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Computer-Supported Training of the Mental Number Line

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Abstract: The human representation of number magnitude is often described by the metaphor of a mental number line. Recent studies repeatedly found the accuracy of children's mental number line representation to be associated with more general arithmetic and mathematic achievement. Therefore, specific number line trainings have been developed. A growing number of intervention studies indicates significantly positive training effect with improvements generalizing from number line accuracy to other numerical (e.g., magnitude comparison) and arithmetical tasks (e.g., mental addition). This holds for both paper-pencil as well as first computer-supported number line trainings. However, with computer technology progressing rapidly it provides increasingly sophisticated possibilities for training the mental number line. In this article we specifically elaborate on the issues of interactive learning environments as well as embodied interactions. With respect to interactivity we discuss its necessity for the adaptivity of the learning environment required to guarantee balanced success rates in multi-learner trainings. As regards embodied interaction we elaborate on new possibilities to develop trainings allowing for bodily experiences of numerical concepts such as the mental number line offered by new digital media. We conclude that the latest development in computer technology opens up new directions for the successful training of numerical competencies that should be pursued as they may be particularly beneficial for those with special needs in numerical / mathematical learning.

Keywords: Mental number line, Computer-supported learning, Numerical development

1. The mental number line

The spatial representation of numbers, often described by the metaphor of a mental number line (MNL), is one of the basic numerical representations (Dehaene, Piazza, Pine, & Cohen, 2003; Booth & Siegler, 2006). Upon this MNL numbers are represented spatially in ascending order corresponding to their magnitude. Importantly, there is evidence on such a spatial representation of number magnitude from infancy on to adulthood (e.g., de Hevia & Spelke, 2009). Moreover, the MNL relates number magnitude and space with a specific spatial directionality. In languages with left-to-right reading and writing smaller magnitudes are associated systematically with the left side and larger magnitudes with the right side of space. Fischer, Warlop, Hill, and Fias, (2004) observed that fixating a digit induces spatial shifts of attention while the spatial direction of head movements predicts number magnitudes produced in random number generation tasks (e.g., Loetscher, Schwarz, Schubiger, & Brugger, 2008). Thus, the association of numbers and space seems automatic, bidirectional and present from early age on.

Importantly, the MNL representation was observed to be associated positively with children's numerical and arithmetical competencies (e.g., Booth & Siegler, 2006). In children, the MNL representation is usually assessed by the number line estimation task in which participants are asked to estimate the location of a given target number (e.g., 17) on a presented number line (ranging for example from 0 to 100, e.g., Siegler & Opfer, 2003). While the spatial-numerical demands of these tasks are undisputed, the processes underlying the task are more controversial (e.g., Moeller, Pixner, Kaufmann, and Nuerk., 2009). Thus, when training number line estimation it is not only the spatial nature of number magnitude representation which is practiced but also other representations recruited to map numbers onto space. Yet, for the development and implementation of successful MNL trainings the continuous spatial character of the MNL seems particularly promising.

2. Training the mental number line

2.1 First approaches

In recent years several number line interventions were proposed. On a quite basic level Opfer and Siegler (2007) gave second graders feedback on their number line estimations and found these to improve performance *„strikingly abrupt, often occurring after a single feedback trial, and impressively broad, affecting estimates over the entire range of numbers from 0 to1000.“* (p. 169). Similarly, Ramani and Siegler (2008) used linear board games, in which children had to move a playing piece along a linear and equidistant number line. The authors observed positive transfer effects of such linear board games on children's performance in magnitude comparison, counting, and numeral identification, which were not observed for an identical game with colors or circular (instead of linear) number board games (Siegler & Ramani, 2009). Furthermore, Ramani and Siegler (2011) found the specific benefits of those linear board games to be present in low-income children in particular. Additionally, Ramani, Siegler and Hitti (2012) also observed these beneficial effects in the classroom setting.

