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Experimental Use of Problem-Posing Exercise System for Efficient Calculation to Promote Relational Interpretation of Numerical Expression

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Abstract: This paper reports a development and experimental use of a computer-supported posing exercise of numerical expressions that can be solved by using efficient calculation methods. In arithmetic, a learner usually interprets the numerical expression as a calculation procedure. In mathematics, relational interpretation of the expressions becomes indispensable. This difference of the interpretations causes of difficulty in introduction phase of mathematics. This exercise is aimed to promote students in an elementary school to change their procedural interpretation to relational interpretation of numerical expressions because the efficient calculation methods request learners to interpret the numerical expressions relationally. The exercise was experimentally conducted for 78 6th grade students in an elementary school. The results of comparative analysis of calculation methods between pre-test and post-test suggest that the exercise promoted the students to interpret the numerical expressions relationally.

Keywords: Procedural Interpretation, Relational Interpretation, Numerical Expression, Efficient Calculation Methods, Efficient Calculation Problem, Problem-Posing

1. Introduction

This paper reports a development and experimental use of a computer-supported posing exercise of numerical expressions that can be solved by using efficient calculation methods. In arithmetic in an elementary school, a numerical expression is a target of a numerical calculation. Therefore, a learner usually interprets the numerical expression as a calculation procedure (Sfard, 1991, Miwa, 1996, Booth, 1998, Miwa, 2001). In a junior high school, however, a mathematical expression usually includes variables and it should be interpreted as an expression of relations among numbers and variables, not as a calculation procedure. In mathematics, a learner is requested to transform the mathematical expression into another expression by using the expressed relations. This difference of the interpretations is one of the most important causes of difficulty for a learner in introduction phase of mathematics in a junior high school (Sfard, 1991, Miwa, 1996, Booth, 1998, Miwa, 2001). In order to relax the difficulty, it is promising to make learners experience relational interpretation of numerical expressions in arithmetic. This Problem-Posing exercise is aimed to promote students in an elementary school to change their procedural interpretation to relational interpretation of numerical expressions because the efficient calculation methods request learners to interpret the numerical expressions relationally.

Efficient calculation methods that make calculation easier by transferring a numerical expression, for example, changing calculation order (Cooper et al., 1996, Heirdsfield, 2004, Suzuki et al., 2010) are one of learning targets in an elementary school. The methods are usually taught and exercised by solving calculation problems that can be applied to the methods. Such problem is called “efficient calculation problem”. Several investigations, however, suggested that the exercise of solving the efficient calculation problem was not enough to promote the learning because the efficient calculation problem could be also solved by general calculation method that was conducted only by interpreting the numerical expression as a procedure (Uesaka et al., 2014, Suzuki &

Ichikawa, 2016). Therefore, it is necessary to adopt an exercise that a learner has to pay attention to the relations in a numerical expression.

Several investigations suggest that posing a problem is a promising exercise to promote learners to think about the relations included in the problem (Silver, 1994, English, 1988). Moreover, in problem-posing, immediate feedback for the posed problems is effective to promote the learning (Nakano et al., 1999, Hirashima et al., 2007, Kojima et al., 2013, Hirashima et al., 2014). Based on these considerations, we have designed and developed a computer-supported posing exercise environment of efficient calculation problems (Enomoto et al., 2018). The environment has ability to automatically diagnose posed numerical expressions and generate feedback for based on the diagnosis results. So, in the exercise, a learner is able to receive immediate feedback for his/her posed problems. The exercise was experimentally conducted for 78 6^{th} grade students in an elementary school. The results of comparative analysis of calculation methods between pre-test and post-test suggest that the exercise promoted the students to interpret the numerical expression relationally.

2. Procedural Interpretation and Relational Interpretation of Numerical Expression

2.1 Numerical Expression and Mathematical Expression

In an arithmetic class in an elementary school, a numerical expression is usually taught as an expression of calculation procedure. For example, in the case of "2 × 7 × 3", the expression is interpreted as a calculation procedure, and then “2 × 7” is calculated at first, and then 14 × 3 is calculated, and the answer of 42 is derived. In a junior high school, however, a numerical expression is extended to a mathematical expression including variables and relational interpretation of it is required. Relational interpretation is to grasp numerical expression as operational relations among numbers or variables in the mathematical expression (for examples, ax + bx + cx = (a + b + c)x). Because of the major change of the interpretation of the expressions at the introductory phase of mathematics in a junior high school, many students feel difficult to learn it (Booth, 1998).

