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A Mathematical Model of Collaborative Learning using Differential Equations

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Abstract: This study presents a mathematical learning model for collaborative learning using differential equations. We first specify initial knowledge about a subject and each student's ability to understand the subject. Then, we analyze the evolution of students' knowledge over time. Our findings show that the effect of collaborative learning depends on how students are grouped together. These results can suggest ways of designing a class for effective collaborative learning.

Keywords: Collaborative learning, Mathematical modeling, Differential equation

1. Introduction

In recent years, numerous studies have shown the effectiveness of collaborative learning. However, most of these are inductive studies in which pedagogical findings are derived by observing students in the classroom. In contrast, a deductive approach leads to a conclusion from a general premise. Therefore, if we had a general framework as a premise for collaborative learning, we could design a class for effective collaborative learning. In this study, we construct a mathematical learning model to function as a general framework for collaborative learning.

Existing mathematical models of the teaching-learning process can be classified into three categories: differential equation modeling (Pritchard et al., 2008), Ising spin modeling (Bordogna et al., 2001, 2003; Yeung, 2006; Yasutake, 2011), and stochastic process modeling (Nitta, 2010). Note that the model of Pritchard et al. (2008) only applies to individual learning. In this study, we extend his model to use differential equations to describe collaborative learning.

2. Mathematical learning model by using different equation

2.1 Pritchard's model

Let us briefly review Pritchard's model. He introduces the following model to describe the evolution of a student's knowledge over time: $dK(t)/dt = \alpha_1\beta(1-K(t))K(t) + \alpha_2(1-\beta)(1-K(t))$. Here, the student's knowledge at time t is given by $K(t)(0 \le K(t) \le 1)$. The first term of the right-hand side of this model is motivated by the constructivist view in which students learn new knowledge by constructing an association between it and some prior knowledge. Here, α_1 is the probability that something taught to a student will be retained in his/her mind for potential learning. The second term is motivated by the tabula rasa theory of learning, and α_2 describes a learning rate similar to α_1. The tabula rasa theory assumes that a student's memory is blank before learning begins. Finally, β is a parameter that determines the ratio of the learning effect of the two terms.

2.2 Our model

We describe the time evolution of the i-th student's knowledge during collaborative learning with another student as follows:

$$
\begin{aligned}
\frac{dKi(t)}{d(t)} &= c\big(\beta\alpha_i K_i(t)(1-K(t)) + (1-\beta)\alpha_i(1-K_i(t))\big) \\
&\quad + d\sum_{i\neq j} f_i\big(K_j(t)-K_i(t)\big)\big(K_j(t)-K_i(t)\big) \\
&\quad + e\sum_{i\neq j} g_i\big(K_i(t)-K_j(t)\big)
\end{aligned}
,
$$

where functions $f_i(x), g_i(x)$ are defined as

$$
f_i(x) = \begin{cases} \alpha_i & x > o \\ 1-\alpha_i & otherwise \end{cases} \qquad g_i(x) = \begin{cases} 1-K_i(t) & x > o \\ 0 & otherwise \end{cases}.
$$

The first term of our model is similar to Pritchard's model. We introduce α_i, which reveals the the *i*-th student's ability to understand, instead of α_1, α_2 in Pritchard's model. The second term represents a learning effect in which students gain knowledge by being taught by other students. When the *j*-th student's knowledge is greater than that of the *i*-th student $\big(K_j(t)-K_i(t)>0\big)$, the higher the value of α_i, the more knowledge the *i*-th student gains. Conversely, when the *j*-th student's knowledge is lower than that of the *i*-th student $\big(K_j(t)-K_i(t)<0\big)$, the higher the value of α_i, the less the negative effect is on the *i*-th student. The third term of our model represents the gain in students' knowledge when they teach something to other students. This effect is revealed only if the *i*-th student's knowledge is greater than that of the *j*-th student. The lower the *i*-th student's knowledge, the greater the learning effect the *i*-th student gains. The ratio of three terms is determined by the values of *c*, *d*, and *e*.

