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Math Creation: Integrating Peer Tutoring for Facilitating the Mathematical Expression and Explanation

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Abstract: The purpose of this study is to contribute insights into how math creation integrated peer tutoring can facilitate students' mathematical expression and explanation. Twenty-five second-grade primary students play the role of teachers to construct their math creations as the teaching materials to teach their partners. Specifically, the students need to solve given word problems and produce three forms of creation—including drawing expression, arithmetic expression, and solution explanation—to justify their solutions. This study shows that peer tutoring can promote students' drawing expression and solution explanation by providing opportunities for students to reexamine their math creation mutually. However, the progress of arithmetic expression does not achieve significantly. Besides, this study also finds that high and low achievers have different sequences in completing math creations and ways of participation. Finally, how computers facilitate and change students' mathematical expression and explanation compared with paper-based environment is also discussed.

Keywords: Math creation, peer tutoring, mathematical explanation, mathematical expression

1. Introduction

During the last decade, the educational reform movement in Taiwan has advocated for increasingly emphasizing students' mathematical expression and explanation. One of the government policies in Ministry of Education of Taiwan (2003) is "to understand the comprehension of the mathematical language, such as symbols, terms, tables, graphs and informal deductions". Furthermore, mathematical expression and explanation, a critical approach to present the mathematical language, may show how people understand mathematical concepts and use those ideas. More specifically, peer explanation may facilitate spontaneous using the mathematical language such as diagrams in solving mathematics word problems (Uesaka & Manalo, 2011).

Teaching mathematical concepts and principles by depicting problems graphically and using peer explanation activities during mathematics instruction had found to be consistently effective (Griffin & Jitendra, 2009). During peer explanation, students' gains in drawing diagrams likely contributed to their diagram construction skills (Uesaka & Manalo, 2011). Besides, the explanation of students as explainers may become more complete because they have to monitor their own misunderstanding and knowledge gaps (King, 1994), which may help them understand (Roy & Chi, 2005). In other words, mathematical explanation may facilitate mathematical comprehension (McNamara, 2004). On the other hand, learning in pairs or small groups has positive learning benefits on children's learning development (Dillenbourg, 1999). Therefore, applying peer explanation to peer tutoring could be an adequate learning approach to foster students' mathematical expression and explanation because tutors' explanations may expose tutees to the information they lacks (VanLehn et al., 2007), which may assist their tutees' learning (Chi, Siler, Jeong, Yamauchi, & Hausmann, 2001).

Nowadays, as most classrooms have provided Internet access and one-computer-per-student, it is possible for students to create their own math creations as an educational strategy for training mathematical expression and explanation. Chi (2009) posits that learners benefit most from active constructing their own artifacts and explanations. Furthermore, peer tutoring is used as a good strategy for the facilitation of mathematical explanations (Berghmans, Neckebroeck, Dochy & Struyven, 2013). Developing mathematical representation and solution explanation by constructing an appropriate math creation for integrating peer tutoring activity is likely to improve the comprehension and completion of the math creation. Students are also more likely to apply these skills through peer tutoring, thus advancing the quality of their works. Taking the literature and current trends together, a question arises if constructing math creation could enhance students' mathematical expression and explanation in a peer tutoring activity. Therefore, the current study is conducted to examine the effects of math creation followed by peer tutoring to improve the performance of second graders in their mathematical expression and explanation. The method for exploring this question is described as follows.

2. Method

This research investigated how students constructed their math creations by integrating peer tutoring for facilitating mathematical expression and explanation. Students had to write down their solutions (arithmetic expression), draw the mathematical representation (drawing expression), as well as explain how and why they solve word problem in their own ways (solution explanation). This section will describe the participants, the activity procedure, the learning system and the supplementary learning sheet, followed by data collection and analysis.