2.2 A Way to Teach Relational Interpretation in Arithmetic Numerical Expression

In order to relax this gap and smoothly connect to mathematics learning, it is promising to let students experience relational interpretation of numerical expression in arithmetic learning. Efficient calculation methods that are one of teaching targets in an elementary school request a learner to interpret a numerical expression as relations. An efficient calculation method suggests a learner to make calculation easier by transforming a numerical expression, for example, changing calculation order. In the case of "2 × 7 × 3", usual calculation method requests a learner to calculate from left side and it is necessary to calculate “14 × 3” that is a little difficult to conduct mental calculation. An efficient calculation method suggests to change the order of calculation to "2 × 3 × 7". By this change, the calculation becomes easier ones, that is, "2 × 3" and "6 × 7". To conduct the change of the numerical expression, it is necessary for a learner to interpret the numerical expression as relations.

2.3 Problem-Posing Exercise to Promote Relational Interpretation

In an elementary school, the efficient calculation methods are usually taught and exercised through solving calculation problems that can be applied to the methods. Such problem is called “efficient calculation problem”. The exercise of solving the efficient calculation problems, however, it is not enough to promote to learn the methods because the efficient calculation problems can be also solved by general calculation method that can be conducted only by interpreting the numerical expression as a procedure (Uesaka et al., 2014, Suzuki & Ichikawa, 2016). Therefore, it is necessary to adopt an exercise that a learner has to pay attention to the relations in a numerical expression.

In this research, we adopt “learning by problem-posing” to promote the relational interpretation of numerical expressions. The problem-posing is a learning method that request a learner to pose problems to understand the problems structurally. Learning by problem-posing is a widely practiced as a promising learning method (Silver, 1994, English, 1998, Nakano et al. 1999, Kojima et al. 2013, Hirashima et al., 2007, Hirashima et al., 2014).

Problem-posing of efficient calculation problems is a learning method which cannot be solved by executing procedural operation, and it is a relational operation of the numerical expressions at the same time. Since numerical expressions are also calculation procedures, in the learning method of solving problems, correct answers can be calculated according to the procedure without applying efficient calculation methods. On the other hand, problem-posing to create calculation problems cannot answer correctly by executing calculation procedure because students must create problems which the efficient calculation methods can be applied based on the understanding the methods. Furthermore, since what kind of efficient calculation methods can be applied corresponds to the structural operation in the numerical expression, creating a calculation problem which efficient calculation strategies can be applied is a structural operation of the learning target. Therefore, making calculation problems promotes students to understand the structure of numerical expressions.

In order to effectively conduct the problem-posing, diagnosis of correct answers and appropriate feedback based on diagnosis are important. Unlike solving problems, multiple correct answers can be considered in problem-posing learning. In order to deal with the multiple correct answers, an interactive learning system with diagnosis function of posed problems is indispensable.

3. Problem-Posing Exercise System

3.1 System Interface

The interface of the system developed in this research is shown in Fig. 1. The system consists of four exercises, (Exercise 1) problem solving exercise that only requests an answer of calculation, (Exercise 2) problem solving exercise that requests to write efficient calculation process, (Exercise 3) problem-posing exercise that requests to pose a problem that can be calculated with the same efficient calculation method used in exercise 2, and (Exercise 4) problem-posing exercise that requests to pose a problem that cannot be applied the efficient calculation method used in the previous exercises. In this exercise system, for one numerical expression, the four exercises are assigned. In Figure 2, the four exercises deal with “27 + 9” as the numeric expression, and targeting efficient calculation methods can be shown as “(27 + 3) + (9 - 3). These exercises, diagnosis and feedback are explained in this section.

