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# The Effect of Integrating Algebraic Thinking in Problem-Based Learning via Virtual Environment among Secondary School Students

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**Abstract:** In this paper, we conducted a study that integrated students' algebraic thinking into the problem-based learning (PBL) process. Three different learning approaches, namely, the conventional approach (CA), the integration of algebraic thinking (AT) and the PBL approach with the integration of algebraic thinking (ATPBL); were implemented in three different schools involving 85 students in a quasi-experimental study. Results showed that the algebraic thinking process of students in ATPBL sessions was enhanced. A parametric test using MANCOVA revealed that the students from the ATPBL group performed better in exploring relationships, generalizing and formalizing, reasoning about and with representations, and using algebra as a tool compared to the students from the AT group. The ATPBL group performed significantly better in the manipulation of symbols and procedures, exploring relationships, generalizing and formalizing, reasoning about and with representations, and using algebra as a tool compared to the students from the CA group. Additionally, there was significant difference in students' algebraic thinking in the AT group compared to the CA group. In conclusion, this study suggests that the PBL approach with the integration of algebraic thinking is able to enhance algebraic thinking among students at lower secondary level.

**Keywords:** Algebraic Thinking, Problem-Based Learning, Virtual Environment

## 1. Introduction

Algebraic thinking is a process by which the students express and build mathematical relationships practically (Soares, Blanton and Kaput, 2006). According to Kieran (2004), algebraic thinking is a way for students to focus on relations, operations, alphanumeric characters, representing and solving problems as well as refocusing the meaning of the equals sign. The foundation of algebraic thinking is developed as the student becomes able to make connections of patterns with the real world (McGarvey, 2012). According to Mason et al. (2005), algebraic thinking develops through the combination of four strands of algebra with mathematical themes and mathematical powers within conjecture conditions. Korea, China, the USA, and Canada are among the countries that have their own curriculum for algebraic thinking skills, which leads to better performance in algebra compared to Malaysia. The mathematics curriculum in Malaysia is only focused on thinking and reasoning in general. However, some elements of algebraic thinking do exist. Therefore, study is required to identify the appropriate strategy to make the learning of algebra more comprehensive and effective (Cai et al., 2005). To encourage students to practice such thinking, proper teaching and learning activities should be designed, and one potential strategy is PBL. PBL is the one of the best approaches that emphasizes problem as a starting point, followed by student-centered and teacher as a facilitator in the learning process. It is also proven that PBL provide positive impact in student's achievement in mathematics. However, there are scarce of studies pertaining algebraic thinking in PBL, whereby algebraic thinking is important as a foundation of success in learning algebra.

## 2. Problem Statement

A number of researchers have investigated the algebraic thinking of middle school students (Ayalon and Even, 2013; Booth et al., 2014). However, not all the characteristics of algebraic thinking have been demonstrated among students. Early algebra emphasizes algebraic thinking, which involves the understanding of arithmetic relationships, generalizing and recognizing variable structure. Early algebra should be differentiated from typical algebra in terms of contents, subjects, and teaching methods (Lee and Pang, 2012). The development of algebraic thinking requires students to (1) help themselves to make a smooth transition between arithmetic and algebra and (2) appreciate the usefulness of generalized algebraic approach in solving various problems (Cai and Moyer, 2008; Cai et al., 2005). It is important to study algebraic thinking because it could make the learning of algebra more comprehensive and allow the development of an algebraic perspective of mathematics. Furthermore, algebraic thinking is able to develop a deeper understanding of the underlying structure of mathematics, dealing with generalizations and ways of thinking that allow results to be expressed across a range of problem forms rather than simply finding a particular answer to a series of individual problems. The importance of algebraic thinking in learning algebra will guide teachers in teaching algebra effectively as well as enhance students' algebraic thinking. Another strategy, as indicated by the literature, is to use PBL, an approach that focuses on the development of thinking. This method has been perceived to be able to facilitate students' knowledge construction and reasoning skills because it uses real-world problems as the starting point in the learning process.

## 3. Research Questions

The study is conducted to investigate;

(i) Is there any significant difference in algebraic thinking for students before and after being taught;
   a. by the conventional approach (CA) compared to the integration of algebraic thinking (AT)?
   b. by conventional approach (CA) compared to the PBL approach with the integration of algebraic thinking (ATPBL)?
   c. with the integration of algebraic thinking (AT) compared to the PBL approach with the integration algebraic thinking (ATPBL)?

