Liu, C.-C. et al. (Eds.) (2014). Proceedings of the 22nd International Conference on Computers in Education. Japan: Asia-Pacific Society for Computers in Education

Learning by Posing Problems Using Illustrations Instead of Words

Takanobu UMETSU[a*], Hikaru ETO[a], Tsukasa HIRASHIMA[b] & Akira TAKEUCHI[a]
[a]*Computer Science and Systems Engineering, Kyushu Institute of Technology, Japan*
[b]*Department of Information Engineering, Hiroshima University, Japan*
*umetsu@ai.kyutech.ac.jp

Abstract: Learning by posing arithmetic word problems is well-recognized as an effective means to mastering the use of solution methods; however, several learners cannot pose problems using words. In problem-solving exercises, if a learner cannot understand what is expressed in a problem statement, an illustration of the scene expressed in the statement can help the learner understand. Therefore, in this paper, we propose learning by posing problems with illustrations instead of words. One of the purposes of learning mathematics is to acquire the ability to derive a numerical relation from a scene and to associate a calculation expression with various scenes. We expect that posing problems via illustration and without words is an effective way for beginners to learn. In the problem-posing exercises, learners construct various types of problems. Teachers have to evaluate each posed problem and advise learners on the basis of the evaluations. This task places a heavy burden on teachers. Therefore, we developed a computer-based environment for posing problems using illustrations. The environment provides a learner with a calculation expression and illustrations. The illustrations can be disassembled into several figures. A learner assembles an illustration that describes a story of arithmetic problem that correspond to the calculation expression by combining the figures. The learner uses no sentences in this problem-posing exercise. The environment diagnoses the illustration problem and produces advice to the learner. We conducted a preliminary evaluation of the environment to confirm that a learner who cannot pose problems via words was able to pose problems using illustrations. Our results were promising, indicating that our environment provides a means for learners who cannot pose problems via words.

Keywords: Problem-posing, interactive learning environment, visualization, arithmetic problem

1. Introduction

In this paper, we describe a computer-based environment for learning by posing problems expressed using illustrations for learners who cannot describe problems using words.

Learning by posing arithmetic word problems is well-recognized as an effective way to master the use of solution methods (Polya, G., 1945) (Ellerton, N. F., 1986) (Yu, F., Liu, Y. H., and Chan, T. W., 2003); however, it can be difficult to provide problem-posing exercises wherein learners construct various types of word problems. Because it is difficult to prepare adequate feedback for every problem that learners construct, teachers are required to evaluate each posed problem and advise learners on the basis of the evaluations, which places puts a heavy burden on teachers.

Given these circumstances, a computer-based learning environment called MONSAKUN was developed for interactive problem posing using arithmetic word problems (Hirashima, Yokoyama, Okamoto, and Takeuchi, 2007) (Hirashima and Kurayama, 2011). MONSAKUN provides learners with disassembled sentences and calculation expressions. The learner is required to pose a problem that is solvable by the calculation expression by selecting some of the sentences and sorting them into their proper order. MONSAKUN diagnoses the combination of the sentences and presents advice to the learner; however, there are several learners who cannot pose problems from the disassembled sentences in MONSAKUN.

In problem-solving exercises, if a learner cannot understand what is expressed in a problem statement, an illustration of the scene expressed in the statement can help the learner understand.

Therefore, in this paper, we propose learning by posing a problem expressed by illustrations instead of words. First, a calculation expression is presented to a learner. Next, the learner poses a problem by creating an illustration that describes a story of increasing, decreasing, or combining illustrated elements that match the calculation expression. The learner uses no sentences in this problem-posing exercise.

One of the purposes of learning mathematics is to acquire the ability to derive a numerical relation from a scene and to associate a calculation expression with various scenes. We expect that posing problems via illustration and without words is an effective way for beginners to learn.

We developed a computer-based environment for posing such illustration problems. This environment provides learners with components of the illustration and a calculation expression. As noted above, the learner assembles a scene that describes a story of increasing, decreasing, or combining elements that correspond to the calculation expression. The illustration is assembled by combining its components. The environment diagnoses the illustration problem and produces advice to the learner. In this paper, we introduce our learning environment and present results of a preliminary evaluation of the environment.

2. Related Work

Arithmetic textbooks in elementary school are replete with illustrations to help learners understand. The visualization approach has generated new enthusiasm for mathematics and has improved problem-solving performance (Moses, 1982). Learning by posing problems using illustrations is based on this viewpoint. We expect posing illustration problem as being a more effective means for learners who cannot describe problems by words.