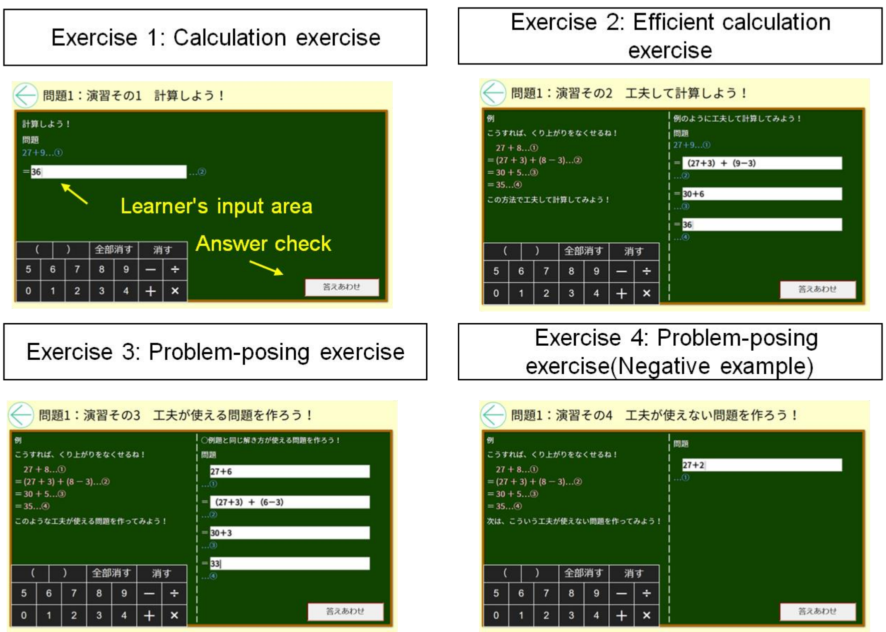

Figure 1. Interfaces of Exercises

3.2 Details of Exercises

Fig. 2 shows examples of the four exercises constituting the problem-posing learning of efficient calculation methods. The first is a calculation exercise (Exercise 1). This calculation exercise requests a learner only a correct answer and does not matter how the answer was derived. In this problem-posing exercise system, it is assumed that a learner is able to correctly answer a numerical expression that used in the four exercises.

Next exercise is an efficient calculation exercise (Exercise 2). The efficient calculation exercise requests a learner to calculate following an example of an efficient calculation that is presented in left side of exercise interface as shown in Figure 1 (Exercise 2). In Exercise 2 in Figure 1, the exemplified efficient calculation problem is "27+8" and its efficient calculation process is "27+8=(27+3)+(8-3)=30+5=35". A learner is requested to solve "27+9" in the same way with the example while writing the process of the efficient calculation. This is an example-based learning. In the example-based learning, because there is a possibility of only superficial mimicry without thinking about meaning of the example, it is important to request a learner to explain why the calculation method is applicable. Such explanation task is called self-explanation. In this research, because target learners are elementary school students and to express the calculation process with language is heavy load task for them, we adopted exercises to pose problems that can be solved by the calculation process. The problem-posing exercises in this research request them to write only numerical expression, but not able to correctly conduct only by imitating the example superficially. So, the problem-posing exercise is expected to be able to conduct, and to promote to recognize the example structurally.