Based on the research questions, the researcher has put forward several null hypotheses ($H_0$) built on the significance level $\alpha = 0.05$ as follows;

- $\mathbf{H_{01}}$ There is no significant difference in students' algebraic thinking before and after being taught using CA compared to AT.
- $\mathbf{H_{02}}$ There is no significant difference in students' algebraic thinking before and after being taught using CA compared to ATPBL.
- $\mathbf{H_{03}}$ There is no significant difference in students' algebraic thinking before and after being taught using AT compared to ATPBL.

## 4. Conceptual Framework

The framework of algebraic thinking put forward by Walkoe (2013) was adopted because its description of algebraic thinking is sufficiently detailed and comprehensive to be implemented for secondary level students. Furthermore, it is suitable for the syllabus of Form Two students in Malaysia. Most of the characteristics of algebraic thinking in Walkoe's (2013) framework, namely the manipulation of symbols and procedures, exploring relationships, using algebra as a tool, reasoning about and with representations and connecting representations, are explicitly taught to Form Two students, the only exceptions being generalizing and formalizing. However, not all of these construct are addressed explicitly, such as justifying, proving, thinking about or with representations of functions such as graphs, table and situations and using one representation to reason about another. The strength of Walkoe's (2013) framework is it expansion and extension of the framework developed by Kieran (1996). Walkoe's (2013) framework is applicable for secondary school students' manipulation of symbols and procedures, using algebra as a tool and connecting representations. However, reasoning about and with representations, exploring relationships and generalizing and formalizing can be further enhanced through a suitable approach. It is important to identify the characteristics of algebraic thinking among students as well as guidance provided by teachers and discussion with peers. This can lead the students to make connections between arithmetic and algebra as well as to think algebraically rather than focusing on computational fluency. Walkoe's (2013) framework consists of manipulation of symbols and procedures, exploring relationships, generalizing and formalizing, using algebra as a tool, reasoning about and with representations and connecting representations. Interventions were implemented for Form Two students using the learning task for the AT group and the learning task for the ATPBL group. The ATPBL group was given two scenario problems in the learning process based on Tan's model (2003) of PBL. Tan's model is suitable to be applied for any subject matter, as it emphasizes problem-solving skills and new areas of learning.

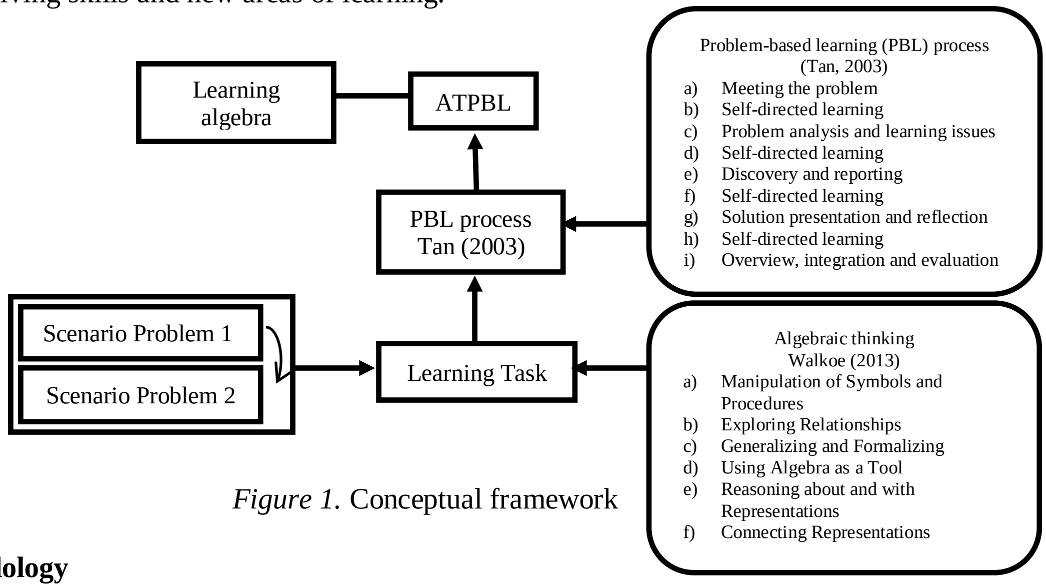


*Figure 1.* Conceptual framework

## 5. Methodology

This study involved two interventions, namely the integration of algebraic thinking (AT) and the problem-based learning (PBL) approach with the integration of algebraic thinking (ATPBL) in learning algebra. The aim was to test whether the interventions are effective within a certain period.

