

Revealing Students' Thinking Process in Problem-Posing Exercises: Analysis of First Sentence Selection

Nur HASANAH^{a*}, Yusuke HAYASHI^a & Tsukasa HIRASHIMA^a

^a*Dept. of Information Engineering, Hiroshima University, Japan*

*nur@lel.hiroshima-u.ac.jp

Abstract: We have developed a computer-based learning environment called MONSAKUN to realize learning by problem-posing, where students pose arithmetical word problems by selecting and arranging several sentence cards. As the next step of MONSAKUN development, we analyze the sentence selection process which is considered to reflect students' thinking process. In the first step of analysis, we focused on first sentence selection. We found that the selection approach changed based on different type of story and students' exercise experience. This result is an important step towards building elaborate process model of problem-posing and adaptive support of the process.

Keywords: Arithmetical word problems, learning analytics, problem posing, reverse thinking problem, sentence integration

1. Introduction

Problem posing practice, one of the central themes in mathematics education, involves the generation of new problems in addition to solving pre-formulated problems (English, 1998). In learning by problem posing, one of the most important issues is the way to assess and give feedback to posed problems. In traditional problem posing method, teachers and students were faced with difficulties to conduct the learning activities effectively. It is not easy for students to pose mathematically correct problems in a given time, and teachers were having problems to assess and give feedback to the wide variation of problems that students pose. The inefficiency of time and available method made problem posing activity less attractive for most mathematics educators.

In order to realize learning by problem-posing in a practical way, we have been investigating a computer-based learning environment to assess and give feedback to problems posed by students (Hirashima et al., 2007). The software, named MONSAKUN (means "Problem-posing Boy" in Japanese), provides an interactive support for learning arithmetical word problems solved by one operation of addition/subtraction. The practical use of MONSAKUN at several elementary schools has been reported in previous studies (Hirashima et al., 2007; Yamamoto et al., 2013).

In this study, we examine how learners pose arithmetical word problems as sentence integration on MONSAKUN. Our assumption is learners do not choose sentence cards randomly - they arrange sentence cards based on some sort of thinking. In the analysis, as the first step toward analyzing problem-posing activity, we especially focus on what kind of sentence card was firstly selected by the learners in different types of story.

2. Analysis of First Sentence Selection from MONSAKUN Log Data

2.1 Modeling of Problem Posing Activity in MONSAKUN

The interface of an assignment in MONSAKUN is shown in Figure 1. A learner is provided with a set of sentence cards and a numerical expression, and then he/she is required to pose an arithmetical word problem using the numerical expression by selecting and arranging appropriate cards.

While it is difficult to trace thinking process in a free problem posing activity, we can trace learners' card selection in MONSAKUN which can be considered to reflect their thinking process. Learner's assignment is to choose appropriate cards from several sentence cards provided by the system in order to fill the requirement of numerical expression and story type.

In MONSAKUN, problem posing can be considered as combinatorial search of sentences. Figure 2 illustrate a search space of problem posing in a MONSAKUN assignment which provides six sentence cards. This search space is a tree structure of combination of cards. Here, the root is the starting point and the numbers represent identifiers of cards. For example, the starting point is empty and the combination of cards 1, 2 and 3 indicates the correct answer at the top left. The links between nodes represent possible paths that learners can follow in selecting cards during their problem posing activity.

Bold links in Figure 2 represent links actually followed by learners in a particular assignment. This indicates that not all paths were followed by learners. From this fact, there is a possibility that learners choose combination of cards based on some sort of thinking processes. The goal of this study is to investigate tendencies of students to choose cards. Especially, this study pays attention to the first card that learners select at the start of each assignment.

2.2 Change of Approach through the Problem Posing Activity

In this research, the analysis of MONSAKUN log data from an experiment of MONSAKUN used by eleven undergraduate students from Faculty of Education is reported. We analyzed the subjects' log data in assignments at Level 1 (lowest difficulty) and Level 5 (highest difficulty) which require the subjects to pose forward-thinking problems and reverse-thinking problems, respectively. Both levels consist of 12 assignments and four story types: combination, increase, decrease, and comparison.

The sentence cards in MONSAKUN contain different number according to the given assignment. Here, cards are distinguished by the order of numbers in the required calculation expression. For example, in an assignment "Make a story problem about "How many are the difference" that can be solved by $7 - 3$ ", where the required calculation is " $7 - 3 = _$ "; a card contains the first number (7) is called "first number card". Similarly, a card contains the second number (3) or the third number (blank) is called "second number card" or "third number card", respectively.

From the analysis, we found that the proportion of each sentence card to be selected firstly is entirely not even. In Level 1, the percentage of first, second, and third number card being firstly selected are 91.8%, 3.3% and 4.9%, respectively. However, in Level 5, the percentage changed to 58.7%, 16.5% and 24.8%, respectively. We found that there is a bias against first selected card, which shows our assumption that the subjects did not choose a card randomly, but with some sort of approach. Furthermore, there is a different trend between Level 1 and 5. We presume that this difference between Level 1 and Level 5 appeared because subjects had different approach to pose either forward-thinking or reverse-thinking problems.