Joya's "fraction block" (Joya, Maeda, and Hirashima, 2013) is similar to our research wherein a learner creates an illustration that describes a numerical relationship. The fraction block reifies characteristics of "ratio fraction" as a pair of numerator and denominator blocks. The block length can be varied by maintaining the same ratio of numerator blocks to denominator blocks. Joya developed a learning environment wherein learners can directly operate these fraction blocks to derive one quantity from another.

Although both Joya's and our environment use illustrations, there is a difference in purpose. The fraction block is a kind of graph. Hanagata (1990) stated that graphs and diagrams help learners understand quantitative relations and solution methods for a variety of problems; however, Hanagata also noted that graphs and diagrams rarely help to understand what is expressed in problem statements in the early stages of mathematical development. A beginner cannot associate abstract figures on the basis of the solution method such as the fraction block with a problem sentence if the beginner is unaccustomed to the abstract figures.

Hanagata suggested that an illustration of the scene expressed in the statement helps learners understand what is expressed more than graphs and diagrams in the early stages of learning, because the learners can directly associate the illustration with a scene in their life. The purpose of our research is to support learners who cannot pose problems in the form of sentences, therefore we do not use graphs and diagrams, but rather than illustrations of the given scenes.

3. Learning by Posing Illustration Problems

In this section, we describe learning by posing problems using illustrations instead of words. First, we describe a structure of problems in our learning environment. Next, we describe how to pose a problem using illustrations in our system.

3.1 Structure of Problem

Our research focuses on problems that are solved by either one addition or subtraction operation. Addition and subtraction operations can be classified into increase-change, decrease-change, combine, and compare (Riley, Greeno, and Heller, 1983). Our learning environment can diagnose increase-change, decrease-change, and combine operations. Increase-change and combine problems are solved via addition formula A + B = C. Decrease-change problems are solved via subtraction formula A − B = C.

The structure of problems in our learning environment is based on MONSAKUN, which is a learning environment for problem posing by words. Yamamoto, Kanabe, Yoshida, Maeda, and Hirashima (2012) explained the structure of problems in MONSAKUN. In MONSAKUN, a problem comprises three sentences. Two of the three sentences are "existence sentences," which express a number of objects. The third sentence is a "relation sentence," which expresses a numerical relation between the objects in the two existence sentences. For example, the following sentence is an increase-change problem: "Tom has three pencils. Tom buys two pencils. Then, Tom has five pencils." The first and third sentences are existence sentences, whereas the second sentence is a relation sentence. The numerical relation in the problem is 3 + 2 = 5.

Much like MONSAKUN, a problem in our system comprises three illustrations. Two of the three illustrations express a number of objects, and the third expresses a numerical relation between the objects.

Our learning environment provides three kinds of objects for existence illustrations, namely: food, money, and people. A person can have food and money. The environment also provides several places wherein people exist. For example, a learner can assemble existence illustrations corresponding to the following sentences: "Taro has three apples"; "Taro has five dollars"; and "Three people are in the sandbox." Figure 1 shows an example of an existence illustration.

Figure 1. Example of an existence illustration

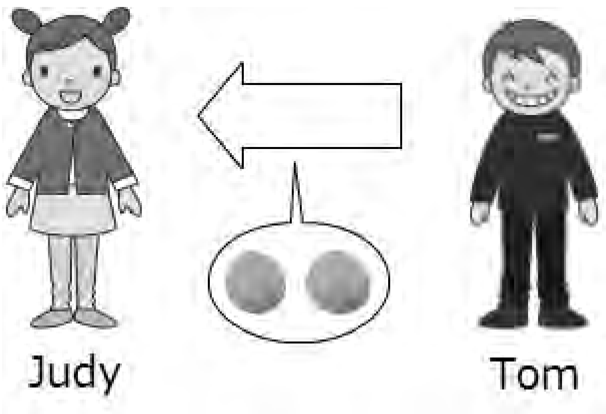

Figure 2. Example of a relation illustration

In our environment, a learner can express "give/get," "buy/sell," "move" and "combine" via relation illustrations. The actions "give," "get," "buy," "sell," and "move" are illustrated using an arrow, as shown in Figure 2. “Give” and “get” are expressed by the same figure, depending only on the different subjects at each arrow endpoint, as are “buy” and “sell.” More specifically, “give/get” is a movement of food and is illustrated by an arrow annotated with food between two people. “Buy/sell” is illustrated by an arrow with money and food between a person and a store. “Move” is a movement of people and is illustrated by an arrow with people between places. We do not provide an illustration for “combine”; however, a learner can assemble a “combine” illustration by combining two existence illustrations. Figure 2 shows an example of a relation illustration in which Tom gives Judy two oranges.