2.1 Participants

Twenty-five second-grade primary students (13 boys and 12 girls) and their teacher from the same class at a public school in the north region of Taiwan participated in this study. The students had their own tablet PCs brought by their parents in schools since first grade and had developed good competence and familiarity in mathematical self-learning. In this study, they were assigned to six groups by their teacher. There were four or five students in each group, where everyone was further paired to each other. The teacher was willing to participate in the activity of math creation, so she encouraged students to engage in peer tutoring and math creation. Besides, she also provided students with many opportunities to show their works and to teach the whole class in front of the classroom.

2.2 The Learning System & Supplementary Learning Materials

The system of math creation was called Little Mathematical Teacher (LMT), which was designed to assist students' learning and to visualize their math creations with a sketch-board and a sharing zone. Besides, the researchers also provided the supplementary learning sheets to scaffold students during the math creation and peer tutoring.

For the purpose of assessing students' mathematical expression and explanations in math creation, students were asked to complete a word problem solving assignment in the sketch-board. The sketch-board was divided into three parts, including drawing expression, arithmetic expression, and solution explanation (see Figure 1). The sketch-board provided a component library, which contained various mathematical components, such as coins, building blocks, and carton images etc., suggested by the teacher in order to meet the needs of primary students. Students can utilize those components to prepare their drawing expression, arithmetic expression and solution explanation for concretely explaining their solution procedures in their math creations.

Meanwhile, different word scaffoldings were used according to students' abilities of mathematical explanation. In initial activities, more word descriptions were provided to help students think related mathematics concepts by completing the keywords of the word problems. Students can learn how to explain their mathematical ideas to others through imitating the similar explanatory patterns. After that, the system only provided simple conjunctions to facilitate students to think appropriate solutions, the ways of solving, and what the most important reason was. For example: *[First, I used]* <u>one calculating method</u>, *[because]* <u>(I) wanted to ....</u> *[Then, I used]* <u>one calculating method</u>, *[because]* <u>…</u>".

Besides, the LMT provided a sharing zone to display all students' math creations. When students finished their math creations and saved them, the creations were uploaded to the sharing zone. Students only had to choose the classmate's number and then his/her creation would show up (see

Figure 2). This function helped students share, observe, and understand the math creations of the whole class. They can thus learn mutually and take advantage of this function to teach their partner about what the mathematical concepts applied and how to solve the problem.

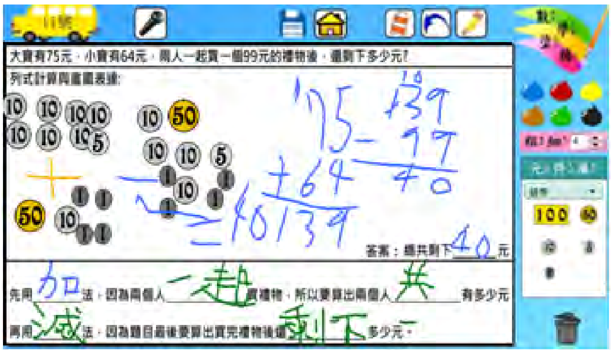

Figure 1. The interface of the sketch-board

Figure 2. Individual's math creation displaying in the sharing zone

To facilitate students' math creations, three supplementary learning sheets were given. Two question sheets embedded in the system were used in different stages. One question sheet listed relevant questions about the assigned word problems. This study adopted two forms of a question sheet revised from Mason (2000) to prompt students' awareness development of mathematical thinking and to allow students to answer questions for themselves or a partner. The two forms were enquiring and testing questions. First, enquiring questions guided learners to understand every meaning through several divided problem solving steps. Hence, this study adopts enquiring questions to guide students explaining the meaning of problems by drawing and arithmetic expression when students created mathematical artifacts. Students can also ask their partner or classmates some questions as those on question sheet. Thus, even if they have not thought of any questions on their own, they can still take advantage of the question sheet to join the peer tutoring activity. The example questions were shown as follows. (1) What does this problem ask for? (2) How do you express six boxes of milk and each box have twelve bottles of milk by drawing expression? (3) How do you express the milk drank by drawing expression? (4) How do you express the milk left by drawing expression? (5) What does each calculation mean?