Table 1 shows the characteristics of first selected card from each assignment at Level 5 that has marginal or significant difference in number of selection from the average. These results were

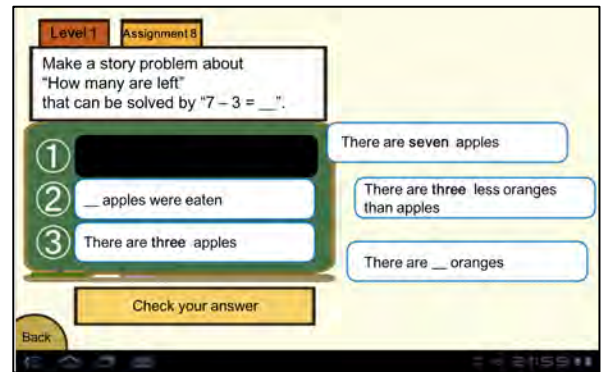


Figure 1. Interface of MONSAKUN

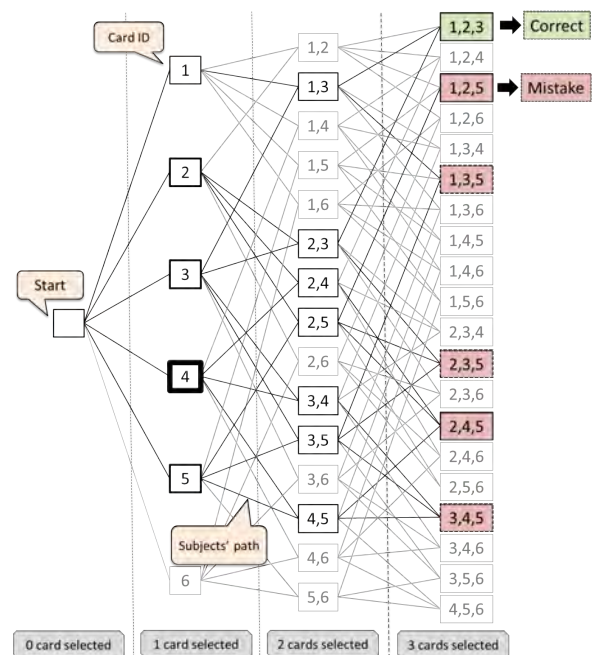


Figure 2. Illustration of a search space in a MONSAKUN assignment

analyzed with binomial test to the amount of each card being firstly selected or not in each assignment. Binomial test is an exact test of the statistical significance of deviations from a theoretically expected distribution of observations into two categories. Based on our assumption that students posed problems by selecting cards through a thinking process, we expect the distribution of first selected cards to have a significant difference in comparison with other cards.

Table 1. Result of binomial test of first selected card in Level 5 assignments

Assignment Number	Type of story	Order of assignment	Type of first selected card	Type of sentence	p-value
1	Combination	1 st	First number card	Existence	7.05×10^{-5} **
2		2 nd	First number card	Relational	1.88×10^{-7} **
3		3 rd	First number card	Relational	1.97×10^{-3} **
4	Increase	1 st	First number card	Existence	1.89×10^{-5} **
5		2 nd	Second number card	Existence	0.0504 +
6		3 rd	First number card	Existence	0.0504 +
7	Decrease	1 st	First number card	Existence	2.35×10^{-4} **
8		2 nd	Second number card	Existence	2.35×10^{-4} **
9		3 rd	Second number card	Existence	2.35×10^{-4} **
10	Comparison	1 st	-	-	-
11		2 nd	Third number card	Relational	0.0266 *
12		3 rd	Third number card	Relational	0.0266 *

**: significant difference ($p < .01$), *: significant difference ($p < .05$), +, marginal difference ($p < .1$)

Subjects are firstly given simple forward-thinking problems to pose at Level 1 of MONSAKUN. Here, we found that they first simply selected an existence sentence card with the first number in the required calculation expression, and then proceeded to choose other appropriate cards. This approach worked well for Level 1. When subjects arrived at Level 5 Assignment 1, they initially approached the assignment with the same way of thinking. However, this did not work well, and they tend to make more mistakes than in the previous levels. We presumed that the subjects were aware that the previous approach did not work for reverse-thinking problems, because as seen in Table 1, in Level 5 Assignment 2 they selected different type of card, which is a relational sentence card containing first number. In a similar way, subjects changed their approach from the first order of assignment in a type of story to the second and third order of assignment in the same story type.

This leads to two findings about changes in subjects' way of thinking through the exercises. The first one is that subjects change their approach to pose problems after they had experienced posing the same story type. The next finding is that the change of approach depends on the story type.

These changes of thinking approach seem to bring a good effect, because in comparison to the first order of assignment in a story type, the average of steps and mistakes in the second and third order of assignments of the same story type are mostly decreased.

3. Concluding Remarks

In this research, we have conducted analysis of MONSAKUN log data of university students to investigate their thinking process in problem-posing activity. We found that depending on type of story and subjects' exercise experience, they selected different first sentence card. These findings proved our assumption that users of MONSAKUN did not chose sentence cards randomly, but with some sort of thinking process.

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