In summary, there is accumulating evidence suggesting that spatial-numerical games can be applied successfully to foster numerical and arithmetic competencies in children. So it is not surprising that components training spatial-numerical associations have also been employed in other multi-componential intervention or prevention programs such as Rightstart (e.g., Griffin, Case, & Siegler, 1994) or Big Math for Little Kids (Greene, Ginsburg, & Baalfanz, 2004). However, their specific effects were usually not evaluated. Finally, it is important to note that all of these interventions were paper-pencil based. Yet, computer-supported methods may add considerably to increase both the motivational appeal but also training efficiency by (i) implementing numerical trainings in general, (ii) ensuring adaptivity, accessibility and interactivity (i.e., collaborative and competitive gaming), and (iii) allowing to experience numerical concepts in an embodied fashion.

2.2 Computer technology to train numerical competencies

Over the last decades the use of computer technology to support mathematical learning has become increasingly popular. Already back in 2000 the National Council of Teachers of Mathematics of the USA claimed that "technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning" (p.11). Recently, Li and Ma (2010) evaluated the actual benefits of computer technology in mathematics education. They not only observed significant positive effects of computer technology on mathematics achievement but also found that effects of computer technology were relatively more pronounced in elementary school and for children with special needs in mathematics learning. This is of particular interest as it suggests that computer technology may be specifically suited to support the acquisition of basic numerical competencies which are most often impaired in children with mathematics learning disabilities (e.g., Dowker, 200). Against this background the MNL is a reasonable candidate to be trained in computer-supported environments.

2.3 Computer-supported MNL training

There have been several approaches to train the MNL representation using computer technology in recent years. In the following, we will first give a brief overview of the range of computer-supported MNL trainings from very circumscribed number line trainings to multi-componential training programs. After this we will then discuss two specific aspects of the use of computer technology important with respect to future applications: Interactivity and adaptivity of training environments and the possibility of task-specific embodied experience of the concepts trained.

2.3.1 Computer-supported number line trainings

Current developments in educational technology strive to use methods of computer-based instruction or – as more advanced methods - of "serious games", "game-based learning" or "digital educational games" (Amory, Naicker, Vincent, & Adams, 2008). Besides general motivational effects, these methods are also able to support learning more specifically by measuring learner performance, providing different levels of task difficulty, and giving fast and individualized *feedback* in rich multimedial environments. So, it is not mainly the use of media itself which supports learning, but their ability to induce or support specific cognitive processes (Kozma, 1991; Clark, 1994).

There are currently a variety of different computer-supported number line trainings and we do not pretend to present an exhaustive list. Nevertheless, most of the existing number line trainings are part of multi-componential numerical trainings for the prevention or remediation of mathematics learning difficulties. Some of these programs are freely available on the internet (e.g., The Number Race: http://www.thenumberrace.com or Calcularis: http://www.calcularis.ch/int) whereas others are commercially oriented (e.g., Number games with Elfe and Mathis: http://www.psychometrica.de/rechenspiele1.html; Number Worlds and Building Blocks: http://www.sranumberworlds.com/; http://gse.buffalo.edu/org/buildingblocks/ which overlap partly). Importantly, all of these programs address basic numerical competencies and are targeted at rather young children from pre-kindergarten through elementary school (see Clements & Sarama, 2011 for a review). However, the reference to as well as the specificity with which the MNL is trained varies considerably between programs. Calcularis, Number Worlds, and Elfe and Mathis use number lines in a quite explicit way with some kind of number line estimation as one component of the intervention. In Calcularis, for instance, a falling cone has to be landed as closely as possible to the target number on a given number line. In contrast, the primary focus in The Number Race is on magnitude comparisons: Whenever one wins against a computer player one is allowed to move a playing piece an according number of steps. Thereby, a number line is referenced to in a similar way as in the board games used by Siegler and collegues (e.g., Siegler & Ramani, 2009).

