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Non Numerical Aspects of School Mathematics

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Abstract: In this short paper, we describe issues resulting from a lack of clarity in understanding the nomenclature of *numeracy* in mathematics education at the school level and consider some of the underlying foundational structures of mathematical thinking. The purpose of the paper is to open a conversation about shifting the focus from the narrower conceptual boundaries concerning *numeracy* by considering theoretical perspectives that describe mathematical thinking as a form of *intelligence* on the one hand, and as a *skill* within the paradigm of 21st century skills, on the other. We identify a number of questions to be considered in teaching mathematics and specifically in contexts where digital technology is utilised.

Keywords: STEM, mathematics, digital technology, sense-making, 21CC

1. Introduction

> When instructions for a task are too directive, then it may be possible to carry them out without actually encountering the intended ideas behind the task.
> (Mason & Johnston-Wilder, 2006, p. 29)

As opposed to solving an issue or problem using rules, a key purpose of mathematics education is to enhance student cognitive ability to connect the non-numerical aspects of the process and enable application of this learning in other situations. It is this aspect of mathematics, however, that is often not effectively conveyed, either explicitly or implicitly, at the school level of the discipline.

The purpose of this paper is to provoke discussion that could assist in shifting focus from *numeracy* – a term given far too much emphasis as a proxy for basic mathematical ability – to other aspects of cognitive facility associated with mathematics at the school level.

Intended learning may fail to be achieved for many reasons and if the purpose of the mathematical tasks is confined or limited to 'solving', rather than 'learning from solving', it is likely that learners may neither learn nor enjoy engaging with the problem. Focusing merely on the numerical aspects of mathematics and problems does not create opportunities for students to generalize the situation in order to apply the mathematical learning in other contexts. Differences in perceptions between teachers and students regarding the purpose of a task may also exacerbate non-effectiveness in the performance of the tasks.

As Stein (1987) has famously pointed out, the intention to 'teach thinking' can easily turn into a set of instructions so that learners do not have to think.

The main purpose of the paper is to provide an overview of issues resulting from an over emphasis upon numeracy in school mathematics education and to consider some of the underlying foundational structures of mathematical thinking. Conveying a sense of the range of thought processes available when working within a problem can help children engage with the logic and ideas which is arguably more at core of mathematics teaching. Doing so could bring the focus back from the algorithmic processes, numerical memorization, and rote learning aspects to a richer palette of sense-making, abstraction, and inquiry. In the words of Anderson (2001): "In trying to connect mathematics with what is learnable, we have disconnected school mathematics from what is genuinely useful."

Now that we are mid-way through the second decade of the 21st century it is also timely to consider how numeracy and mathematics education relate to the discourse on *21st century skills* or *21st century competencies* (21CC). This discourse has arisen largely as a consequence of ongoing innovation in digital technology and the proliferation of networks (global and local) as an organizing principle of society that cuts across most political configurations. The key question here is: *how do mathematical thinking and numeracy skills fit within the various frameworks of 21st century skills?*

2. Numeracy – a Misnomer?

2.1 Origins

The term *Numeracy* was first used in 1959 in *The Crowther Report* developed by the Central Advisory Council for Education (England) to consider the changing social and industrial needs of society and, in particular, to consider the balance at various levels of general and specialized studies between students aged 15-18, and to examine the inter-relationship of the various stages of education.

Numeracy had been defined as a term with similar utility to that of *Literacy* – as a means of thinking about the world in a quantitative way, to realise problems as problems of the degree even as they appear to be problems of the kind, and to point toward the scientific approach to the study of phenomena: observation, hypothesis, experimentation and verification. Justification for this was articulated by Cockcroft (1982) as: "Statistical ignorance and statistical fallacies are widespread and quite as dangerous as the logical fallacies that come under the heading of illiteracy" (p. 11).