Second, testing questions were used to ask students to explain their own mathematical thinking in order to evaluate their understanding, and help peer mutually examine the correctness of their drawing expression, arithmetic expression, and word explanation. Additionally, students can ask relevant questions if they have any unclear or doubtable part, for example:

(1) If there is something unclear or incorrect in the instructor's drawing expression, please question him/her.

(2) Is his/her arithmetic expression correct? If there is something wrong, please find out and tell him/her the correct arithmetic expressions.

(3) Is his/her explanation correct? If there is something wrong, please find out and tell him/her how to modify it.

Many students spent most of their creating time in thinking about how to draw the mathematical representation to express their solutions. Therefore, the researchers provided an additional supplementary learning sheet with six examples of mathematical representation in paper form to assist students' drawing expressions.

2.3 Learning Procedure

This study was conducted for 13 weeks. All students participated in 13 times of learning activities, and each learning activity took two class periods, approximately 80 minutes. Before the experiment, the researchers explained the usage of the sketch-board to the students and asked them to construct their own math creations for teaching their students (*i.e.* paired members). These creating steps were chosen and proceed in the sequence based on Polya's (1957) four stages of problem solving: understanding the problem, devising a plan, carrying out the plan, and looking back. However, the final step, *i.e.* checking calculations, was revised to solution explanation to help students re-think how they solved the problem. Furthermore, in order to promote the activity of math creation smoothly, the first step, *i.e.* understanding the problem, was conducted as an independent session by discussing solutions with their partners to understand the condition given and the problem asked. Therefore, four learning sessions were described as follows.

Session 1: Understanding the word problem. In the first session, each student received a word problem and a question sheet (the details were in the Supplementary Learning Sheet). Students read the word problem and, as a group, tried to understand it by discussing what the problem meant and how to explain its solution with their partners. They, furthermore, tried to connect with the requirement of solving such problem and think about what was the proper strategy of the problem. Students thus got joint understanding of problem required. This session usually took five minutes.

The students worked in pairs. In the first to seventh activities, students were provided the same word problem in order to help them familiar with the activity and discussed with each other more easily. In addition, they were able to learn how their peers solved the same problem with different strategies and explanations. After that, students were given parallel word problems in the same pair to enhance their learning and thinking more other word problems with similar concept and structure, but had different scenarios and numbers for avoiding students feeling bored with the same word problems, which may lead to reduce the benefits of peer tutoring. The examples of parallel word problems were shown as follows.

(1) Liz invited 26 classmates to her home for eating cakes. Every cake was divided into 13 pieces. Liz prepared 4 cakes, and everyone could have one piece. How many pieces of cakes left at last?

(2) Mei prepared 7 cakes, and each cake was divided into 10 pieces. She invited 26 classmates to her home for eating the cakes. Everyone had 2 pieces. How many pieces of cakes left at last?

Session 2: Prepare one's math creation. All students prepared their math creations as the teaching materials individually in the classroom. More specifically, s/he solved a problem on his/her own and preparing his/her math creation on the sketch-board, including: (1) Drawing expression: Students used words, symbols, models and manipulative materials as their mathematical representations to devise a plan and convey their ideas and communicative information; (2) Arithmetic expression: Students wrote down their mathematical equations for solving this problem as carrying out the plan; (3) Solution explanation: Students reflected how and why they solved the problem and wrote their solution explanation for looking back. Students had to try an understandable approach in his/her drawings and explanation for others because one's drawing created the mental representations of the solution. After finishing the math creation, the student could save it and then view other math creations made by their classmates in the online sharing zone.

Session 3: Peer tutoring. Students were given ten minutes for tutoring the partner in each group. The researcher asked students to use a recorder as a microphone. For sharing ideas with a peer, students may hold the recorders to play the role of little math teacher and debrief the representation of his/her thinking, such as how s/he solved the problem and what s/he considered. And then the partner had to take over the recorder and play the role of learner to ask the little math teacher some relevant questions. The little math teacher had to answer these questions. At the time, the learner was easy to check with the solution of the little math teacher, because s/he might discover some incorrect parts of the solution during debriefing. In another turn, the paired students had to switch their roles.

