

# Scaffolding of Thinking about Structure with Kit-Building Task

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**Abstract:** In this paper, we introduced interactive environment for “learning by problem-posing” in arithmetic word problems. This problem-posing task is designed based on a structure model of arithmetic word problem and the task is realized by selection and combination task of provided sentences. This research expects that a learner is promoted to think about structure of arithmetic word problem through the activity of the problem-posing. We call this task “kit-building task”. We think that design “kit-building task” based on “structure model” is a promising approach to realize promotion “thinking about structure”. In this paper, we introduce learning by problem-posing as an example of this approach. Through presentation and discussion about this approach in TELoTS, we would like to make clear the relation between “Thinking Skills” and this research.

**Keywords:** Thinking about Structure, Kit-Build Task, Learning by Problem-Posing, Thinking Skills

## 1 Introduction

This paper describes a method to promote thinking about structure [Hirashima et al. 2016] with kit-building task [Hirashima et al. 2015a]. Problem-solving exercise is a very popular activity for learning. However, through the exercise, although a learner works on mastering the way to derive an answer by using a solution method as the purpose of the exercise, the learner often doesn't pay attention to understand the problem or solution method. Without such understanding about the problem or solution method, the learner is usually able to derive a correct answer for the problem with the solution method. Therefore, if we set a goal of learning at such understanding, problem-solving is not an enough task. In this research, we propose kit-building task that requests a learner to compose an understanding target by using components that are provided beforehand. It is possible to say the combination of the components is the structure of the target. Therefore, the activity to compose the target by the components requires a learner to think about the structure of the target in various ways. We call this thinking “thinking about structure” and the task “kit-building task”. In this paper, as targeting arithmetic word problems, a structure model of the word problems called “triplet sentence model” [Hirashima et al. 2014], and “learning by problem-posing” [Hirashima et al. 2000] as a kit-building task to promote “thinking about structure” are introduced. Through presentation and discussion in TELoTS, we would like to make clear the relation between “Thinking Skills” [Beyer 88] and a series of our researches.

## 2 Thinking about Structure through Problem-Posing

### 2.1 Learning by Problem-Posing in Arithmetic Word Problems

In this subsection, problem-posing task of arithmetic word problems designed on “triple sentence model” is introduced. An interactive problem-posing environment of arithmetic word problems [Hirashima et al. 2007] (we call it MONSAKUN, that is, Problem-Posing Kid in Japanese) has been developed and practically used in arithmetic classes in several elementary schools at the first grade (addition and subtraction) [Yamamoto et al. 2012], the second grade (multiplication) [Yamamoto et al. 2013], and the third grade (multiplication and division) [Yamamoto et al. 2014].

Several investigations have already indicated that problem-posing of arithmetic word problems are promising learning activity [Ellerton et al. 1986]. However, this activity gives heavy load to both a

learner and a teacher. It is usually hard for a learner to make sentences from scratch. The learner often feels difficult how to write sentences, select story or numbers that are not so important from arithmetical point of view. Although posed problems and their posed processes are different in each learner, it is impossible for a teacher to diagnose posed individual problems in real time. Therefore the learner is not able to receive individual support. Even if the teacher gives up diagnosing the posed problems in real time, to diagnose all problems posed by a set of learners is a time-consuming task. Moreover, it is not easy for the teacher to use the diagnosed results for teaching effectively because of time lag between the class of problem-posing and the class of feedback based on the results. Because of these difficulties, it was rare that a class of learning by problem-posing was carried out. Agent-assessment is a solution of this issue [Hirashima et al. 2000]. To realize the agent assessment, kit-build approach is a promising approach.

