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Development of a Gamified Number Line App for Teaching Estimation and Number Sense in Grades 1 to 7

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Abstract: Fraction knowledge is known to be a gatekeeper to more advanced mathematical learning. On the basis of the literature on early number learning, a number line mobile application called *Catch the Carrot* was designed to develop students' knowledge of whole number and fraction magnitude. This paper aims to describe the design of the *Catch the Carrot* app and discusses the rationale for using number lines as representational scaffolds for developing children's understanding of numbers, particularly their estimation and number sense skills. The gamification features of the app, as well as strategies for integration in a classroom are also presented. This app, which utilizes number line representations, is expected to deliver the benefits associated with using number lines in instruction as suggested by the literature. The next step is to investigate the extent to which the gamified number line app promotes number sense and mathematical competence.

Keywords: Mathematical app, number sense, estimation, number line, STEM

1. Introduction

Entry into STEM-related fields requires competence in mathematical reasoning. Fraction knowledge is known to be a gatekeeper to more advanced mathematical learning. In fact, it has been shown that children's knowledge of number magnitude, particularly fraction magnitude, predicts algebraic performance (Booth & Newton, 2012) and various aspects of mathematical competence (Schneider, Merz, Stricker, De Smedt, Torbeyns, Verschaffel, & Luwel, 2018). As such, it is important to cultivate students' knowledge of fraction magnitude to facilitate success in mathematics and reinforce motivation to enter science-related fields. This necessitates knowledge of whole numbers as well, since both whole numbers and fractions are equally important in numerical development (Siegler, 2016).

To support students' critical thinking and problem solving, which are the twin goals of mathematics education as stated in the Philippine K-12 Mathematics framework (Department of Education, 2016), the authors designed a suite of applications, or apps, through the ongoing project "Technology Innovations for Mathematical Reasoning, Statistical Thinking and Assessment" (https://mathplusresources.wordpress.com/about/). This project is supported by the Philippine Department of Science and Technology-Philippine Council for Industry, Energy and Emerging Technology Research and Development (DOST-PCIEERD). The apps are aligned with the Philippine Department of Education's most essential learning competencies that were identified in response to the pandemic (Department of Education, 2020). The apps are free and internet connection is needed only once upon download. The apps can run on mobile phones, tablets, laptops, or computers.

The design of the apps was based on research in mathematical learning. The apps are not electronic versions of textbooks or worksheets whose primary functions are to provide drill and practice. Rather, the apps were designed to promote mathematical reasoning and visual thinking so that children can learn to think by and for themselves. Further, the apps were designed to be engaging, interactive and easy-to-play. Thus, the apps can be used even with minimal supervision, and are very apt for remote and independent learning of students. As most instruction in elementary and high schools focuses on theory rather than on experiential learning (Vahidy, 2019), these apps were designed to increase student engagement and support STEM learning outcomes.

This aim of the paper is to discuss one particular mathematical app, called *Catch the Carrot*. It discusses the pedagogical basis of the app, followed by its description and game-like factors, and suggestions for integration in the classroom. This app, which harnesses a number line representation of whole numbers, fractions, decimals, and integers, can be used for Grades 1 to 7 mathematics. The app is based on locating numbers on a number line, which has been shown to improve arithmetic knowledge (Maertens, De Smedt, Sasanguie, Elen, & Reynvoet, 2016) and mathematical competence (Schneider et al., 2018).

The app highlights the impact of technological tools in studying mathematics, namely Edutainment: the use of tools in a game-like environment to master mathematical concepts, develop estimation skills, and harness the aptitude for visualization and exploration. Educational apps are frequently game-based (Murray & Olcese, 2011), so an important feature of mathematics education apps is that the learning of mathematics is embedded in the playing of the game (Moore-Russo et al., 2015).

2. Pedagogical Basis: Number Line Representation

Children's experience of mathematics typically begins from the act of counting. Connecting the counting process to a spatial representation of these numbers, such as in a number line, is beneficial for further mathematical learning (Wu, 2010). A number line is a particularly useful tool for developing students' understanding of cardinality and the ordinal relationship of numbers (Woods, Geller, & Basaraba, 2018) and mathematical operations (Diezmann & Lowrie, 2006; Wu, 2010). There are various other spatial representations of numbers, but the unidimensional number line has been shown to be more effective in improving number sense. For example, Gunderson, Hamdan, Hildebrand, and Bartek (2019) found that the use of the number line resulted in better performance in magnitude comparison tasks as compared to area or square models. Training on number line estimation has been shown to have a positive effect on arithmetic for young children (Maertens et al., 2016). Further, number line estimation proficiency is associated with multiple aspects of mathematical competence (Schneider et al., 2018).