Some of the multi-componential intervention programs are evaluated scientifically indicating their positive effects on children's numerical development in general and their initial arithmetic achievment in particular (Number Worlds: e.g., Griffin, 2004; Building Blocks: e.g., Clements & Sarama, 2007; Number games with Elfe and Mathis: Lenhard, Lenhard, Schug, & Kowalski, 2011) - while others still await evaluation (e.g., Calcularis). Nevertheless, evaluation did usually not differentiate between single components of the programs so that it is hard to infer the effectiveness of the number line components in particular. This is different for The Number Race as it specifically aims "to emphasize the association between representations of number and space (Wilson et al., 2006a, p. 2). There are several empirical studies indicating its positive effects (Wilson et al., 2006b; 2009). Importantly, however, only Kucian et al. (2011) employed and evaluated a specific number line training. The authors used an advance version of the Calcularis program. The evaluation of this number line training not only involved behavioral measures but also their neural correlates. Behaviorally significant improvements of children's number line accuracy as well as arithmetic performance were found. On the neural level this was accompanied by reduced recruitment of brain regions associated with number magnitude manipulations after the training due to automatization of cognitive processes.

Moreover, there are also attempts to use the MNL concept for training more advanced mathematical competencies. Still on a basic level The Number Catcher (http://www.thenumbercatcher.com) addresses the base-10 place-value structure of the Arabic number system and multi-digit numbers. Moreover, the ARILAB II environment contains a training of fraction knowledge by systematic use of number line information (e.g., Psycharis, Latsi, & Kynigos, 2009). Finally, Olson (1995) developed a framework using number lines in the teaching of algebraic functions.

2.3.2 Interactive number line trainings

In most of above number line trainings learners interact only with the learning environment. Yet, it is known that players in digital games seem to favor the opportunity to challenge and compete with one another (e.g., Weibel, Wissmath, Habegger, Steiner, & Groner, 2008). Such a competitive reward structure (Johnson & Johnson, 1989) seems adequate for simple and speed-related tasks which train automatic processes (Clifford, 1972), as it is the case for training the MNL. However, it also requires a homogeneous group of learners, because otherwise it would be demotivating for low-achieving learners (Ames, Ames & Felker, 1977). However, as a homogeneous group of learners cannot be guaranteed there are specific demands on interactivity and adaptivity of the learning environment.

In this vein, we recently developed a web-platform allowing for interactive browser games addressing numerical and orthographical competencies (http://dbwiscam.informatik.uni-tuebingen.de/). It is possible to register at the platform and create an account before entering the game zone. Amongst other games training arithmetic (e.g., addition, multiplication, etc.) a number line game was implemented in which the players have to indicate the correct position of a given target number within a pre-defined number range. The number line game comprised two number ranges (i.e., 0 to 100 and 0 to 1,000) and two difficulty levels (i.e., 0 as start and 100 or 1,000 as endpoint vs. odd ranges, e.g., positioning 37 on a number line from 12 to 64). Importantly, the number line game allows interactive gaming in either single- or multi-player modus with up to five players in which a player can play against other human or computer-controlled opponents. Generally, it is not possible that a player places his/her estimation on the same position as another player because the marker used to indicate ones estimations covers about 5% of the entire number line. This way, players are encouraged to make spontaneous estimations. After all players have set their mark the player with the estimation closest to the correct position of the target number (indicated by a green bar) wins the game. Importantly, a first field study showed that because numerical skill levels of players are very heterogeneous playing against students with a higher skill level was frustrating. However, skill levels need to be matched so that players apply more effort and play longer periods of time (Liu, Li, & Santhaman, 2013). Therefore, adaptivity of the training environment is the crucial aspect which will make or break the whole idea of interactive number line training (see also Wilson et al., 2006a, for adaptivity in The Number Race). In the further development of the number line game and the web-platform adaptivity will be implemented in several ways. First, with every game played each registered player will receive an automatically updated indication of his/her proficiency level to selectively pair comparable game partners. Second, we will introduce the possibility of individual handicaps (e.g., time delays) to a balance the chance of winning. Finally, based on the players' previous estimation performance the activity of the computer-controlled player shall be adapted to allow for an individual success rate of at least 50%. Thereby, it should be possible to balance success rates and keep the motivation of the individual players high regardless of their skill level with the ultimate goal of interventions tailored to the individual's needs.