Table 1
*Research design*

| Groups | Pre-test | Intervention | Post-test |
|---|---|---|---|
| Experimental 2 | $0_1$ | $X_2$ | $0_2$ |
| Experimental 1 | $0_1$ | $X_1$ | $0_2$ |
| Control | $0_1$ | - | $0_2$ |

*X1 = AT, X2 = ATPBL

A pre-test was administered before the treatment to the two experimental groups and a control group. The first experimental group consists of students who were taught with the integration of algebraic thinking (AT), while the second experimental group experienced learning algebra through a PBL approach with the integration of algebraic thinking (ATPBL). The control group went through the conventional approach (CA) to learning algebra. The purpose of the test was to identify the characteristics of the students' algebraic thinking before treatment. A post-test was given after the treatment for the two experimental groups and the control group taught using the conventional approach. The aim of this post-test was to identify the same characteristics of algebraic thinking as those in the pre-test. The same algebraic thinking test was given to the students to all groups. A reliability test was conducted to establish test-retest reliability. The fifteen selected students were given a pre-test and a post-test three weeks later. The test was administrated for one hour and thirty minutes. A Pearson's correlation was computed to assess test-retest reliability of the algebraic thinking test scores, r (15) = .796. This is considered significant; reliability coefficients should be positive and greater than .70 (Leech, Barrett and Morgan, 2005).

The teaching of algebra involving the ATPBL implemented two scenario problems through the PBL model. The process of teaching and learning was conducted in a computer lab, which was not in the academic building. This was to provide the students with internet access and because of the lab's proximity to the library, where they could easily find resources. The teacher who volunteered to conduct PBL was using virtual environment (*Frog platform*). She was briefed on how to conduct PBL. She also received a module to guide her in running lessons on algebraic thinking using the PBL approach. The learning materials, which are the learning tasks of ATPBL, were uploaded to *Frog platform*.

### *5.1 Sampling*

In this study, purposive sampling was used to select boarding schools in one state in Malaysia based on homogeneity of scores in the algebraic thinking test. Three of these boarding schools were selected based on algebraic thinking test scores. These students were selected due to possession of equivalent ability of algebraic thinking. The three selected boarding schools were randomly assigned as two experiment groups and one control group. There are five classes in each of the selected boarding schools with mixture boys and girls students. However, only one class was selected from each school. The classes were selected based on permission from the schools' administrators. The total number of participants involved in this study was 85 students.

## 6. Findings and Discussion

### *6.1 Difference in Algebraic Thinking in Students before and After Being Taught Using the Conventional Approach (CA)*

This section presents the difference in algebraic thinking for students before and after being taught using the CA. A pre-test and post-test were conducted with 25 students to identify the difference in students' algebraic thinking before and after being taught using the CA. Table 2 shows the results of the paired sample t-test and Table 3 shows the effect size value for dependent means for the CA group.

Table 2
Paired samples t-test for before and after CA

| | Paired Differences | | | | |
|---|---|---|---|---|---|
| Pair 1 Pre-Test- Post-Test | Mean | SD | T | df | Sig. (2-tailed) |
| | 16.12 | 66.07 | 1.22 | 24 | .234 |

**n* =25, students, α = 0.05

Table 3
The effect size value for dependent means

| | Mean | SD | Effect size, Cohen *d* |
|---|---|---|---|
| Post-Test | 208.60 | 77.78 | .21 |
| Pre-Test | 224.72 | -73.76 | |

A paired-samples t-test was conducted to evaluate the effect of CA on students' algebraic thinking. From Table 2, the results of the paired samples t-test indicate that the null hypothesis cannot be rejected. A decrease in Algebraic Thinking Test scores from pre-test (M = 224.72, SD = 73.76) to post-test (M = 208.60, SD = 77.78), t (24) = 1.22, p = .234, with an effect size of d = .21. The guidelines proposed by Cohen (1992) for interpreting this value are as follows; .20 = small effect, .50 = medium, .80 = large effect. Given that d = .21, this indicated that there was a small effect, with a difference in the algebraic thinking test scores obtained in the pre-test and post-test.