Another rationale for using number lines is it can illustrate the magnitude of numbers. Number magnitude is an integral concept that underpins knowledge of the real number system (including rational and irrational numbers) (Siegler, Thompson, & Schneider, 2011). Number magnitude knowledge of whole and rational numbers is correlational with and predictive of arithmetic, general mathematical achievement (Siegler, 2016), and algebraic performance (Booth, Newton, & Twiss-Garrity, 2014).

The number line also has an advantage as it can be used to represent all real numbers and the four arithmetic operations (addition, subtraction, multiplication, and division) (Wu, 2010). It is also a natural representation for the mathematical definition of a fraction. That is, the fraction $1/b$ is defined to be a whole divided into b equal parts, and the fraction a/b is defined to be a copies of $1/b$ (Wu, 2010). By contrast, other models are appropriate in only particular contexts. A pie representation can be used to model fraction addition, but it is not appropriate for representing two-thirds divided by three-halves. Ultimately, a fraction has to be learned as a *number*, and not as pieces of a pie.

To summarize, number line tasks promote estimation skills and magnitude knowledge, which are associated with increased mathematical competence. As such, an app based on number line representations is expected to deliver the benefits associated with using number lines in instruction. Further, a number line can represent both whole and rational numbers, as well as the four fundamental operations. For these reasons, the number line provides a consistent instructional medium for mathematical learning over several years.

3. The *Catch the Carrot* App

3.1 App Description

The *Catch the Carrot* app presents a random number and a number line or rectangle on the screen (Figure 1). A player has to estimate the location of the number on the number line or rectangle. The goal is to locate as many numbers as possible within the given time. Depending on the student's Grade Level, s/he may choose to play with whole numbers (Figures 1a and 1b), fractions (Figure 1c and 1d), decimals (Figure 1e) or integers (Figure 1f). There are also multiple given lengths for each number line. For example, Figure 1a shows a number line that starts from 0 to 10, while Figure 1b shows a number line that starts from 0 to 1,000,000. The number lines for fractions also utilize a hybrid unidimensional number line (Figure 1c) for younger students and a pure unidimensional number line (Figure 1d) for older ones. The hybrid unidimensional number line is particularly helpful because the space between the hash marks can draw students' attention to the unit or the whole (Gunderson et al., 2019), which is an important aspect of fraction understanding.