If a problem includes a relation of “give/get,” “buy/sell,” or “move”, there is a temporal ordering between the three illustrations. One illustration is anterior to the increase or decrease. The other is posterior to the increase or decrease. In the case of word problems, existence sentences are arranged in a temporal order. For example, in the following word problem, Taro had three pencils before buying and therefore has five pencils after the buying: “Taro has three pencils. Taro buys two pencils. Taro has five pencils.”

In posing illustrations, we provide morning, noon, and evening illustrations to express temporal order. If a learner wants to pose an increase/decrease problem, the problem is composed of an existence illustration in the morning, a relation illustration at noon, and an existence illustration in the evening.

3.2 Interface for Posing Illustration Problems

In this subsection, we describe how to pose a problem using illustrations in our system. Figure 3 shows an interface of our learning environment. First, the environment provides a learner with a calculation expression (upper left portion of Figure 3) and illustrations (right portion of Figure 3). The illustrations can be disassembled into several figures. A learner assembles an illustration problem that corresponds to the calculation expression by combining the figures. The lower left portion of Figure 3 shows the illustration problem in the process of assembly.

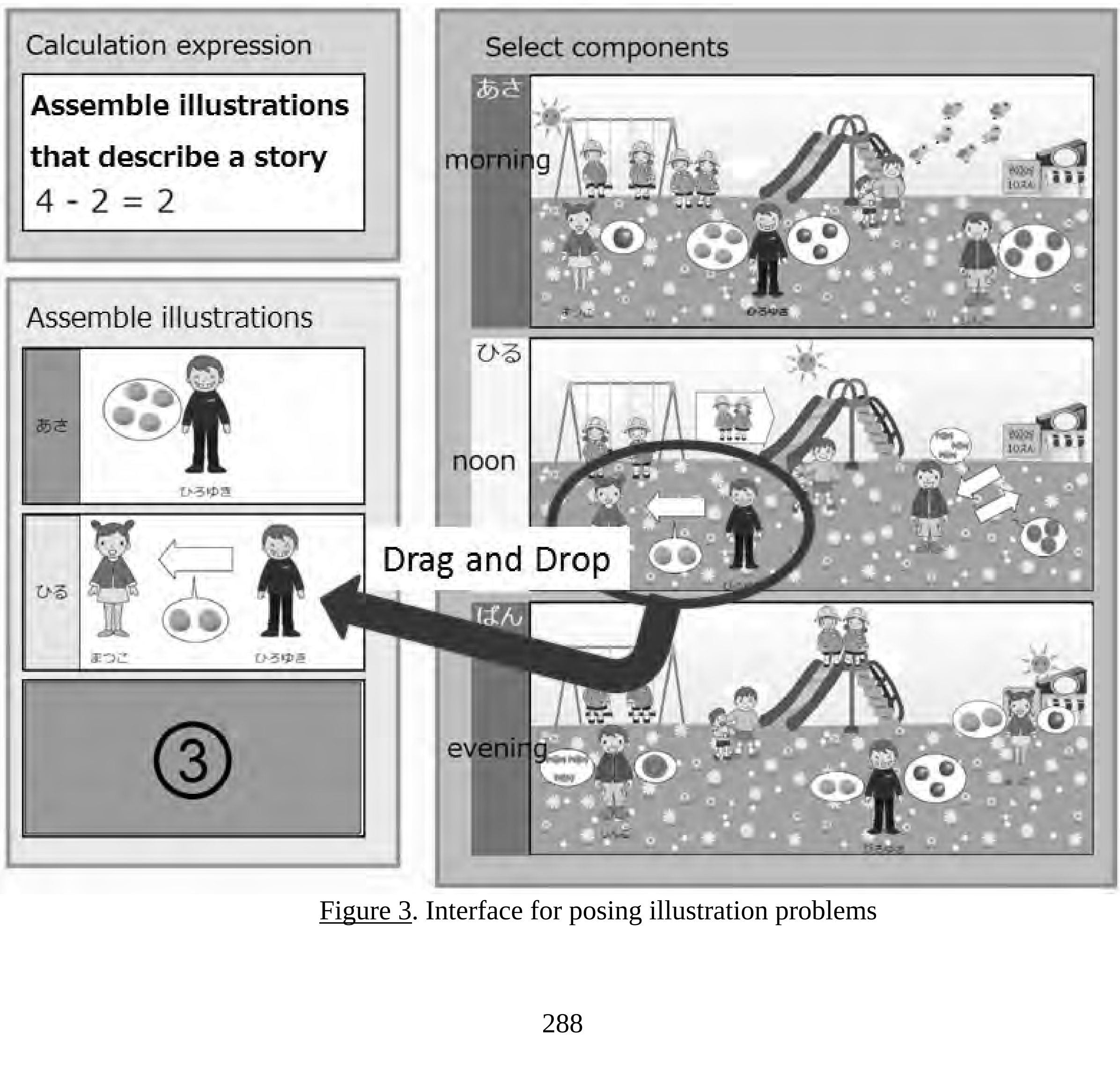

Figure 3. Interface for posing illustration problems

The illustrations provided comprise all types of illustration problems: “give/get,” “buy/sell,” “move,” and various combine problems. A learner selects one illustration problem that corresponds to the calculation expression from the provided illustrations. The system provides the illustrations by combining various existence illustrations and “give/get,” “buy/sell,” and “move” relation illustrations so as not to logically contradict. If a problem includes an increase or decrease relation, there is a temporal order as noted above. Therefore, the provided illustrations are composed of morning, noon, and evening illustrations. The system combines various existence and relation illustration based on the temporal order.

In the illustrations provided in the right portion of Figure 3, there are 20 illustration problems: three increase-change problems, three decrease-change problems, 14 combine problems. Two of the 20 illustration problems corresponds to“4 – 2.” The illustration in the morning in the upper portion of Figure 3 comprises six existence illustrations: “Four people are by the swing”; “Two people are by the slide”; “Matsuko has an apple”; “Hiroyuki has four oranges”; “Hiroyuki has three apples”; and “Shingo has 40 yen.” The illustration at noon in the middle portion of Figure 3 comprises four relation illustrations: “Two people move from the swing to the slider”; “Matsuko gets two oranges from Hiroyuki (Hiroyuki gives