The workspace of the problem-posing activity is shown in Figure 1, and Figure 2 shows a scene where a student is using MONSAKUN for problem-posing. In the upper left side of the interface, a calculation “8-6” and a type of arithmetic story (an increase story) [Riley et al. 1983] are assigned (these words were translated from Japanese into English). A learner is required to pose a problem that can be solved by the calculation and belongs to the specified type of arithmetic problem by using sentence cards provided in the right side of the interface. The set of sentence cards includes not only the necessary ones but also unnecessary ones (the unnecessary card is called dummy card). In the lower left side, there are three blanks where a learner puts sentence cards in order to complete a problem. In Figure 1, two cards have been put in the blanks. In this case, correct problem is {(1) “There are 6 apples.”, (2) “Several apples are given.”, (3) “There are 8 apples.”}. The story of this problem is “increase” that can be expressed as “ $6+?=8$ ”, and then, the calculation is  $8-6$ . In the case of Figure 1, the two cards are wrongly put into the first and the second blank. From the two cards, although it is possible to pose a problem that is solved by “8-6”, the story of the problem is “decrease” that is expressed as “ $8-6=?$ ” and there is no necessary card to complete the problem, that is, “there are several apples”. By pushing the “Check the problem” button, the posed problem is diagnosed and the learner is able to receive feedback based on the diagnosis.

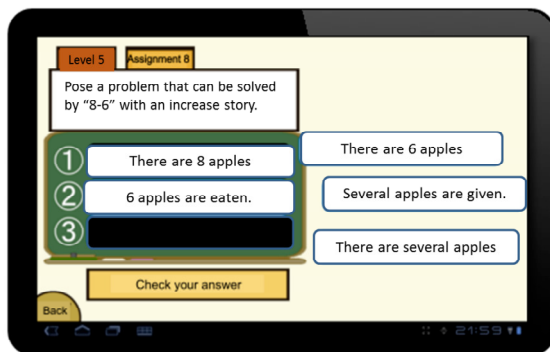


Figure 1. Workspace of Problem-Posing.

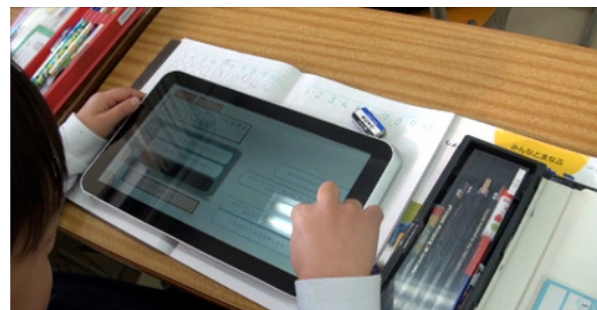


Figure 2. A Scene of Practical Use.

In MONSAKUN, the task to pose a problem is externalized as (1) selection of sentence cards and (2) ordering the selected sentence cards based on the model of information structure of arithmetic word problem. In other words, a learner can operate structure of an arithmetic word problem by operating sentence cards. In usual problem-posing situation, only a posed problem appears as the result of thinking of a learner. Therefore, the problem-posing is a typical thinking task in mind. In contrast, in MONSAKUN, components of a problem are visualized for a learner as operatable ones. Then, the learner is able to compose a problem through visual operations of the components. Therefore, it is possible to say that the MONSAKUN realizes a kit-building task to promote thinking about structure.

## 2.2 Triplet Sentence Model

In MONSAKUN, problem-posing task is designed based on a model of an arithmetic word problem called “triplet sentence model” [Hirashima et al.2014]. In the model, a basic arithmetic word problem is composed of three sentences. Then, the problem-posing task is designed as combination of the sentences. An arithmetic word problem should be solved using one or more basic arithmetic operations. A problem that can be solved by one basic operation is called basic arithmetic word problem. Since one operation composed of three numerical values, that is, two operands and one result, the basic arithmetic word problem includes three arithmetic concepts corresponding to the three numerical values. By writing a pair of the arithmetic concept and its value in a sentence, it is possible to express a basic problem by using three sentences. In the triplet sentence model, the sentences are categorized into two types, that is, (1) existence sentence and (2) relation sentence. The existence sentence includes an independently existing arithmetic concept and its numerical value. For example, “there are six apples” or “there are two dishes” are existence sentences. The relation sentence includes an arithmetic concept and its value that expresses a relation between other two existence sentences. For example, “two apples are eaten” expresses the relation between the number of apples before eating and after eating. “There are six apples” and “there are four apples” is able to be connected by the relation sentence. By arranging the three sentences in the following order: “there are six apples, “two apples are eaten” and “there are four apples”, an arithmetic story is formed. The story is transformed to a problem by changing a numerical value to unknown one and requesting to derive the value from other two values.