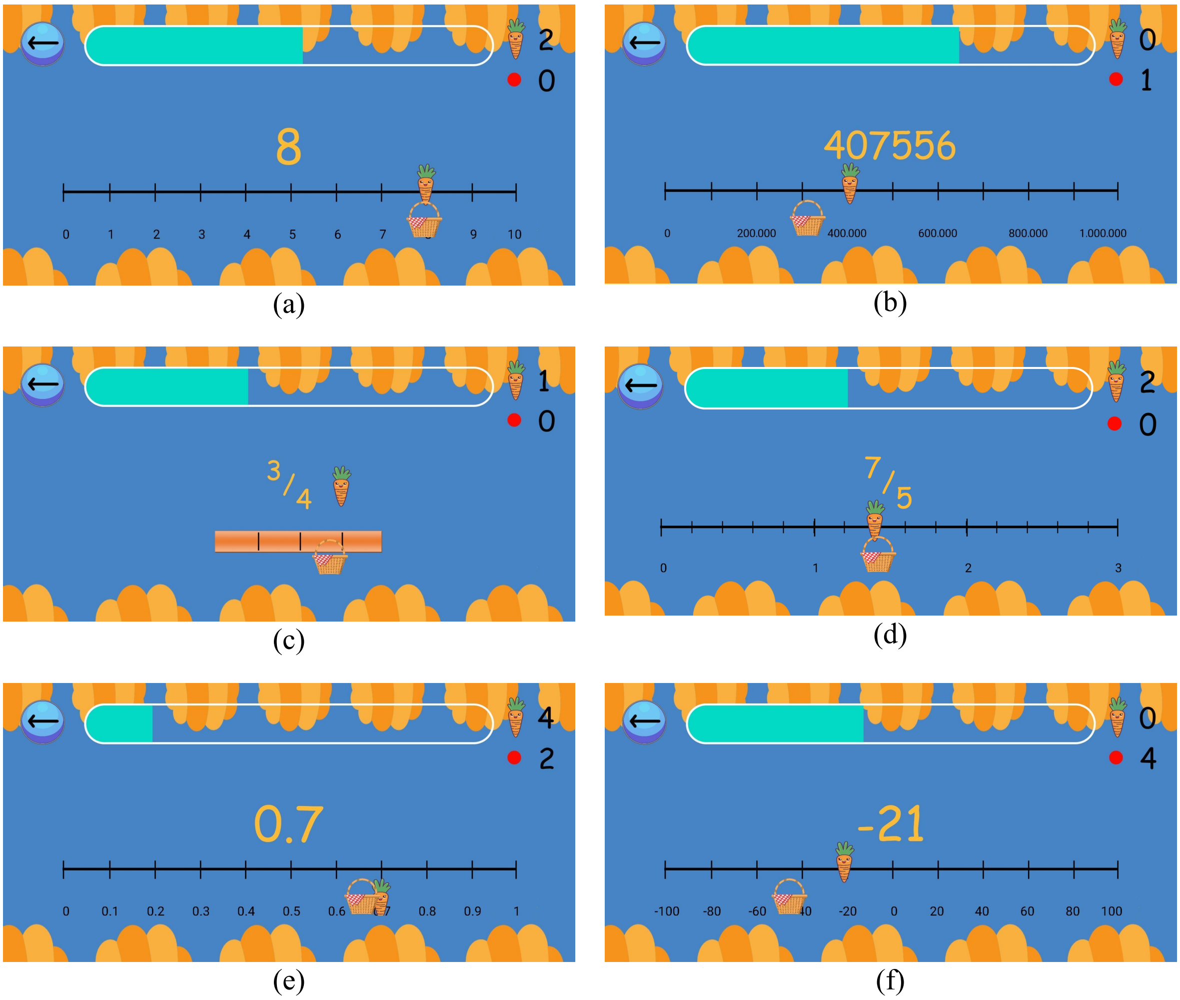
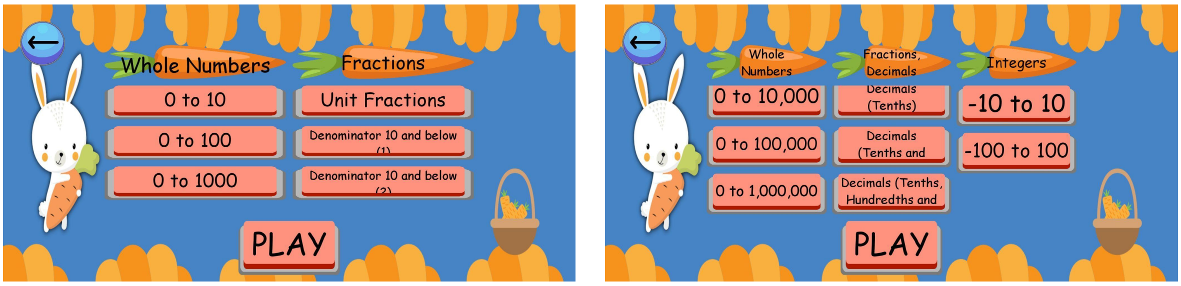


Figure 1. Different Levels of *Catch the Carrot.*

The levels and challenges present in the app are sequenced to align with the type and range of numbers that are specified in the official most essential learning competencies (MELCs) mandated by the Philippine Department of Education (Department of Education, 2020). For example, in Grade 2, a student is expected to learn whole numbers in the range 0 to 1000, as well as fractions less than 1 with denominators 10 and below. Thus, the app provides options "0 to 10", "0 to 100" or "0 to 1000" for whole numbers and "unit fractions" or "denominator 10 and below" for fractions (Figure 2a). On the other hand, the options for Grade 6 include "0 to 1,000,000" for whole numbers, "tenths, hundredths and thousandths” for decimals, and “–100 to 100” for integers, to align with the expected learning competencies for Grade 6 (Figure 2b). For Grade 7, the official curriculum includes a review of arranging rational numbers on the number line. Thus, the optional levels for Grade 7 include all the prior competencies from Grades 1 to 6. Across all levels, students are expected to be able to locate whole numbers and fractions on a number line, according to the scope of numbers specified in the official curriculum, to develop their skills in estimating number magnitude.



(a) Options for Grade 2 (b) Options for Grade 6

Figure 2. Number Types and Levels.