Session 4: Presenting solution. Students had to teach the whole classmates by presenting their drawing and solutions of math creation in the electronic whiteboard in the final session. However, due to the time limit, the teacher only chose seven or eight students as the presenters in each activity. Then the other students may ask the presenter some questions about their solutions. After the solution presentation, s/he may reflect on his/her math creation and oral explanation for improving next time.

2.4 Data collection and analysis

To examine how students advanced the mathematical expression and explanation in LMT, data collection included students' math creations, class observations, and interview. The researchers calculated the scores of three sub-abilities in students' math creation, which included: drawing expression, arithmetic expression, and solution explanation. Each sub-abilities was 5 points at most, so the total score is 15 points. We scored the expression and explanation by considering if the key concept was involved in the description and if the relationship of conditions problem given were showed. The evaluation criteria of drawing expression were shown as follows. (a) 1=incorrect representation, which using concrete objects referred to the problem, but the representation was incomprehensible; (b) 2=correct but drawing objects without marking meanings; (c) 3=calculation form of calculating representation; (d) 4=correct but incomplete schematic representation; (e) 5=schematic visual representation, which could express the spatial relations between objects in a problem for explaining their solution strategy. To ensure the reliability of scores of math creation, the scores was evaluated by two raters simultaneously. The inter-rater reliability of was 0.91，$p < .000$. Besides, that classroom note focused on the nature and type of tutoring undertaken by primary students. In addition, the semi-structured interviews were conducted to interview the class teacher and the high, moderate, and low achieving students according to their performance of math creations. These qualitative data was transcribed, coded, categorized, and compared in multiple ways for emerging meaningful themes. The classroom notes were used throughout the data analysis process for the purposes of triangulation.

3. Result

3.1 The progress of the expression and explanation in the math creations

The aim of this study was to investigate the mathematical expression and explanation advancement. We examined and marked the online math creation of whole class to gain better understanding of the students' learning process. Figure 3 provided the information about how the means of students' expression and explanation in their math creations changed in 13 times. Overall, it was clear that their mathematical expression and explanation had upward trends.

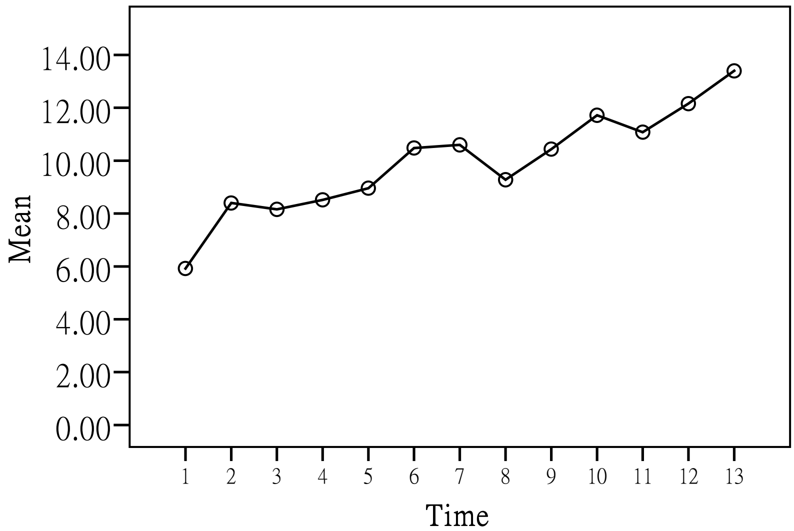

Figure 3. The means of students' expression and explanations of math creations in each time.