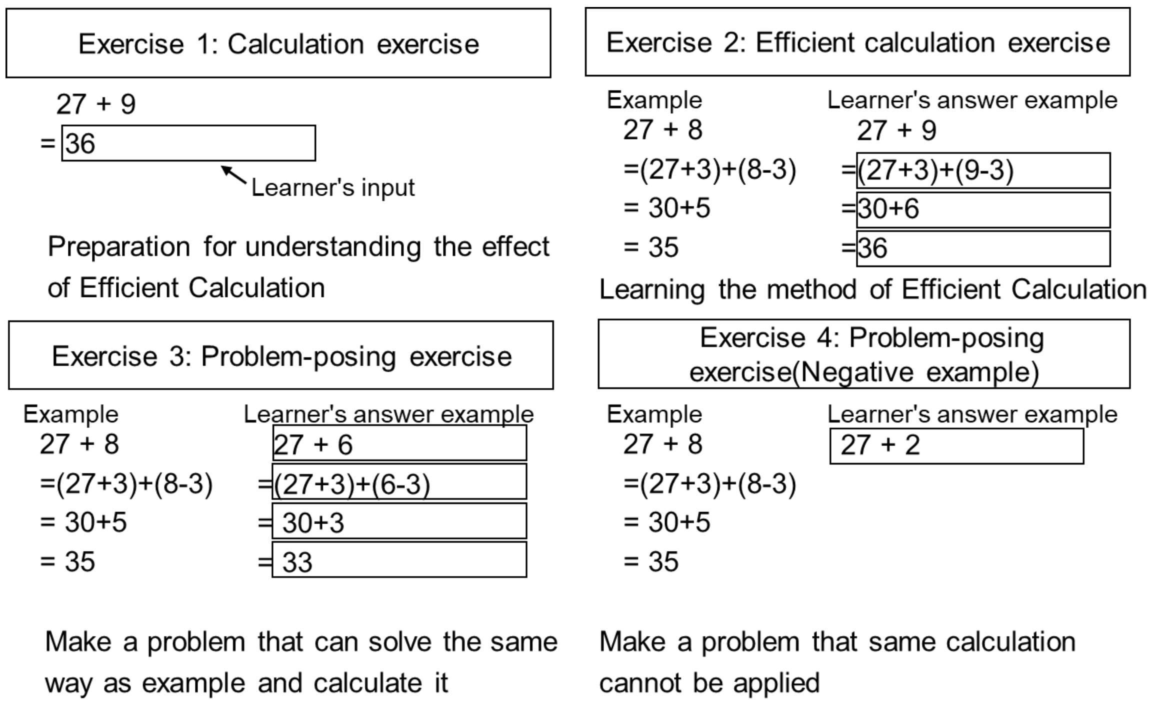

Figure 2. Details of Exercises

The third exercise is a solution-based problem-posing exercise (Exercise 3). The efficient calculation method of the learning target is presented as an example, and students pose a problem that can be solved by exemplified efficient calculation method. If the problem itself, the calculation process, and the final answer are all correct, the posed problem is regarded as a correct answer. For example, the efficient calculation problem that can be solved by the same way as "27+8", for example, "27+6". In this exercise, students must make a correct problem and correct calculation process and correct final answer, for example, "27+6=(27+3)+(6-3)=30+3=33".

Here, if a learner performs only the superficial imitation of the presented example, it is expected that the learner creates an inappropriate problem which is not applicable the efficient calculation method, like "27+2". If students created a problem that can be solved by the same way as the presented example, it is suggested that the learner comprehends the presented example relationally.

Last exercise is efficient calculation problem-posing (negative example) (Exercise 4). Unlike the efficient calculation problem-posing exercise, let the learner pose a problem that the efficient calculation method is not applicable, for example, in this case, "27+2" is a correctly posed problem. Combination of both Exercise 3 and 4, it is expected to promote a learner to interpret numerical expression structurally. In this exercise, since it is a problem-posing that the target efficient calculation method is not applicable, students only pose a problem and do not make the calculation process and final answer.

3.3 Diagnosis and Feedback

In each exercise, the system automatically determines whether the answer inputted by the learner is true or false. In addition, the system interactively returns feedback to the learner. If it is wrong, feedback according to the contents of the error.

Table 1 shows several examples of true / false diagnosis and feedback for the answer inputted by the learner. In the first example, the process of the calculation is correct but the answer is wrong. Therefore, the feedback indicates the correctness of the calculation process and the wrongness of the answer. In the second example, although the answer is correct, but his/her process does not follow the efficient calculation method of the shown example. Therefore, the feedback indicates that the answer is correct but the calculation process is not fit for the example.

Table 1

Diagnosis and Feedback

Exercise	Upper: Example Lower: Leaner's Answer	Diagnosis and Feedback
3	27+8=(27+3)+(8-3)=30+5=35 27+6=(27+3)+(6-3)=30+3=34	"Answer is Incorrect. Let's recalculate again."
3	27+8=(27+3)+(8-3)=30+5=35 27+6=(27+3)+3=30+3=33	"Answer is correct but calculation is incorrect. Let's look the way of the example once more."
4	27+8=(27+3)+(8-3)=30+5=35 27+9	"Incorrect. The way of the example can use. Let's look the way of the example once more.
4	27+8=(27+3)+(8-3)=30+5=35 27+2	"Correct. The way of the example cannot use."