The *Catch the Carrot* app was designed with the premise that numerical development is dependent on knowledge of estimation and number magnitude (Siegler & Lortie-Forgues, 2014). By playing the game, students learn to anchor their fraction images with fraction magnitude and the definition of a fraction. For example, Figure 1d shows that the fraction 7/5 is equal to 7 copies of 1/5 (or 7 segments of 1/5 away from 0).

The design of the *Catch the Carrot* app was also informed by the theory of representational systems as a key goal of mathematics education (Goldin, 1998). Goldin (1998) proposed a unified psychological model of mathematical learning and problem solving based on a theory of representations. He stated, “the overarching goal should be to foster in students the *construction of powerful, internal systems of representation*” (p. 159; italics in original). In this connection, the app was designed to provide manipulable representations and visual corrective feedback (Bouck, Long, & Park, 2020). For *Catch the Carrot*, the manipulable representation is shown in the way a student can freely adjust the position of the basket along the number line. For visual feedback, the student’s answer is verified by the animation of a falling carrot; the answer is correct if the carrot falls into the basket positioned by the student. Through this dynamic feedback, students are allowed to learn from their mistakes and develop a better understanding of number magnitude.

3.2 Game-design Factors

With the intention of maximizing the potential of learning activities, gamification is sometimes employed as an approach to increase students’ engagement and motivation (Dichev & Dicheva, 2017). Moreover, gaming can serve to capture children’s interests towards studying and develop important skills in STEM education such as analytical thinking, strategizing, and problem-solving (Vahidy, 2019). In addition, given the current online or distance learning set-up in the Philippines coupled with the popularity of mobile games among Filipinos (Elliott, 2020), the gamification approach, especially when using mobile technology, may prove to be timely and apt.

Gamification has been applied in instructional design for elementary mathematics. For example, Hu and Shang (2018) transformed traditional teaching activities for learning elementary mathematics into gamified teaching activities using mobile applications. Following this approach, the development of *Catch the Carrot* involved the gamification of number estimation activities. While traditional activities on number estimation involve board work and paper worksheets, *Catch the Carrot* situates the activities in a game environment with the aim of increasing students’ engagement and motivation, and allowing them to play with minimal instruction. Furthermore, the app was designed so that it can be one of the mathematical apps that are “not only aligned with the curriculum but are also creative, fun and provide opportunities to learn” (Miller, 2018, p. 9).

The development of *Catch the Carrot* adheres to the GBL Design Model (Shi & Shih, 2015) that involves 11 interrelated game-design factors: game goals, game mechanism, game fantasy, game value, interaction, freedom, narrative, sensation, challenges, sociality, and mystery. The game-design factors of *Catch the Carrot* are presented in Figure 3, following the layout used by Shi & Shih for the GBL Design Model (Shi & Shih, 2015). Each game design feature is discussed in the succeeding paragraphs.

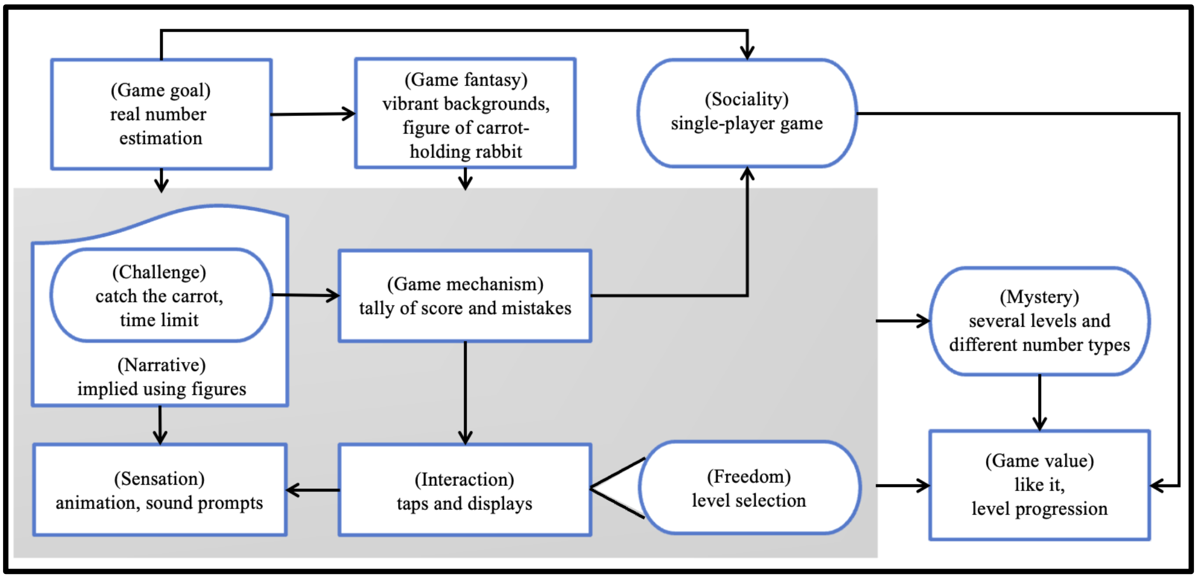
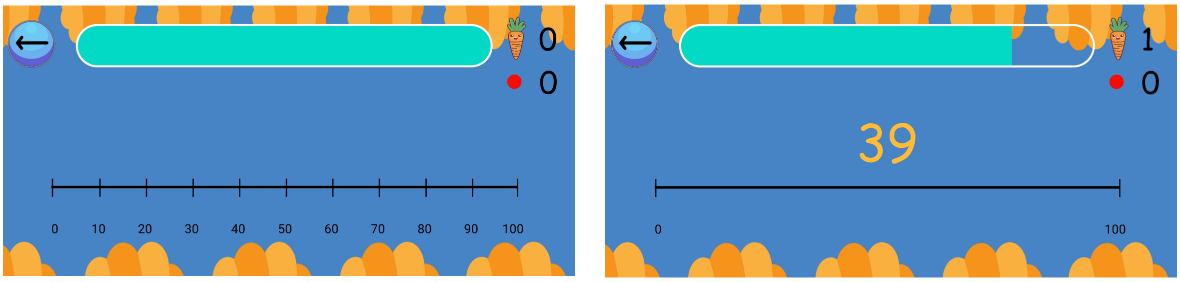