Moreover, Figure 4 showed the information about how the means altered by the three sub-abilities in their math creations among 13 times. The score of arithmetic expression was only 2.68 points at the first time. Because some students thought drawing expression was equal to the arithmetic expression and thus did not write the part. However, the mean was a rapid climb to 4.64 points as students caught the key in the second time. After the third time, the following means were a steady increase scores from 4.56 to 5 points. As for the scores of drawing expression, it started at 2 points at first time and after a slight fluctuation reached to just less than 5 points in thirteenth.

It was notable that the scores of both drawing expression and solution explanation had similar patterns. In contrast with the relatively stable performance in the arithmetic expression, the scores of both drawing expression and solution explanation were matched from first to fourth time with 2 points, after a slight increased at sixth time, then dropped to 2.4 points at the eighth time because students had to deal with an advanced learning topic. More specifically, the topic shifted from addition and subtraction mixed problems to addition, subtraction, and multiplication mixed problems. Students had to use new explanatory approach. Meanwhile, students were given parallel word problems from eighth time. But their scores later sustained increased and reached to the highest score at thirteenth time. To identify the relationship between drawing expression and solution explanation, Pearson correlation coefficient was applied. There was a positive correlation between the two variables, $r = 0.894$, $p = 0.000$, which mean that drawing expression and solution explanation had a high, positive correlation. Increases in overall performance of the drawing expression were correlated with increases in performance of the solution explanation.

Overall, students' scores in drawing expression and solution explanation were lower than arithmetic expression from beginning to the end, which meant that the two sub-abilities needed further enhancement. However, through the activity, students gradually improved their drawing expression and solution explanation, which were closed to the level of arithmetic expression in the end.

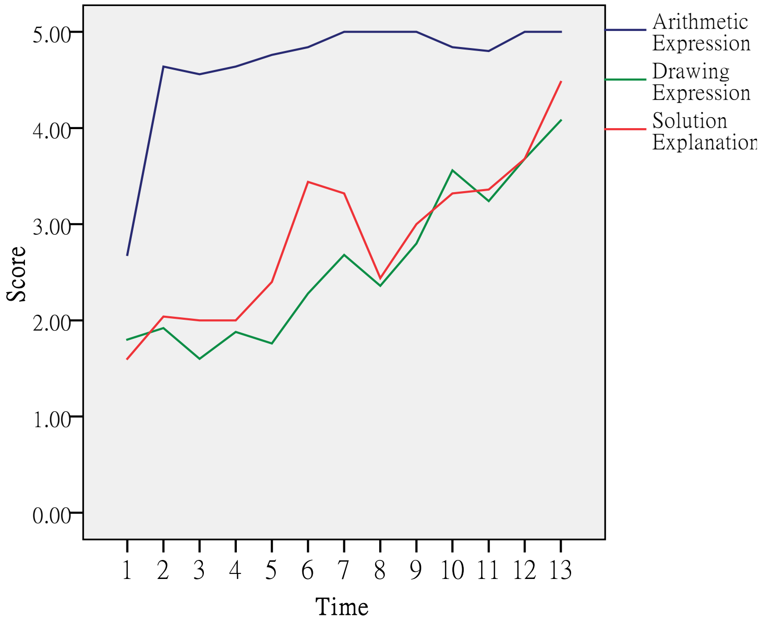

Figure 4. Means of each dimension in students' math creation.

Nevertheless, this study did not have the control group and thus needed more evidence to confirm the improvement of math creation. The researchers compared the initial and final score of math creation to understand the degree of advancement. As mentioned earlier, students were unfamiliar the system operation and rules of math creation at first. However, once students were proficient with system and peer tutoring procedures, they can devote their full attention to the actual content of the lessons (Mathes, Howard, Allen, & Fuchs, 1998). So this study used paired samples t-test to compare the means of second and thirteen math creation. The students achieved mean scores in second math creation (Mean = 8.40, SD =2.52) and thirteen math creation (Mean = 13.40, SD = 3.06) (see Table 1). The result showed a significant difference between the total scores of the two math creations ($t(24) = -10.74$, $SE = .47$, $p = .00$).