4. Experimental Use and Results

4.1 Experimental Use at Elementary School

In order to verify whether the developed system can be used in the class and there is learning effect for efficient calculation methods, we conducted an experimental use of the system. The participants of experimental use are 78 students of 3 classes in an elementary school 6^{th} graders. The system was used in one class time (45 minutes). The time schedule of the class is as follows: general introduction (5 minutes), explanation of efficient calculation methods by the teacher (5 minutes), use of the system (30 minutes), questionnaire (5 minutes). Pre-test was conducted four or five days before the experimental use, and post-test was conducted four or five days after the experimental use. By keeping an interval of several days, we tried to check whether the educational effect by the system was established.

4.2 Questionnaire

Questionnaire on 6 items and 1 free description regarding the usability and effectiveness of the system was conducted. 74 children's data were regarded as valid data. Positive opinions were obtained from over 80% of students for the following questions: (1) The system was easy to use, (2) When a calculation mistake occurs, mistake points were understood immediately, (3) When a calculation mistake occurs, mistake contents were understood immediately, (4) Posing problems that same efficient calculation method is applicable as example was easy, (5) By posing problems same as example, how to calculate efficiently was understood well, (6) Posing problems not the same as example was easy. From this, it is suggested that problem-posing exercises of efficient calculation methods realized in this system were accepted as useful activities for students. Among the few children who answered, "Strongly disagree", there were some children who were unable to complete the exercises, and there were some improvements in support of system use. On the other hand, in free description, there was also answer that the children taught each other.

Table 2

Questionnaire results

Question	Strongly agree	Agree	Disagree	Strongly disagree
1	41	27	6	0
2	39	25	8	2
3	40	28	4	2
4	45	17	10	2
5	45	26	2	1
6	42	24	5	3

4.3 Pre-test and Post-test

Table 3 shows the categorization of problems used in the pre-test and the post-test. The problems are classified into three categories as follows. (1) Non-efficient calculation problem that does not require to use an efficient calculation method. (2) The learned problem is an efficient calculation problem that is solved by the method learned in the system. (3) The transfer problem is an efficient calculation problem but that is not learned by the system. The problems used in the pre-test and post-test were the same ones but the order of them in the tests were different. Then, the problems themselves are not so difficult for the 6th grade students. Based on these conditions, we assumed that the changes of calculation methods were reflecting the changes of learners' interpretation for numerical expressions.

Table 3

Pre-test and Post-test Problems

Problem Classification	Problems
Non-Efficient Calculation Problems (4 Problems)	17+2, 3+27, 11+12, 191+4
Learned Problems (3 Problems)	86+7, 98+5, 197+8
Transfer Problems (3 Problems)	19+17, 150+37+50, 19+22+18+21

4.4 Changes of Calculation Styles

Table 4 shows the change of calculation styles in pre-test and post-test. Here, we categorized calculation styles into following three: calculation by writing, efficient calculation and mental calculation. Calculation by writing and efficient calculation were specified by checking the description that the learners wrote on the test papers. Figure 3 is an example of calculation by writing. If there was only an answer and no other description, the calculation was categorized into mental calculation. Table 3 shows the ratio of each calculation styles in a problem category. Due to absence of one student, n = 77.

$$\begin{array}{r} 27 \\ +\ \ 9 \\ \hline 36 \end{array}$$

Figure 3. Calculation by Writing

In Table 4, as for Non-Efficient Calculation Problems, the number of calculation by writing decreased, the number of efficient calculation increased, and the number of mental arithmetic also increased. Here, efficient calculation for Non-Efficient Calculation Problems means that the calculation was conducted with an efficient calculation method but it did not make the calculation easier. So, a learner who conducted this calculation would not understand the efficient calculation method.

Regarding to Learned Problems, calculation by writing decreased, efficient calculation increased, and mental arithmetic increased. Regarding to Transfer Problems, calculation by writing decreased, efficient calculation increased, and mental arithmetic increased. Summarizing the changes in the calculation style before and after system use, calculation by writing decreased, efficient calculation increased, and mental arithmetic increased.

Wilcoxon's signed rank sum test was conducted to analyze the differences of the values in Table 4. There were statistically significant differences in decreasing the number of “calculation by writing” in Learning problems and Transfer problems. Here, Bonferroni adjustment was performed because multiple comparison was conducted.