3. Results and discussion

3.1 Calculation results

We set the initial knowledge about a subject when a student starts studying and each student's ability to understand the subject as a control parameter. Under these conditions, we analyze the evolution over time of a student's knowledge in collaborative learning with three other students. Here, we show two characteristic results of our study. One result is obtained for case (a) $K_1(0)=0.1, \alpha_1=0.1$, $K_2(0)=0.3, \alpha_2=0.3$, $K_3(0)=0.6, \alpha_3=0.6$, and $K_4(0)=0.8, \alpha_4=0.8$, for student 1, 2, 3, and 4, respectively. The other result is obtained for case (b) $K_1(0)=0.1, \alpha_1=0.5$, $K_2(0)=0.2, \alpha_2=0.6$, $K_3(0)=0.3, \alpha_3=0.7$, and $K_4(0)=0.4, \alpha_4=0.8$, for student 1, 2, 3, and 4, respectively.

Those two results are shown in Figure 1 (a) and (b). The red lines denote collaborative learning and the blue lines denote individual learning. In case (a), collaborative learning is less effective for those students who have a high level of initial knowledge or a high ability to understand the subject. In case (b), collaborative learning is effective for all students.

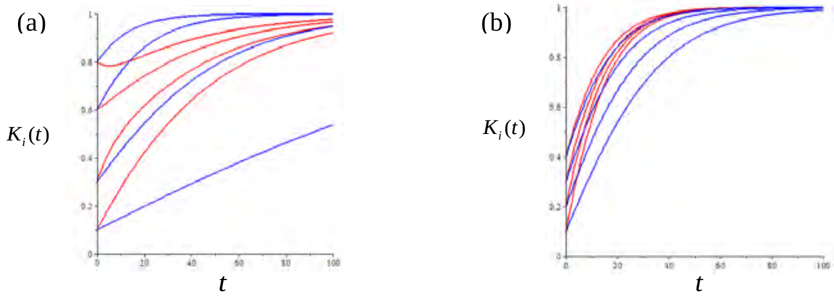

Figure 1. The time evolution of students' knowledge during collaborative learning (red lines), with $\beta=0.5, c=d=e=0.1$, and individual learning (blue lines), with $\beta=0.5, c=0.1, d=e=0$.

3.2 Comparison between inductive research and deductive research

Our model could apply to a variety of learning situations. Therefore, we compare the inductive research method, such as observing students in a classroom, to our deductive mathematical model. Nitta introduces stochastic process modeling (Nitta (2010)). He observed students in a classroom solving multiple-choice questions about introductory physics. The students solved a question, then solved the same question again soon after discussing it with other students. He calculated the percentage of students choosing the correct answer before and after collaborative learning took place. We investigated the initial knowledge and transitional knowledge at time t = 10 for each student, which corresponded with the students' knowledge before and after discussing the problem with other students. Then, we compared our results to his. Figure 2 shows that the two results are qualitatively similar. Therefore, we can say our model is appropriate.

(c) Inductive research (Nitta's research) (d) Deductive research (Calculation of our model)

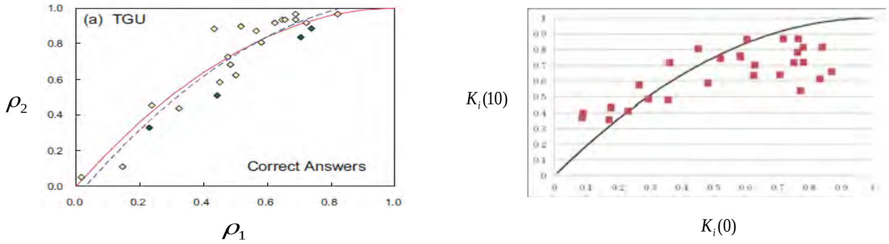

Figure 2. (c) Relationship of percentage of students choosing correct answer before (ρ_1) and after (ρ_2) collaborative learning [Reprint from Nitta (2010)]; (d) Relationship between initial knowledge and transitional knowledge at time t = 10 in the case of collaborative learning.

4. Conclusion

Our findings show that the effect of collaborative learning depends on the way students are grouped. In case (b), collaborative learning is more effective than individual learning. Therefore, collaborative learning appears to be more effective when students who have a high ability to understand learn a new subject. However, according to our model, in some cases (e.g., case (a)), the learning effect of collaborative learning is limited when students who have a high amount of initial knowledge or ability to understand engage in collaborative learning. Mathematical learning models are important, because they enable us to discuss the effect of collaborative learning in a variety of learning situations. Finally, we compared our simulation result with actual classroom data. However, our mathematical model needs to reflect how learning takes place in a classroom in more detail.

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