Based on this model, it is possible to derive several problems from one existence sentence as shown in Figure 3. Bold rectangles are relation sentences and others are existence sentences. An existence sentence can be used in all kinds of stories/problems although its role is different depending on the type of stories/problems. The types of arithmetic word stories/problems solved by addition or subtraction are categorized into following four stories: change-increase, change-decrease, combine, compare. Compare stories are often classified into compare-more story and compare-less story. As for the stories/problems solved by multiplication or division, there is only one story and the story is composed of three factors, that is, “base quantity”, “proportion quantity”, and “comparison quantity”. Relation between them are expressed “base quantity” \* “proportion quantity” = “comparison quantity”. In both multiplication story and division story, three arithmetical concepts are assigned to one of them. In the story of multiplication or division, an existence sentence plays a role of the proportion quantity or the comparison quantity. The base quantity is assigned only relation sentence. In Figure 3, “one apple is 80 cents” and “2 apples on one dish” are relation sentences that play the role of base quantity. The existence sentence “there are 6 apples” expresses the portion quantity when the relation sentence is “one apple is 80 cents”, and it expresses the comparison quantity when the relation sentence is “2 apples on one dish”.

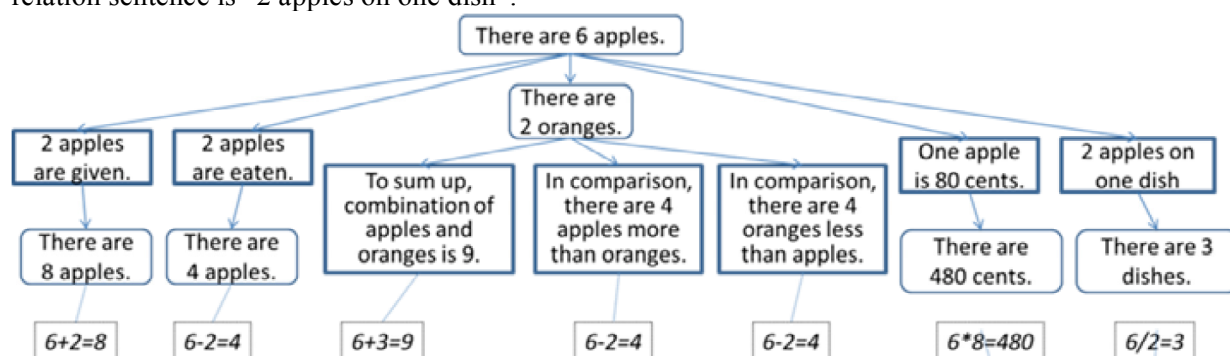


Figure 3. Various Problems Derived by the Same Existence Sentence.

Based on this model, it is possible to design various kinds of activities to manipulate the structure of arithmetic word problems and to implement diagnosis and feedback function for the activities. Through practical use of MONSAKUN, it has been confirmed that operating the sentence cards doesn't disturb learners' thinking process and promotes the task of problem-posing. Moreover, learning effect for structural comprehension of arithmetic word problems has been observed.

Therefore, it is possible to say that the investigation about MONSAKUN is a promising example of externalization of thinking tasks. These researches deal with a basic problem that can be solved by one arithmetic operation. In order to deal with more complex problem that can be solved several operations, triangle block model and interactive learning environment based on the model have been investigated [Hirashima et al. 2015b]. Because of page limitation, they are not introduced in this paper.

### 3. Conclusion Remarks

In this paper, interactive environment for “learning by problem-posing” in arithmetic word problems was described. In the presentation in workshop, we will introduce practical example of promotion of “thinking about structure”. We also discuss other promising challenge for promotion of “thinking about structure” with “analogy” and “reading comprehension” [Hirashima et al. 2016]. Through this presentation and discussion, we would like consider contributions of current our researches to “Thinking Skills”.

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