two oranges to Matsuko)”; “Shingo buys three candies from the shop (The shop sells three candies to Shingo)”; and “Shingo buys the candies for 30 yen from the shop (The shop sells the candies for 30 yen to Shingo).” The illustration in the evening in the lower portion of Figure 3 comprises eight existence illustrations: “Two people are by the swing”; “Four people are by the slider”; “Shingo has three candies”; “Shingo has 10 yen”; “Hiroyuki has two oranges”; “Hiroyuki has three apples”; “Matsuko has two oranges”; and “Matsuko has an apple.” As noted above, there is no “combine” relation illustration in Figure 3, because a learner can assemble a “combine” illustration by combining two existence illustrations.

Each existence and relation illustration can be moved by drag-and-drop freely in the interface. A learner selects two existence illustrations and one relation illustration and then moves them into the blanks in the lower left portion of the interface (see Figure 3) to pose a problem. Time information is also moved into the blanks using the drag-and-drop. Note that there are three blanks because a problem comprises two existence illustrations and one relation illustration. If a learner wants to express a combine relation, the learner moves two existence illustrations into the same blank, means calculating a total number of objects in the two existence illustrations.

Next, our environment diagnoses the illustration problem and outputs whether it is correct. If incorrect, the reason or reasons why is displayed. The problem sentence represented by the illustration problem is also displayed. This problem sentence helps a learner understand the meaning of the assembled illustrations and see how to describe a problem.

If an illustration problem includes a figure consisting of an arrow with food, the system outputs two problem sentences. Because the arrow with food is translated into “give” and “get,” the system describes the problem in two ways (i.e., the “give” relation and the “get” relation). Similarly when a figure consists of an arrow with money and food, the system outputs both “buy” and “sell” sentences. In short, the illustration problem can be transformed into two problem sentences, and the two problem sentences correspond to the same calculation expression.

4. Implementation

In this section, we describe how our system diagnoses an illustration problem. First, we describe information comprising illustrations for the diagnosis. Next, we describe how to diagnose a problem with the information in our system.

4.1 Illustration Information

An existence illustration contains the following information: the type of illustration, the corresponding verb, the object name, the number of objects, the owner or place, and the temporal order. The type of illustration is used to distinguish between existence and relation. The verb is used to translate the illustration into a problem sentence. Figure 4 shows an example of this set of information.