2.3.3 Embodied number line trainings

In recent years, digital media such as dance mats and later infrared- (e.g., Nintendo Wii) and camera-based systems such as the XBOX Kinect sensor started to provide the additional feature of an "embodied interaction" (Dourish, 2001) by measuring user movements and/or gestures. Yet, besides the general motivational appeal of embodied interaction, movements and thus embodied features can also be used to support learning more specifically - given the task to be learned also has a spatial dimension. In case of numerical tasks the movement of a learner may be congruent or incongruent to the internal MNL representation. When any learning task activates the MNL, it may be expected that movements which are congruent to this internal representation should corroborate task performance and learning.

As a first example of specific embodied number line training Fischer, Moeller, Bientzle, Cress, and Nuerk (2011) used a digital dance mat to train kindergarteners on a systematic left-smaller and right-larger association. In a magnitude comparison task children had to take a step to the left when a presented target number was smaller than a simultaneously presented standard and a step to the right when the target number was larger than the standard. Importantly, the results corroborated the hypothesis that an embodied number line training should be more effective than a control training without physical-spatial elements. Children not only got specifically more accurate in positioning target numbers in number line estimation but the advantage of the embodied over the control training also generalized to children's counting principles, which were not trained directly. Finally, the results of a mediation analysis indicated that children's improvement in counting was mediated by their improvement in number line estimation and thus by an increase in number line accuracy.

Interestingly, these differential training effects favoring the embodied over the control training were found for the case of a categorical training of number line direction (i.e., one step to either the left or right). However, so far the association between the MNL and numerical/mathematical performance was even more pronounced for tasks assessing the continuous nature of the MNL (i.e., number line estimation). Therefore, it seems promising to expand the concept of embodied number line training to continuous movements along a number line. Possible computer technology to allow for this is now available with the increasing width of SMARTBoards® or camera-based systems such as the XBOX® Kinect sensor. Accordingly, Link, Moeller, Huber, Fischer, & Nuerk (in press) pursued this idea using the kinect sensor in first graders who had to estimate the position of a target number by walking along and stop at the correct position on an up to 3 meter long number line taped on the floor. Compared to a non-embodied control condition they observed that children improved more strongly not only in number line estimation but also in a mental addition task. Again this indicates an advantage of the embodied training condition. Finally, so far these embodied approaches to number line learning are restricted to single user environments. However, when implementing above described attempts to balance success rates competitive trainings on the dance mat are well conceivable. Moreover, refined tracking procedures for camera-based systems now allow for tracking more than one person at a time and thus a competitive version of the continuous number line training as used by Link et al. (in press).

3. Summary and Conclusions

In the current article we described the idea of a MNL and aimed at evaluating in what way the development of children's MNL representation can be corroborated by computer-supported learning environments. Please note that there already are both paper-pencil but also computer-supported approaches to number line trainings for which scientific evaluation indicated significant training effects. Nevertheless, we argue that with the rapid progress in computer technology increasingly sophisticated possibilities for training the MNL are provided. With respect to interactive learning environments we suggest that adaptivity will be the crucial aspect of any learning environment aiming at allowing interactive and competitive number line trainings. Mechanisms implemented to guarantee balanced success rates in multi-learner trainings seem mandatory to keep even those with specific needs motivated. As regards the possibility of embodied interactions with the learning material we propose trainings allowing for bodily experiences of the MNL to be particularly fruitful. New media devices such as the Kinect sensor allow for full-body movements along the MNL with the isomorphism between full-body movements in physical and number space corroborating children's understanding of numerical quantities. In this context, recent results from research on children with special needs in mathematics education are of specific relevance. Studies by Ramani and Siegler (2011; 2012, see also Wilson et al., 2009, for similar evidence for The Number Race) clearly indicate that training the number line by playing linear board games seems to be even more effective for children from underprivileged families. Against this background exploiting the possibilities of recent progress in computer technology seems specifically reasonable as it opens up new directions for the successful computer-supported training of the MNL targeted at those with special needs in numerical / mathematical learning.

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