### *6.2 Difference in Algebraic Thinking in Students before and After Being Taught Using the Integration of Algebraic Thinking (AT)*

This is to identify the difference in students' algebraic thinking before and after being taught using AT. The pre-test and post-test were conducted before and after the intervention. Thirty students took the pre-test and post-test. Table 4 shows the result of paired sample t-test and Table 5 shows the effect size value for dependent means for the AT group.

Table 4
Paired sample t-test for before and after AT

| | Paired Differences | | | | |
|---|---|---|---|---|---|
| Pair 1 re-Test- ost-Test | Mean | SD | T | df | Sig. (2-tailed) |
| | -100.80 | 66.06 | -8.36 | 29 | .000 |

Table 5
The effect size value for dependent means

| | Mean | SD | Effect size, Cohen *d* |
|---|---|---|---|
| Post-Test | 337.33 | 58.36 | 1.73 |
| Pre-Test | 236.53 | 68.88 | |

A paired-samples t-test was conducted to evaluate the effect of AT on students' algebraic thinking. From Table 4, the results of paired samples t-test indicated that the null hypothesis should be rejected. There was a statistically significant increase in Algebraic Thinking Test scores from pre-test (M = 236.53, SD = 68.88) to post-test (M = 337.33, SD = 58.36), t (29) = -8.36, p = .000, with a large effect size (d = 1.73), indicating a substantial difference in the Algebraic Thinking Test scores obtained in the pre-test and post-test.

### *6.3 Difference in Algebraic Thinking in Students Before and After Being Taught Using the Problem-Based Learning (PBL) Approach with the Integration of Algebraic Thinking (ATPBL)*

This section attempts to identify whether there was a significant difference in students' algebraic thinking before and after being taught using ATPBL. The pre-test and post-test were conducted before and after the intervention with 30 students. Table 6 shows the result of the paired sample t-test and Table 7 shows the effect size value for the dependent means for the ATPBL group.

Table 6
Paired sample t-test for before and after AT

| | Paired Differences | | | | |
|---|---|---|---|---|---|
| Pair 1 | Mean | SD | t | df | Sig. (2-tailed) |
| Pre-Test-Post-Test | -49.20 | 53.88 | -5.00 | 29 | .000 |

**n* =30, students, α = 0.05

Table 7
The effect size value for dependent means

| | Mean | SD | Effect size, Cohen *d* |
|---|---|---|---|
| Post-Test | 317.70 | 59.07 | .83 |
| Pre-Test | 268.50 | 63.91 | |

A paired-samples t-test was conducted to evaluate the effect of ATPBL on students' algebraic thinking. From Table 6, the results of the paired samples t-test indicated that the null hypothesis should be rejected. There was a statistically significant increase in Algebraic Thinking Test scores from the pre-test (M = 268.50, SD = 63.91) to the post-test (M = 317.70, SD = 59.07), t (29) = -5.00, p = .000, with a large effect size (d = .83).

### *6.4 Difference in Algebraic Thinking in Students Taught Using the Integration of Algebraic Thinking (AT) Compared to the Problem-Based Learning (PBL) Approach with the Integration of Algebraic Thinking (ATPBL)*

This section will describe the difference between AT and ATPBL based on outcomes from the MANCOVA. These findings are set out in Table 8, which compares students' algebraic thinking between the AT group and the ATPBL group.

Table 8
Comparison between AT and ATPBL

| | AT | | | | | |
|---|---|---|---|---|---|---|
| ATPBL | MSP | ER | GF | UA | RA | CR |
| MSP | .001 | | | | | |
| ER | | .979 | | | | |
| GF | | | .939 | | | |
| UA | | | | .427 | | |
| RA | | | | | .611 | |
| CR | | | | | | .003 |

The results are significantly different for two characteristics of algebraic thinking, namely manipulation of symbols and procedures and connecting representations, for which the p - value is less than .05. However, the p - values for exploring relationships, generalizing and formalizing, using algebra as a tool and reasoning about and with representations are bigger than .05. The null hypothesis thus cannot be rejected and the researcher concludes that there was no significant difference in students' algebraic thinking when they were taught by ATPBL compared to AT except in the manipulation of symbols and procedures and connecting representations.