Table 1: Students' performances on the second and thirteenth math creation.

Math creation	Second artifacts		Thirteenth artifacts	
	Mean	SD	Mean	SD
Arithmetic expression	4.64	1.15	5.00	0.00
Drawing expression	1.92	1.91	4.08	1.29
Solution explanation	2.04	0.61	4.48	1.08
Total	8.40	2.52	13.40	3.06

Furthermore, in order to investigate the effect of these sub-abilities in the math creation individually, further analysis was conducted. The results showed that integrating peer tutoring for facilitating mathematical expression and explanation resulted in significant improvement on the drawing expression ($t(24) = -2.28$, $SE = .2$, $p = .00$), on the solution explanation ($t(24) = -14.03$, $SE = .17$, $p = .00$). However, there was no significant difference in the arithmetic expression ($t(24) = -1.57$, *SE*=.23, *p* = .13). The findings suggested that these sub-abilities of drawing expression and solution explanation in math creation could be fostered at the same time when they were trained through integrating peer tutoring, while most of their arithmetic expression were very close to the full score therefore it did not show significant improvement.

Apart from the statistic data from the scores of students' math creation, the researchers also interviewed the class teacher to get her perception about students' improvement. The teacher responded "*some students, such as S5, when he found out an appropriate drawing expression like number line; he would adopt this method afterwards and solve problems accurately. Besides, as student introduced his/er math creation, I could find their confidence never showed before.*" As long as students found out their ways to overcome their difficulties of mathematical expression and explanation, they could perform well in the later math creations and have more confident in peer tutoring. They were also more willing to raise their hand for presenting to the whole class. After doing so, they got more feedback from other classmates and teacher, and thus modify their math creations to make a positive loop.

3.2 Progressive examples of the math creations

The progress of math creation was observed by different achieving students. A high achieving student S15 said: "*I didn't know how to explain or teach others about how to solve the problem at first, but now I learned how to explain why I use the addition, multiplication, and subtraction.*" The case of S15 reflected that students had got the point on how to explain the solving method for helping others understand in his solution explanation of math creation through peer tutoring. In addition, S5, a moderate achieving student mentioned that "*I could learn another method [for problem solving] and solve the mathematical problem better*" implying that peer tutoring allowed students to talk over mutual solutions, and students were able to view other classmates' math creations in the sharing zone. Through sharing knowledge and assimilate each other's ideas, students learned new problem-solving strategies and used those for refining their expression and explanation of the math creation. Similarly, another low achieving student S2 stated "*My classmates and I checked the answer mutually to make sure that our equations and answers were correct.*" Students' behavior suggested that integrating peer tutoring into math creation facilitated students not only to provide helps but also to receive aid from their partners.

Additionally, this study provided a typical example among students to show the progress of students' math creations. In the early period, most children's drawing expression only adopted coins to reflect their calculation process. Figure 5 was a typical example. It showed that student did not understand how to map the details of the problem onto his drawing expression. The word problem was "*Da-bao had 75 dollars, and Xiao-bao had 64 dollars, they bought the gift with 99 dollars together, how much money did they leave?*" S5 used many coins as representations in his drawing to replace the number in calculation. Besides, he used apples as the left money, and each apple represent 10 dollars. More specifically, he only put all coins together. All the numbers in the problem were showed in the drawing, but the relationship between the money was not clear, such as which is addend, which is minuend, or subtrahend. In addition, the student misused two different things, coins and apples as the same representations: money. Although the student could calculate by using the information the problem provided as his arithmetic expression, and also fill correct key words in the blanks as the solution explanation, the representation implied that he did not really understand how to draw the mathematical meaning of addition and subtraction mixed problem or could not convey it. The drawing was not correct for other students to understand his thinking and the concept of problem solving.