Figure 3. The GBL Design Model for *Catch the Carrot.*

The *game goal* of *Catch the Carrot* is to correctly estimate the location of a given real number in the real number line. This goal was a key consideration in the design of the app as it affected the game's mechanics, context, and structure. The game's *narrative* was designed to be simple. As the game is primarily intended for Grades 1 to 7 students, a complicated or text-heavy context might not be appropriate for the game. The narrative is presented through the figures of a rabbit, a basket, and carrots (Figure 4); that is, the implied narrative of the game is to help the rabbit by placing the basket correctly to catch a falling carrot. The figure of the carrot-holding rabbit and the game's vibrant background were designed to enrich the *game fantasy.*

Figure 4. The home screen of *Catch the Carrot* depicts an implied narrative for the game.

The *challenge* in the game design is demonstrated by showing the markings on the real number line only at the beginning of a round (Figure 5a). Once the game starts, the tick marks along the number line disappear (Figure 5b). Another challenge is the timer, which is shown at the top of the screen (Figure 5a and 5b) as a light blue bar that slowly decreases in length.



(a) Tick marks are shown at the beginning of a round. (b) Tick marks disappear as the game progresses.

Figure 5. The game screen of *Catch the Carrot*.

The game's *mechanism* is a simple tally, shown in Figure 6, of successful and unsuccessful carrot catches. The score tally is displayed after each round is played. While the mechanism was designed to be simple, it can be effective for both teachers and students alike, for tracking students' progress with number estimation.

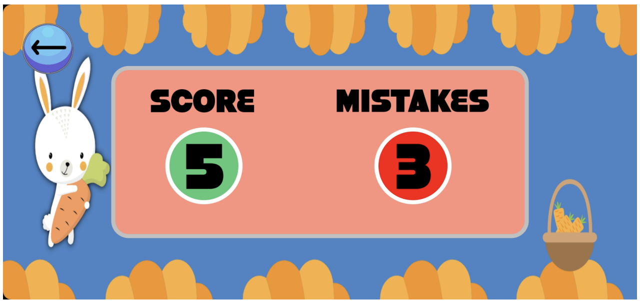


Figure 6. The score tally after one round of *Catch the Carrot*.

For the *interaction* and *sensation* game-design factors, it was important to take advantage of the capabilities of the digital environment without making the game appear or sound too complicated and chaotic. Thus, the game combines simple tap/click interactions with appropriate graphics, animations, and sound prompts. More specifically, after a player estimates the location of a given number on the number line, a basket appears at the tapped location and an animation of a falling carrot is shown. A bell sound is played if the carrot falls into the basket while a buzz sound is played otherwise. To provide the game some *mystery*, the app was designed to have separate levels for Grades 1-7. Within each grade level, the player has the *freedom* to choose the difficulty level based on number magnitudes and number types. The availability of these multiple levels is intended to capture the players' curiosity about the harder levels or the levels involving other number types. In addition, these levels were made available to align with the students' grade level and to give students the freedom to progress to higher levels once they are not anymore challenged by earlier levels.