Table 4

Changes of Calculation Styles

Classification	Calculation by Writing	Efficient Calculation	Mental Arithmetic
Non-Efficient Calculation Problems	6.5(0.229)	2.3(0.141)	91.2(0.309)
	2.6(0.159)	1.3(0.089)	96.1(0.180)
	p=0.5625	*p* >1	*p* =0.2812
Learned Problems	15.2(0.320)	3.9(0.152)	81.0(0.389)
	6.5(0.234)	5.6(0.224)	87.9(0.313)
	p =0.0307	*p* >1	*p* =0.3433
Transfer Problems	15.2(0.297)	22.9(0.305)	61.9(0.379)
	6.9(0.217)	26.0(0.287)	67.1(0.325)
	p =3.5703e-03	*p* >1	*p* >1

Ratio (%) of Calculation for Each Problem Category, SD (Upper: Pre-test, Lower: Post-test), and Bonferroni adjusted p value

4.5 Analysis of Change of Calculation Style

In this research, it was assumed that if the problem-posing exercise promoted learners to interpret numerical expressions structurally, the calculation style the learners used could change. We expected that we could observe following three valuable calculation style changes: Change-1: efficient calculation increases, Change-2: calculation by writing decreases, and Change-3: mental arithmetic increases.

As for Change-1, since the learning target in this system is efficient calculation problems, this change is valuable. As for Change-2, because calculation by writing is a general calculation style to interpret a numerical expression as a calculation procedure, this change is valuable. Because the metal arithmetic is usually heavy load calculation style, when a numerical expression is complex, it is not easy to conduct calculation as it is. So, Change-3, that is, increase of mental calculation, is also valuable.

4.6 Calculation Style Transformation

Figure 4 shows the changes of calculation styles between pre-test and post-test. This examined the answers of students in each test and organized how the individual problems of pre-test were solved by post-test. "CW 20" means that 20 answers were derived by calculation by writing (CW) in the pretest. Then, “8” means that 8 answers were derived by calculation by writing (that is, the same style with the pretest) in the post test, “1” means that 1 answer was derived by efficient calculation style (EC), “11” means that 11 answers were derived by mental arithmetic (MA). Although there is only a small part of the decrease in calculation quality (shift from mental arithmetic to calculation by writing), overall it is shown that the calculation style is transferring in the direction that the calculation quality improves.

Table 5 shows the calculation method transformation summary. From this table, it can be seen that transformation of calculation styles between pre-test and post-test occurs with maintenance and improvement computational quality with a few exceptions.