In the end of the study, the students began to think different explanatory methods and examine the process and reason one step after another. Figure 6 illustrates the students' mathematical expression and explanation in the final math creation. In this example, the word problem was that "*A packet of biscuits costed 15 dollars, Miu bought 5 packets, she paid 100 dollars, how much change should she get?*" S5 drew the box represented the money of a packet and marked 15 in each box, so there were five boxes, and he wrote the total boxes meant 75 dollars. This drawing corresponds with the representation of the problem identified in the first step. Next he drew two row blocks. There were ten blocks in the upper row and seven blocks with half size of block in the lower row. It's obvious that S5 combined the information of the same unit onto the relevant diagram and flagged the goal needed to be found in the problem by using a question mark. More specifically, he drew ten blocks of ten as the paid money, one hundred dollars, then seven blocks and half size of block of ten as the total money needed to pay, and finally used a question mark to represent the unknown money given back. The student's understanding was demonstrated by how he mapped the details of the problem onto his drawing expression.

Furthermore, S5 knew the reason why he used the multiplication and subtraction and the representing meaning of each number. He explained that "*[First, I used] multiplication, [because] one packet cost 15 dollars, and Miu bought 5 packets (calculated the price of five packets). [Then, I used] subtraction, [because] Miu should paid 75 dollars, but she paid 100 dollars (and the change was the price).*" It was clear that the student properly used the representation and explained his solutions.

Figure 5. The initial math creation.

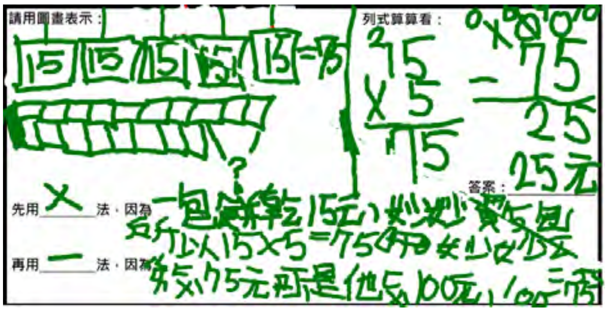

Figure 6. The final math creation.

The difference of two math creations showed the improvement of mathematical explanation from the beginning to the end. It was obvious that constructing students' own math creation and peer tutoring may facilitate students' mathematical explanatory strategies, enhance spontaneously constructing more appropriate diagrams, absorb the mathematical expressing methods of their classmates, and then contribute to his own mathematical expression and explanation.

3.3 The sequences in completing math creation and ways of participation

The teacher and most students had positive attitudes toward constructing their own math creations and engaging in using their works to the peer tutoring activity. However, the classroom observation and interview showed that some students encountered obstacles and tried different ways to involve themselves in the learning activity. The researchers interviewed two low achievement students S13 and S14 and two high achievement students S1 and S15 to further inquiry their perceptions and habits for math creations. Both low achieving students preferred to finish the arithmetic expressions first. S13 emphasized that *"Drawing is a little bit difficult, so I write the arithmetic first. I can calculate the answer"*. It reflected that S13 might not able to draw his solution, but at least he could finish the arithmetic expression in his math creation whether the answer was right or not. Similarly, S14 usually tried many different drawing expressions and then erased them repeatedly, but left the arithmetic expression without drawing expression and solution explanation in the end. Many low achievers had the same behaviors with S14. Such phenomenon was changed gradually after the learning sheet of drawing examples was provided for them to rely on and imitate. Students started to try drawing expressions and saved them.

The teacher usually guided low achievers by monitoring their progress of math creation from the sharing-zone and reminding them to utilize clear marks and descriptions on their drawings expression and solution explanations. Some of them might thus modify the two parts; nevertheless, due to the abilities and time limitation, students might not always complete their drawing expression and solution explanations among the remaining time. However, they would still try to explain their solutions in the peer tutoring but focus on explaining how to calculate the mathematical equations instead of why they solved word problems in that way.