Type of illustration: existence
Verb: has
Object name: apple
Number of objects: 2
Owner: Tom
Temporal order: morning

Figure 4. The Example of information comprising an existence illustration

An increase/decrease relation illustration contains the following information: the type of illustration, the corresponding verb, the increased/decreased object name, the increase/decrease number, the owner whose objects are increased, the owner whose objects are decreased, and the temporal order. Figure 5 shows an example of this set of information. There is no information for the combine relation, because we do not provide illustrations for combining.

Type of illustration: relation
Verb: give/get
Increased/decreased object: apple
Increase/decrease number: 2
Owner whose objects are increased: Taro
Owner whose objects are decreased: Judy
Temporal order: noon

Figure 5. The Example of information comprising a relation illustration

4.2 Diagnosis of a Posed Illustration

Our environment first checks whether the illustration problem is composed of two existence illustrations and one relation illustration. Otherwise, the system displays “It is not an arithmetic problem. A calculation expression is not formed. The first (or second/third) illustration is incorrect.”

If the relation illustration is an increase/decrease relation, the environment checks that the object names in the three illustrations are the same. Otherwise, the system displays “There is no relation between <object name A> and <object name B>.” Furthermore, the environment checks that the temporal order in one existence illustration is morning, the other existence illustration has a temporal order of evening, and the temporal order in the relation illustration is noon. Otherwise, the system displays “An illustration before (or after) the giving (or buying/moving) is required. Select an illustration in the morning (or evening).” Next, the environment checks that the owner of objects in one existence illustration is the same as the owner whose objects are increased, and the owner of objects in the other existence illustration is the same as the owner whose objects are decreased. Otherwise, the system displays “There is no relation between <owner A> and <owner B>.”

Finally, the environment derives a calculation expression from the illustrations and ensures the derived calculation expression matches the provided calculation expression. If the owner of objects in the morning illustration is the same as the owner whose objects are increased, the calculation expression is “number of objects in the morning illustration” + “increase number in the relation illustration” = “number of objects in the evening illustration.” Conversely, if the owner of objects in the morning illustration is the same as the owner whose objects are decreased, the calculation expression is “number of objects in the morning illustration” − “increase number in the relation illustration” = “number of objects in the evening illustration.” If the derived calculation expression is different from the provided calculation expression, the system displays “These illustrations do not correspond to the calculation expression. The calculation expression of these illustrations is <derived calculation expression>”

If the relation illustration is a combine relation, the environment checks that the temporal order of all illustrations is the same. Otherwise, the system displays “There is no relation between a number of

<object name> in the <temporal order A> and a number of <object name> in the <temporal order B>." Next, the environment checks that the relation illustration comprises the other illustrations. Otherwise, the system displays "There is no relation between first (or third) illustration and second illustration." Furthermore, the environment checks that the object types in the three illustrations are the same. For example, it is meaningless operation that a number of food plus an amount of money. If one illustration includes food, the others must include food. If the object types are different, the system displays "It is meaningless operation that a number of <object name A> plus a number of <object name B>."

Finally, the environment derives a calculation expression from the illustrations and ensures the derived calculation expression matches the provided calculation expression. The calculation expression is "number of objects in one existing illustration" + "number of objects in the other existing illustration" = "total number of objects in the two existence illustrations." If the derived calculation expression is different from the provided calculation expression, the system displays "These illustrations do not correspond to the calculation expression. The calculation expression of these illustrations is <derived calculation expression>"

5. Evaluation

We conducted a preliminary evaluation of our environment to confirm that a learner who cannot pose problems via words was able to pose problems using illustrations. The participants of our study were four seven-year-old children. They were divided into two groups. Two of the four participants were first posed problems using words via MONSAKUN and then posed problems using illustrations via our environment. The other two participants first used our environment and then used MONSAKUN.

We started by first explaining problem posing to all four participants. Next, we explained how to use MONSAKUN to the two participants. These two participants used MONSAKUN for 15 minutes. After using MONSAKUN, we explained how to use our environment. These two participants then used our environment for 15 minutes. For the other two, we first explained how to use our environment, and they used our environment for 15 minutes. After using our environment, we explained how to use MONSAKUN, and they used MONSAKUN for 15 minutes.