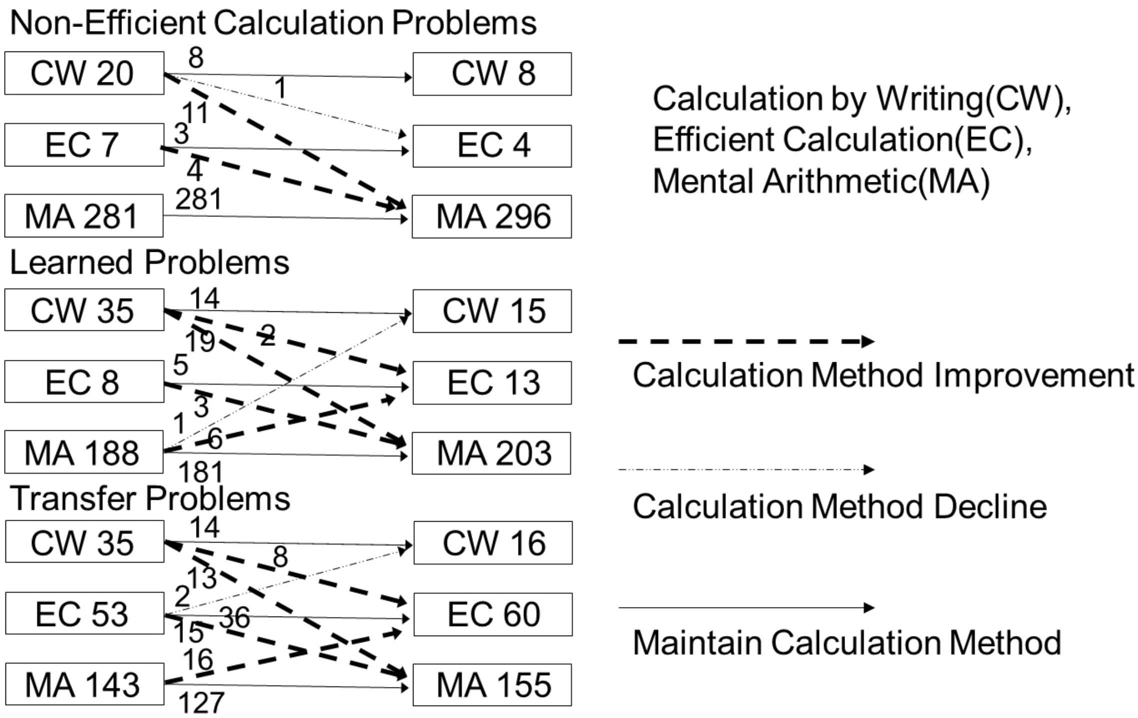

Figure 4. Calculation Method Transformation

Table 5

Calculation Method Transformation Summary

Calculation methods between Pre and Post	Improvement	Decline	Maintain
Non-Efficient Calculation Problems	15	1	292
Learned Problems	30	1	200
Transfer Problems	52	2	177

4.7 Calculation Style Transformation for Each Student

Table 6 shows the calculation style changes by students in pre-test and post-test. We examined how the pre-test and post-test problems were solved and evaluated the transformation of calculation styles to improve, maintain, decline. Furthermore, we summarize the relationship between those calculation styles transfer and correct answer rates. This figure shows that except for a few exceptional children (one each for Learned problems and one Transfer problems), the transformation of the calculation style occurs in the direction of maintenance and improvement calculation styles. Furthermore, the transition of these calculation styles involves maintaining and improving the correct answer rate of pre-test and post-test. In addition, no students answered all pre-test and post-test problems by efficient calculation style. Therefore, number of students who can still improve in calculation method is 77.

Table 6

Evaluation of Calculation Method Transformation by Students

Learned Problems		Correct Answer Rate		
		Improve	Maintain	Decline
Calculation style Improvement	16 students	0	16	0
Calculation style Maintained	53 students *	2	42	9
	7 students	0	7	0
Calculation style Decline	1 student	0	1	0
Transfer Problems		Correct Answer Rate		
		Improve	Maintain	Decline
Calculation style Improvement	36 students	2	31	3
Calculation style Maintained	21 students *	3	15	3
	19 students	2	15	2
Calculation style Decline	1 student	0	1	0

**: calculated all problems by mental arithmetic*

4.8 Summary of Results

In pre-test and post-test which executed before and after learning Efficient calculation methods by the system, presented calculation changes occurred in the calculation method of students, and the change was accompanied by some significant differences. This change does not occur randomly, but mostly it is a change from calculate by writing to efficient calculation and mental arithmetic, and there was almost no change in the opposite direction. Furthermore, the change in these calculation methods accompanied by retaining and improving the correct answer rate in the post-test, and the precision of the calculation did not decrease after efficient calculation learning. Based on the above, the data of this research suggests the validity of change of calculation method by Efficient calculation strategies learning: increase of Efficient calculation, decrease of calculation by writing and increase of mental arithmetic presented as a hypothesis. From this, it is judged that a certain effect has been found in the Efficient calculation methods learning using this system, in terms of generating a change to calculation method of students in the direction expected by learning the efficient calculation methods.

5. Conclusions

In this research, we designed and developed a problem-posing system for arithmetic Efficient calculation problems and experimentally used in classes at an elementary school 6^{th} graders. As in this research, there is no report that the problem-posing of efficient calculation problems was carried out in the class, and it was the significance of this research that it was able to show that this is possible. Moreover, as a hypothesis, it was shown that the changes of calculation method of the efficient calculation increase, calculation by writing decrease, and mental arithmetic increase occurs as the effect of the learning of the efficient calculation, and the effect of the system can be shown by the validity verification.

Tasks in the future, since the system use target was a 6th grade of elementary school where efficient calculation methods was already learned, it is necessary to verify the learning effect in lower grade, learning efficient calculation methods from now, and quantitative effect measurement method at that time. Regarding the practical use period, this research was a short-term class use, but from the viewpoint of whether it has an effect on connection from arithmetic to mathematics, verification with long-term use will become a future task. Although it is difficult in class, it is also an issue to conduct experiments with control groups and to verify the effect in comparison.

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