of Representational Guidance and Reflective Assessment on Scientific Inquiry. *Science Education*, 86(2), 264–286.

Wang, M., Wu, B., Kinshuk, Chen, N. S., & Spector, J. M. (2013). Connecting problem-solving and knowledge-construction processes in a visualization-based learning environment. *Computers and Education*, 68, 293–306.