This early definition thus encompasses metacognition and logical understanding. Since then this term has attracted alternative definitions numerous times with several related terms having been devised to replace it, e.g., Mathematical literacy, statistical literacy, mathematical skills, working mathematically etc. (Cavanagh, 2006; Jablonka, 2003; Hoyles, Wolf, Molyneux-Hodgson, & Kent, 2002; Kilpatrick, 2001; Wallman, 1993).

A simple search on Google reveals that a common interpretation of this term is *the ability to understand and work with numbers*. The Australian Curriculum defines Numeracy as "the ability to recognise and understand the role of mathematics in the world and having the dispositions and capacities to use mathematical knowledge and skills purposefully" (Numeracy, 2012). In the United Kingdom being Numerate means having the confidence and skill to use numbers and mathematical approaches in all aspects of life (National Numeracy, 2015).

With the proliferation of different definitions the word Numeracy can become quite subjective. For some people it may mean being able to do number operations, being able to do budgeting, being competent with shopping, or performing practical tasks that involve numbers. Some decades ago Riley (1984) analyzed the term and found that it had been distorted and over-simplified. Penny (1984) also questioned the definitions surrounding and she pondered whether the term might mean a checklist of coping skills. In her view, the term should reflect the ability to understand and use mathematical skills as a means of communication closely linked to individual needs. Castle (1992) also observed that approaches to teaching numeracy tend to place emphasis on coping and survival skills and on the needs to accommodate prevailing social and economic circumstances rather than pursuing social change through questioning and challenging structural inequalities.

It is significant to note that the various frameworks associated with 21st century skills or 21CC do not offer re-conceived notions of *numeracy* for the digital age; however, they do this for *literacy* as "information literacy", "technological literacy", and "ICT literacy" (Griffin, McGraw, & Care, 2012; Voot, et al., 2013; Hanover, 2011). Arguably, however, the presence of *critical thinking* and *problem solving* within this discourse – while not explicitly invoking mathematics – can be seen as foundational aspect of mathematical thinking.

2.2 What numbers mean and what lies beyond?

Several research papers on mathematics include in their title '*Beyond the numbers'* (Greenhalgh, & Taylor, 1997; Yoder, 1994; Slovic, 1991; Marr, & Hagston, 2007). However, this has not been the subject of much research in Mathematics Education at the school level.

On one hand numbers quantify, express and explain a measure of quantities while on the other they are just symbols expressing abstractions of the commonality of objects that have the same count. It is important to see beyond the number symbols and understand the generality they express.

In all school curriculums the aspects of literacy and numeracy are integrated and interlinked and in order to understand the rules and learn to apply them it is necessary that we deconstruct the relationship between language, numbers and literacy so we can then focus on distinctive aspects of mathematics teaching and problem solving.

Thus, even in the simplest of the examples with minimal language and maximum numerical involvement such as 13 + 5 =? or 4 x 7 =? the idea that needs focus and clarity in student minds is the *purpose* of these questions – students need to make sense of mathematical statements and symbols, which is not the same as memorization nor just rote learning of the rules. The purpose may be communicated implicitly by asking:

- What is the meaning of these operations?
- Can these problems be relevant?
- In which situations may these be relevant?
- Why do we need to know this?
- What other ideas can we extract from these? (e.g., 13 + 5 = 5 + 13; or 4 x 7 = (2+2) x 7)

The next level of thinking may begin from questions like: *What if …?* for example: *what if … 12 + 1 = 1? In what circumstances might this logic hold true?* Such a problem intentionally collides with conventional common sense yet in mathematics such a problem may lead to ideas such as clock arithmetic which, in turn, may mark modular arithmetic making sense later in the curriculum.

As teachers, we need plan to teach and think: what are the ideas that numbers or numerical data cannot or do not capture in the problem as stated?

From a theoretical perspective, Gardner (1983) offers useful perspective in defining “logical mathematical intelligence” as one of seven types of intelligence and a term to describe the capacity for analyzing and solving problems logically, performing mathematical operations, and to think scientifically while also engaging in activities such as pattern recognition, abstract modeling and computation. By focusing upon *intelligence* as a disposition as well as something to be cultivated Gardner brings to the fore both breadth and depth to mathematical thinking – way beyond numeracy.