Contrary to the low achieving students, the high achieving students had different creating procedures. S15 preferred drawing first; she said *"I think drawing expression is harder. I can't merely draw the arithmetic; I have to consider how to correspond to the problem and my solution. It would take more time to finish, so I usually draw first, and the following arithmetic expression and solution explanation will be finished very soon."* In addition, S1 chose the alternation of drawing and arithmetic in his math creation. S1 thought "*drawing helps me think about how to solve the problem. I write my arithmetic expression after drawing, and then repeat the procedure [to my solution]*". Hence we knew that high achieving students preferred to draw the mathematical representation first although they also felt that drawing expressions were more difficult. Besides, it seemed that the high achiever tended to use the drawing expression as an assistant approach for arithmetic expression rather than distinguished the two expressions as unrelated elements.

Overall, students spent more time on drawing expression and solution explanation rather than arithmetic expression. Most students tended to finish the arithmetic expression and then back to think about how to draw the mathematical representation and solution explanation except for the high achievers. Low achievers usually only completed the arithmetic expression without examples to rely on. Students had learning tension on withdrawing their drawing expression and solution explanation or not because they could not confirm the correctness of the two parts. Besides, compared with writing the solution explanation, students preferred to explain how to solve problems by oral presentation because their oral explanation was always provided more and sooner than the written explanation.

4. Discussion

This study intends to facilitate students' mathematical expression and explanation by integrating peer tutoring into math creation. Traditional mathematics education emphasizes on the correctness of repeated calculation, so that students may perform well in the arithmetic expression. However, the same procedure (e.g., subtraction) used to solve all problems within a chapter (Bonotto, 2013), students may not have the opportunity to distinguish among problems which need different solutions. This study tries to help students understand the meaning and structure of a problem and asks them to address their mathematical knowledge and solving processes by math creations. Students may fail to develop their mathematical expression and explanation at first, once they gain access to math creation to internalize mathematical concepts or relationships, their improvements, whether in drawing expression, arithmetic expression or solution explanation, were seen clearly.

Computers provide an easier and even instant access with a wider audience for students' math creations compared to similar activities with prepared papers. The sharing zone in the LMT supports not only students' mathematical written expression and explanation, but also oral explanation and feedback during the peer tutoring. The sharing zone demonstrates students' math creation and helps students reflect why they construct the expression and explanation in the ways. Besides, teachers can monitor students' progress, analyze students' problems, and examine the knowledge status of students. Subsequently, both teachers and peers can provide real-time feedback and recommend revisions. Students who share their math creations are able to take advantage of the availability of classmates' works to communicate with each other to get more perspectives and suggestions. Therefore they can draw on the strong points to offset the weaknesses for modifying and refining their math creations, and further expand their capacity to interpret their mathematical concepts.

In contrast, the second-grade students loved to draw creative expression. Many students drew a great deal of irrelevant details in their mathematical representations. Through sharing math creation, students may know more than one way to express their mathematical representations. However, after students chose more efficient and effective representations by observing classmates' artifacts in the sharing zone, they may thus become less creative. Their mathematical representation specifically changed from concrete materials mixed with default components to more abstract self-drawing pictures. Although this was a normal development and also an important goal for primary mathematics, this study still reflected that developing the ability of problem solving and creativity simultaneously from initial to the end was not easy in practical classrooms.

5. Conclusion and Future Work

The aim of this study was to improve students' mathematical expression and explanation by integrating peer tutoring into math creation. We analyzed students' math creation of the whole learning activity and compared the performance of second and final artifacts to identify students' advancement. The drawing expression and solution explanation achieved significant difference, except the arithmetic expression. In addition, this study carried out the interview and collected student's math creation in different activity sessions as learning evidences in terms of mathematical expressions and explanation. The low and high achievers' creating sequence in math creation were also explored. However, some questions still needed to be explored. For example, spending two classes to solve one word problem may be considered time-consuming for many teachers. Therefore, how to spend less time on completing such learning activities is still a question. Besides, how students' mathematical representation advanced and how they mapped the details of the problem onto his drawing expression in different stages during the thirteen activities deserved further inquiry, which will need future studies.

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