Sociality was not woven into the game as it was mainly set up as a single-player game. However, with teacher intervention, it is possible to set up different scenarios where students may socialize with each other while playing the game. For instance, a teacher may group students and have the members of each group play the game simultaneously so that they can give their ideas about the correct location to tap. In an online setting, this can be done using breakout rooms with one student sharing his/her game screen.

All the factors mentioned above provide value to the game. In addition, attaining high scores in one level and moving to the next (i.e., level progression) may also add to student motivation. Moreover, the app has also been designed to be easy to understand and to use so that students can play it without the supervision of a teacher or a guardian. This makes the app ideal for asynchronous and independent learning.

4. Integration and Use of *Catch the Carrot*

The design of *Catch the Carrot* app was informed by research on number line estimation and knowledge of number magnitude, which have been shown to improve counting, arithmetic, algebraic knowledge, and general mathematical competence. On this basis, its integration in the elementary mathematics instruction is warranted (Woods et al., 2018). It can be used in presenting a lesson on visualization, representation, and estimation of a number, providing opportunities for practice and assessment. Woods et al. (2018) offered the following steps to integrate number lines in teaching.

Step 1. Determine learning goals

There is a need to determine if *Catch the Carrot* is appropriate and useful in attaining the learning goals. Number lines can model cardinality, ordinality, quantity comparisons, and operations. *Catch the Carrot* can be an effective tool for learning goals that involve these concepts.

Step 2. Plan Ahead

The teacher should prepare in advance the logistical considerations for the use of the app, which is free for download. Teachers should instruct the students on the proper download of the app on an Android device or a desktop. If the app is downloaded on a desktop, the app needs to be played using an Android emulator.

Step 3. Present and Practice

This step involves the presentation of the lesson where the teacher demonstrates how the app works and how it can be played. After this, the teacher may model the process of estimating the fraction (or number) on the rectangle (or number line). The teacher may pose questions such as “Should the fraction be near the left end of the rectangle?” or “When is the location towards the right end?” These guide questions could prevent students from relying on guesswork as a strategy to locate the fraction (or number). As previously suggested, if the online set-up has a provision for break-out groups or rooms, students can work together and collaborate. During this step, students gain opportunities to practice and be familiar with the app.

Step 4. Process and Verbalize

Finally, it is important to process the activity with open-ended questions that can challenge students to explain their thinking and reasoning about the lesson. The teacher can facilitate students’ thinking with reflection questions such as, “What helps you locate the position of the fraction?,” “How do you know that the fraction is near the right (or left) end of the rectangle?” and “Why do you think the fraction is in the middle portion?”

The teacher may give instructions to the students to use the app asynchronously or outside class time for practice and mastery. If a teacher would like to check the students’ progress, s/he may ask them to send screenshots of their outputs over a period of time. A follow-up class discussion can be facilitated in the next synchronous session. During this meeting, the teacher can assess the depth of the students’ understanding and determine their difficulties and weaknesses in estimation and visualization of numbers.

An earlier version of *Catch the Carrot*, involving arranging rational numbers in a number line, was part of a set of mathematical apps introduced to high school teachers in a training workshop in October 2018 (De las Peñas et al, 2019). The teachers were asked to give feedback on the apps and the responses were collected at the end of the workshop. Overall, the teachers gave a positive evaluation of the mathematical apps, especially in relation to increasing students’ interest in the topic. Moreover, teachers stated that the mobile apps provide a more interactive means of instruction (as opposed to traditional paper-based media). They mentioned that the apps show potential in helping students’ retention of lessons. They further responded that the apps provide a novel and interesting way of approaching classroom discussions and help increase student motivation in studying their lessons.

5. Conclusion and Future Direction

This paper described *Catch the Carrot*, a number line app designed to develop estimation and magnitude knowledge. These skills are predictive of algebraic knowledge and facilitate access to more advanced mathematical competencies in STEM fields. The app is an innovation in the form of gamifying the number line to represent number types and magnitudes, which are topics that comprise the curriculum from Grades 1 to 7. The paper discussed the mathematical pedagogies and gamification features that informed the fundamentals of the app creation and offered strategies for its integration in a blended learning environment. The next step is to investigate the effectiveness of the tool in a physical or online classroom, to confirm the anticipated benefits suggested by the literature.

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