3. Context, Connections and Experiences

There could be a range of situations where the development of this behaviour (to see beyond numbers) would enable students to function more effectively. Government policies, taxation, climate change and health issues all need a conscious understanding to discover what lies beyond displayed numbers. A genuine understanding requires a critical view that can only be developed if an explicit training is provided so that we do not get overwhelmed by the amount of tables, graphs and large numbers used to explain as well as persuade. For example, statistics and trend graphs are often presented in political contexts in ways that can prevent us looking beyond, and people believing them as factual without questioning their qualitative nature.

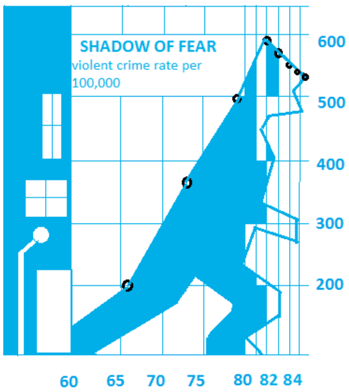

Figure 1. Rising Crimes 1 (OECD, 2009)

A better understanding of the underlying structures and the role of contextual factors can only be arrived at by improved mathematical thinking, along with an understanding of the nature of the situation in which numbers are being applied or drawn from. A good example that depicts this is from the International PISA 2009 Assessment Framework (OECD, 2009, p. 103), as shown in Figure 1 and Figure 2 where the same data is used to convey different messages.

Figure 1 shows the number of reported crimes per 100,000 inhabitants, starting with five-year intervals, then changing to one-year intervals. From this graph the following question is suggested as a task for students to answer: *How many reported crimes per 100,000 were there in 1960?* Using the same data manufacturers of alarm systems produced the graph in Figure 2:

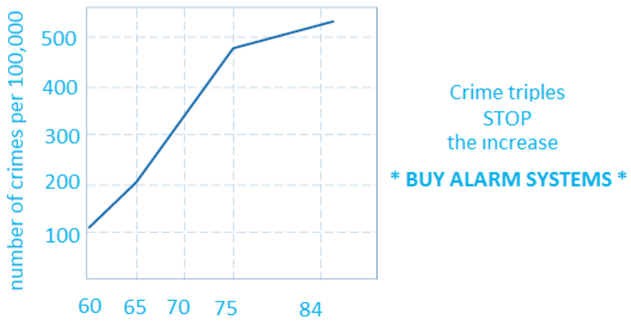

Figure 2: Rising Crimes 2 (OECD, 2009)

A follow-on question is posed: *How did the designers come up with this graph and why?* The police were not too happy with the graph from the alarm systems manufacturers because they wanted to show how successful crime fighting has been. It is then suggested that students be set a task to design a graph to be used by the police to demonstrate that crime has decreased recently. These figures provide an excellent example of the critical relationship between data, context and sense that mathematics in our classrooms needs to provide. It's about making a conscious effort to see what is not shown.

3.1 Conscious questioning - that uses no numbers

Does the articulation and scaffolding of mathematical tasks provide opportunities to students to apprehend vocabulary, definitions, data and numbers and refining these 'seeing' aspects of a problem situation into 'thinking' 'understanding' then 'solving' and then 'connecting' cycle? What questions need be asked to enhance these abilities? Indeed, such questions beg a deeper question concerning the nature of inquiry – *in what ways do or can our students make sense of mathematics?* And, in an era where digital technology is a given, though typically dominated by a "search paradigm" (Mason, 2014), *how can our digital tools be used effectively in ways that promote student questioning?*

3.2 What constitutes mathematical thinking?

Mathematical thinking often begins with some vague questioning to inquire about a situation in order to understand the context. The process of inquiry and answering cycles moves in a particular way in which exact and precise connections within language, symbols and syllogism are maintained. Ideally, no assumptions are made about the unknown and unproven. It is best described in the following tale.