MONSAKUN can provide various types of problem-posing exercises. In this evaluation, the participants posed the easiest type. MONSAKUN provided the participants with a calculation expression without the result of the calculation. The participants were required to pose a word problem that is solvable by the calculation expression by selecting two positive sentences and one interrogative sentence and sorting them into their proper order. For example, MONSAKUN provided the participants with "3 + 2" and the participants posed the following word problem: "Tom has three pencils. Tom buys two pencils. Then, how many pencils does Tom have?" Conversely, our environment provided a calculation expression with the result of the calculation as noted above. One existence illustration expressed the result of the calculation, instead of the interrogative sentence.

Table 1: Results of experimental use.

	Participant	MONSAKUN		Our environment	
		Correct problems	Incorrect problems	Correct problems	Incorrect problems
MONSAKUN –> our environment	A	2	9	5	1
	B	6	1	6	0
Our environment –> MONSAKUN	C	0	2	1	3
	D	2	2	4	1

Table 1 shows the results of our study. The four participants are indicated by letters A B, C, and D. Participants A and B posed problems via words before posing problems via illustrations; conversely, participants C and D posed problems via illustrations before words. In the table, the number of correct problems is the number of problems correctly posed by the participant. The number of incorrect problems is the number of problems incorrectly posed by the participant.

Participants A, C, and D were not good at posing problems via words. Although participant C was also not good at posing problems via illustrations, participants A and D could pose problems via illustrations more effectively than via words. Our initial results here suggest that learners who cannot pose problems by words can pose problems by illustrations. Participant B could pose problems by both words and by illustrations. We expect that a learner who understands problem-posing via words can also pose problems via illustrations in the same way.

6. Conclusion

Learning by posing arithmetic word problems is well-recognized as an effective way to master the use of solution methods; however, there are several learners who cannot pose problems via words. Therefore, in this paper, we proposed learning by posing problems using illustrations instead of words and developed a computer-based learning environment for such a task. The environment provides a learner with a calculation expression and illustrations. The illustrations can be disassembled into several figures. A learner assembles an illustration that describes a story of arithmetic problem that correspond to the calculation expression by combining the figures. The learner uses no sentences in this problem-posing exercise. The environment diagnoses the illustration problem and produces advice to the learner. One of the purposes of learning mathematics is to acquire the ability to derive a numerical relation from a scene and to associate a calculation expression with various scenes. We expect that posing problems via illustration and without words is an effective way for beginners to learn.

We also presented results of our preliminary evaluation using four seven-year-old participants. Our results were promising, indicating that our environment provides a means for learners who cannot pose problems via words. In future work, we hope to conduct more evaluations; we require more participants to confirm the effectiveness of our environment. Furthermore, we also plan to implement the "compare" relation, as well as multiplication and division.

References

Ellerton, N. F. (1986). Children's Made Up Mathematics Problems: A New Perspective on Talented Mathematicians, *Educational Studies in Mathematics*, Vol.17, pp. 261-271.

Hangata, E. (1990). Summarizing Fractions by Children, Practical Studies on Elementary School Mathematics in 1990s. *Journal of Japan Society of Mathematical Education*, Vol. 82, No. 10, pp. 102-105 (in Japanese).

Hirashima, T. and Kurayama, M. (2011). Learning by Problem-Posing for Reverse-Thinking Problems, *AIED2011*, pp. 123-130.

Hirashima, T., Yokoyama, T., Okamoto, M., and Takeuchi, A. (2007). Learning by Problem-Posing as Sentence-Integration and Experimental Use, *AIED2007*, pp. 254-261.

Joya, A., Maeda, K., Hirashima, T. (2013). Fraction Block as a Tool for Learning & Teaching Fraction and Its Experimental Use in an Elementary School. *Proc of ICCE2013*, pp. 103-109.

Moses, B. (1982). Visualization: A different approach to problem solving. *School Science and Mathematics*, 82, pp. 141-147.

Polya, G. (1945). How to Solve It, Princeton University Press.

Rliey, M.S., Greeno, J.G, Heller, J.I. (1983). Development of Children's Problem-Solving Ability in Arithmetic. The Development of Mathematical Thinking, Ginsburg H. (ed.), Academic Press, pp. 153-196.

Yamamoto, S., Kanabe, T., Yoshida, Y., Maeda, K., Hirashima, T. (2012). A Case Study of Learning by Problem-Posing in Introductory Phase of Arithmetic Word Problems, *Proc. of ICCE2012, Main Conference E-Book*, pp. 25-32.

Yu, F., Liu, Y. H., Chan, T. W. (2003). A Networked Question-Posing and Peer Assessment Learning System: a Cognitive Enhancing Tool, *Journal of Educational Technology Systems*, Vol.32, No.2, pp. 211 -226.