## 7. Conclusion

This study shows there are significant differences in students' algebraic thinking before and after being taught using AT and ATPBL approach. Furthermore, there are substantial differences in students' algebraic thinking after being taught using AT compared to CA and after being taught using ATPBL compared to CA. Despite that, "statistically significant" does not mean practical significance since the result is unlikely due to chance. The performance of students might be lower or higher based on the significant difference. Therefore, the researcher indicates the results for mean values of post-test. The results showed that students in the ATPBL group performed better in exploring relationships, generalizing and formalizing, using algebra as a tool and reasoning about and with representations compared to the AT group.

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### References

Ayalon, M., and Even, R. (2013). Students' opportunities to engage in transformational algebraic activity in different beginning algebra topics and classes. *International Journal of Science and Mathematics Education. 13*(November), 285–307.

Cai, J., Lew, H. C., Morris, A., Moyer, J.C , Ng, S. F., and Schmittau, J. (2005). The Development of Students ' Algebraic Thinking in Earlier Grades : A Cross-Cultural Comparative Perspective. *ZDM.37(*1), 5–15.

Cai, J., and Moyer, J. C. (2008). *Developing Algebraic Thinking in Earlier Grades : Some Insights from International Comparative Studies*. NCTM Yearbook.

Fraenkel, J. R., and Wallen, N.E. (2009). *How to Design and Evaluate Research in Education*. (7th ed.) McGraw-Hill Higher Education.

Hamid, D. H. T. A. H. (2006). Reka Bentuk dan Keberkesanan Perisian Multimedia Membaca-Faham Berasaskan Gambaran Visual Bagi Kanak-kanak Prasekolah. Ijazah Doktor Falsafah, Universiti Teknologi Malaysia, Skudai.

Hertzog, M. A.(2008). Considerations in Determining Sample Size for Pilot Studies. *Research in Nursing & Health. 31(*2), 180–191.

Hung, W. (2003). An Investigation of the Role of Causal Reasoning Methods in Facilitating Conceptual Understanding of College Students in Physics. Doctor Philosophy, University of Missouri-Columbia.

Kieran, C. (2004). Algebraic Thinking in the Early Grades : What Is It ? *The Mathematics Educator. 8*(1), 139–151.

Lee, J., and Pang, J. (2012). An Analysis of Fraction Operation Sense To Enhance Early Algebraic Thinking. *Journal of the Korea Society of Mathematical Education.16 (*4), 217–232.

Leech, N. L., Barrett, K.C., and Morgan, G.A. (2005). *SPSS For Intermediate Statistics Use and Interpretation*. (2nd ed.) Lawrence Erlbaum Associates, Publishers.

Mason, J., Graham, A., and Johnston-Wilder, S. (2005). *Developing Thinking in Algebra*. (1st ed.) Paul Chapman Publishing.

McGarvey, L. M. (2012). What Is a Pattern? Criteria Used by Teachers and Young Children. *Mathematical Thinking and Learning. 14(*4), 310–337.

Olaoye, O., and Adu, E. O. (2015). Problem-based Learning Strategies and Gender as Determinant of Grade 9 Students' Academic Achievement in Algebra. I*nternational Journal of Education Science*, 8(3), 485–492.

Osman, K., and Kaur, S. J. (2014). Evaluating Biology Achievement Scores in an ICT integrated PBL Environment. *Eurasia Journal of Mathematics, Science & Technology Education*. 10(3), 185–194.

Reaves, C.C. (1992). *Quantitative Research for the Behavioral Sciences*. (K. Dubno, Ed.) (1st ed.) John Wiley & Sons, Inc.

Sekaran, U., and Bougie, R. (2010). *Research Methods for Business A Skill Building Approach*. (5th ed.) Wiley, A John Wiley and Sons, Ltd, Publication.

Soares, J., Blanton, M. L., and Kaput, J.J. (2006). Thinking Algebraically across the Elementary School Curriculum. *Teaching Children Mathematics. 12*(5), 228–235.

Tan, O. S. (2003). *Problem-based Learning Innovation Using problems to power learning in the 21st century*. (1st ed.) Thomson.

Walkoe, J. D. K. (2013). Investigating Teacher Noticing of Student Algebraic Thinking. Doctor Philosophy, Northwestern University.