> An Astrophysicist, a physicist, and a mathematician were on a train heading north and had just crossed the border into Scotland. The Astrophysicist looked out of the window, saw a black sheep and said "Look! Scottish sheep are black!" The physicist said, "No, no, you are wrong! At LEAST ONE Scottish sheep is black." The mathematician looked irritated. "Both of you are not right in your observations!! There is at least one field in Scotland, containing at least one sheep, of which at least one side is black." (Anonymous)

This short joke has been used many times to convey that mathematics is a precise branch of science – it cannot proceed without evidence or precision. Computers are likewise precise in the way they operate and they are certainly less forgiving than people are in daily discourse. Yet, for mathematics education, they also need to deal with semantics and syntax in ways that extend beyond daily conventions of sense-making and, in some senses, perhaps be more forgiving.

4. Digital Technology

Technologies in the form of talking drums, pen and paper, or the Internet have all been instrumental to human communication and the evolution of education (Gleick, 2011). In a word, technology has generally made it easier for communication to proceed even though technological innovation has not always been welcome. Debates about the use of digital technology in mathematics education have taken many forms and in recent decades the efficacy or otherwise of slide rules, calculators, laptops and the Internet in developing mathematical skills have all been challenged. Given that technology has made it easier to create illusions and hype through numbers, graphs and animated visuals, it is increasingly more important to consider the skills necessary for our next generation so they will not be misled and misdirected by such innovations. For example, 'zoom in' and 'zoom out' features may allow a change of perspective and context and if used effectively could assist students develop the power of imagination and seeing the unseen. The ability to translate and rotate may also help this perspective. Conversely, it is important that students realize that data collected and presented is never independent of personal bias and the context in which it is collected.

Generally, all school mathematics curriculums now require use of a range of digital resources and skills. They ask for the creative and effective integration of these tools to produce a challenging and creative mathematics syllabus. But what might the underlying assumptions be concerning digital technology? To what extent can mathematical thinking be scaffolded by technology? Questions concerning the scope of engagement with digital technology and its integration in developing curiosity and critical thinking also require investigation. Computers have the power to test conjectures, produce counter arguments, and execute fast calculations – seeing this power for what it is may enable students to start seeing beyond numbers to visualise the underlying mathematics could be highly motivating.

Thus, as we move through various stages of the digital revolution (from early forms of computer-based training to now include digital games, mobile devices, computer-aided explanation, learning analytics, intelligent computing, and numerous innovations in the design of software and applications) an increasing diversity of options is available for innovative educators to consider. For mathematics education, as well as STEM more broadly, the era of data-intensive computing sometimes described as the "fourth paradigm" of science has arrived – and it brings with it many implications (Hey, Tansley, & Tolle, 2009). In thinking about what might help shift the focus from numbers and numeracy is there a clue here? Given that the visualisation of data principally occurs as a result of mathematical operations do we now need to think of what might constitute "data literacy" within a STEM or mathematics paradigm for the 21st century?

5. Conclusion

One of the challenges for mathematics education concerns effective communication of its scope. While it is arguably the case that in early years of education foundational skill development based upon *numeracy* and *literacy* are essential there are other considerations as we move forward into 21st century teaching and learning – in particular, there many new skills are demanded that extend well beyond notions of numeracy.

In briefly considering theoretical perspectives that describe mathematical thinking as a form of *intelligence* on the one hand and as a *skill* within the paradigm of 21st century skills on the other, this short paper provides some basic parameters for shifting the focus on numeracy in mathematics education. While non-numerical aspects of mathematics can be thought of in terms of a capacity for abstraction, critical thinking, and problem solving, it appears that the scope of mathematics education could extend further. With data ubiquity now characterizing the age we live in what are the questions we must ask in order to precisely determine this scope? Might data literacy emerge as a form of mathematical thinking or an extension of notions of literacy?

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