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A Dynamic Approach to Teaching Trigonometry

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Abstract. In this paper we describe a highly interactive touch-based application to teach the basics oftrigonometry to secondary school students. The application, called Ziggy, lets students “touch” and“push” triangles, dynamically modifying the shape and size, and observe the effect on the angles, sidesand the trigonometric ratios. An early version of Ziggy has been tested in small-scaleexperiments in the classroom.

Keywords: trigonometry, math, interactive learning environments, touch devices

1. Introduction

Basic trigonometryis hard for most learners. One of the main obstacles students experience is relating the graphical representation of a triangle and the underlying mathematics(Blackett& Tall,1991). Interactive environmentscan help visualize this relationship by showing how changing the shape of a triangleaffects the correspondingvalues of the angles and lengths of the sides.There are many interactive environments that support learning geometry (e.g., GeoGebra, Cinderella, Geometer’s Sketchpad) but, to the best of our knowledge, thereare currently no interactive tools offering a comprehensive instruction on basictrigonometry. Touch devices are becoming increasingly popular in schools, and they make it possible to transform mathematical objects and functions into graspable objects that can be manipulated, transformed, and explored in an intuitive manner.Real-world motion, as used in touch devices, can help students to recognize these kind of patterns more readily(Ottmar, Landy, & Goldstone, 2012).Moreover, successful math students oftenmake use of perceptual and visual patterns to solve problems (Kellman, Massey, et al. 2008).

This paper describes an interactive touch-based application,called Ziggy[1], to teach the basics of trigonometryto secondary school students.The application contains both instructional materialand exercises covering the Pythagorean theorem, the 180º rule and the trigonometric relations sine, cosine andtangent. Ziggy offers the ability todrag numbers from the pictorial representation of anexercise, toalgebraic equations and emphasizes the dynamic relationship of the trigonometric functions. It wasexpected that that this type ofinteraction with triangles fosters a deeper understanding of the underlying principles.

2. The application

Figure 1 illustrates the interface of Ziggy. On the left is the graphical representationof an exercise. The area on the right is used for mathematicalequations related to the exercise, and the instructions are at the bottom.Students can drag the angles indicated with a semi-circle.The values displayed along the sides are updated dynamically while the student is dragging, asare the corresponding values in the equation.

[1]Ziggy is an acronym for “Zeerinteractievegoniometrie”, “Very interactive trigonometry” in English.It is available at http://ziggy.gw.utwente.nl.

Figure1. Screen layout for exercises.

To compute the unknown angle A, students choosethe appropriate rule (e.g., sin) by clicking on one of the buttons in the upper right corner of the screen. An empty equation appears and students solve the exercise by dragging values fromthe triangle to the empty slots in the equation. Next, the equation iscomputedbyrearranging the equations and changing the locations of values. This method, called pushing symbols, is based on the technique used in Algebra Touch (Ottmar, Landy, & Goldstone, 2012). Figure 2 shows how a student drags the denominator (8.05) across the "=" (middle). This gesture represents multiplying both sides by 8.05. Amultiplication sign appears and a hole is made for the value.

$$\cos 37° = \frac{a}{8.05} \qquad \cos 37° = \frac{a}{8.05} \;\; 8.05 \qquad \cos 37° * 8.05 = a \;\; 8.05$$

Figure 2. Illustration of the pushing symbols technique introduced in Algebra Touch as used in Ziggy.

Ziggy immediately offers feedback when students make mistakes tryingto solve exercises. Students are not allowed to drag values to a mathematically incorrect location inan equation. For example, angles cannot appear in locations where a side is expected. When a student selects the wrong rule or when values are incorrectly entered into a formula, Ziggy blocks the actions of the student and displays an error message.A more detailed (technical) description of Ziggy can be found in Anjewierden, Kamp,& de Jong (2013).

3. Ongoing Research

In a pilot study we compared education using Ziggy with traditional education. A prototype ofZiggy was implemented in a real classroom setting at a secondary school in the Netherlands.

3.1. Participants

Participants were 28 third grade high school students, who followed a track for pre-university education. The group consisted of 16 males and 12 female students from two different classes. The non iPad class with 8 females and 6 males scored 5.99 (SD 1.024) on average on math. The iPad class consisted of 4 females and 10 males and had an average math score of 5.52 (SD 1.132). This group was familiar with operating an iPad, which they had been using in the classroom since the beginning of the school year.

3.2 Procedure

Students participated in two lessons of 45 minutes. Both lessons started with instruction on the five rules of basic trigonometry:Pythagorean Theorem, the 180ºrule, and the sine, cosine and tangent. This instruction was thesame for both groups. In the control group static pictures were used to illustrate the underlyingprinciples as opposed to the dynamic and interactive triangles of Ziggy in the experimentalgroup. In the second part of the lessons students were offered a set of exercises in which anunknown value of a triangle had to be calculated using two given values and one of the fiveabovementioned rules. The Ziggy group used interactive and dynamic triangles, and solved theequations using the pushing symbols technique described. The control group was offered similarexercises on paper, using static pictures of triangles, and they used pen and paper and a calculator to compute their answers. The second lesson started with refreshing knowledge learned in the previous lesson. After ten minutes of practice with simple triangles, more difficult exercises were introduced using composite triangles. Apost-test was used todetermine knowledge gain. The first part assessed conceptual knowledge and consisted of thirteen items. Students were asked to predict the effect of changes in shape and size of triangles on the corresponding numerical values of the sine, cosine and tangent. The second part of the test measured procedural knowledge and consisted out of fourteen item in which students had to solve traditionaltrigonometrically problems.

3.3 Results

Preliminary findings do not support our assumption that the use of Ziggy will increase learningeffects. Both groups in the second study showed similar learning outcomes on the post-test. At the time of the study, however, Ziggy was stillunder development and results can be partly attributed to flaws in the prototype. For example,students in the Ziggy group were not able to look up information in the instruction pages duringpractice. We also feel that some of the support Ziggy offers might actually be too much.Students quickly thought they understood the material, and processing of underlying principlesseemed to remain shallow.

4. Conclusion and future work

We currently focus on improving the navigation of Ziggy and fixing remaining bugs. We also aim to improve the feedback offered by Ziggy. As indicated we feel that the feedback should be more subtle.We are working on feedback that will stimulate students to think about problems, like the feedback used by Stampfer,Long, Aleven&Koedinger (2011). In a new version of Ziggy, exercises have been added which areespecially designed to challenge students' understanding of dynamic relationships between trigfunctions and stimulate a deeper understanding of the material. Future research is necessary to check if these adaptations will lead to improvedlearning results.

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