Iyer, S. et al. (Eds.) (2022). Proceedings of the 30th International Conference on Computers in Education.
Asia-Pacific Society for Computers in Education

# Analysis of CSCL for Mathematical Proof Based on The Log Data of Learners' Verbal and Nonverbal Communications

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**Abstract:** In this study, we investigate the process by which university students produced mathematical proofs in a CSCL environment. Students discussed how to prove the addition theorem for trigonometric function while manipulating mathematical content generated by a dynamic geometry system. The log data of their utterances and manipulations was stored and visualized. Besides the annotated text data of their communication and videotaped images of their activity, we analyzed the fine-grained log data. Analyzing how students produce proofs is difficult because we must recognize the critical moment of the proof by observing their activities while they are still engaged in the proof work. Furthermore, their insufficient background in mathematics prevents them from fully explaining their thinking processes. The result of this study indicates that the comparison of qualitative data of learners' communication and those multimodal log data can help us unfold their convergent conceptual change.

**Keywords:** Mathematical Proof, Dynamic Geometry, Log Data of manipulations, Log Data of Utterances, Multimodal Analysis on the Collaborative Work

## 1. Introduction

This study explores the process of how students prove the addition formula for trigonometric function in CSCL environment. As seen in some CSCL studies in mathematics (Oner, 2013; Schwarz, et al., 2018), using a dynamic geometry system can help learners visualize and simulate mathematical objects. The system used in this study is CindyJS (https://cindyjs.org/) which enables us to manipulate mathematical content on a web browser. A Moodle plug-in of CindyJS has been implemented to store the log data of learners' manipulations (Kaneko, et al., 2020). We also utilize another system to record the sound of each learner's utterance and store the log data of its volume (Ishikawa et al., 2019). The resources for mathematical activities are composed of many artifacts including gestures as a conceptual metaphor, written modes, spoken discourse, and visualization on digital media (Lakoff & Nunez, 2001). From the commognitive perspective, learners' discursive action makes them individualize the newly signified mathematical objects which are realized in those modalities. In the case of this study, learners experienced some commognitive conflict while generalizing their prior knowledge to newly emerged situation. Learners' participation in some discursive processes (metadiscourse) to handle the relevant commognitive conflict leads to the reification of those processes through which a new object is constructed (Sfard, 2008). From the commognitive perspective, learners' convergent conceptual change is identified with the change in pattern of their discursive process. Since the subtleties in those changes tend to be missed in simple field observations, fine-grained transcriptions of their communication and repeated viewings/hearings of recordings are needed (Koschmann & Zemel, 2009). This study proposes that high resolution multimodal data of learners' verbal and nonverbal communications can supplement their transcribed discussion.

## 2. Research Method and Results

The CindyJS content used in this study is shown in Figure 1 (left). By moving the red points on the unit circle, the angles $\alpha$(red) and $\beta$(green) can be changed. The log data of the fluctuation of these angles are stored on Moodle with a time stamp. Three students were asked to manipulate the content on an iPad collaboratively. They were three first grade female students in a Japanese university. They had already learned the proof of the addition formula in the rudimentary case of when all angles are acute. Since only one student could manipulate the content at a time, they had to discuss each manipulation and each step of the proof. Using wearable microphones each student's utterances were recorded separately. Moreover, the students' activities around the iPad were videotaped. Before their collaborative work, they were asked to fill in a worksheet including the static figure in Figure 1 (right) to review that rudimentary case separately. The time they needed to fill in the worksheet was measured to estimate their prior skill level. The utterances and behaviors of students were annotated chronologically. The log data of the volume of each student's utterance was graphed with the log data of their manipulations. A previous study indicated that an utterance produced in unison might reflect the fact that it was the most obvious way for participants to describe the phenomenon as they understood it (Koschman & Zemel, 2009). In view of this suggestion, the concordance of some participants' utterances was specified. Using the manipulation data, the process through which students led to those utterances was examined. Simultaneously, using the sound data derived when some participants produced concordant utterances, the power balance between them together with their intention was presumed.

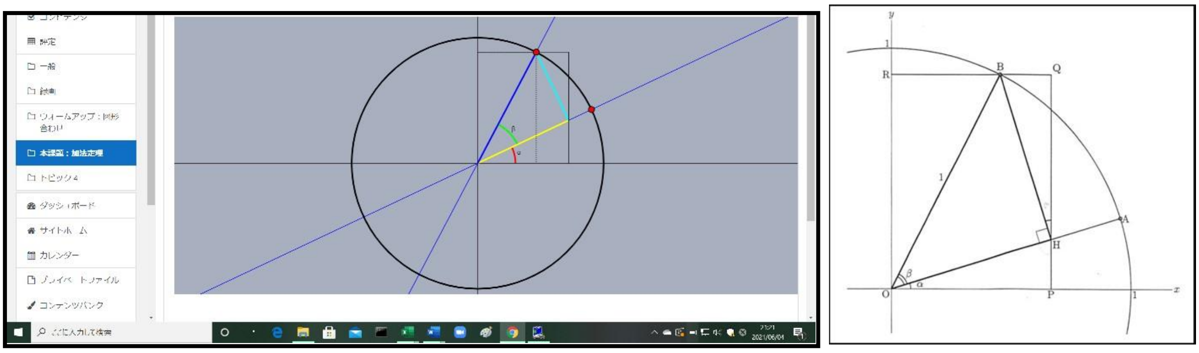


*Figure 1.* The CindyJS content used in this study (left) and static drawing in printed material (right)

The log data of the students' manipulations is visualized in Figure 2 (left). The horizontal axis pointing to the right represents the passage of time and the fluctuation of the angles $\alpha$ and $\alpha+\beta$ is represented in black and red respectively. Comparing this data to the annotated text of communication, it can be observed that participants' discussion had reached an impasse after completing the first step of the proof where the angle $\alpha+\beta$ was obtuse while the angles $\alpha$ and $\beta$were kept acute. In fact, the reason for the impasse was that the participants tried the complex case in which the angle $\beta$ was negative and the instructor gave them some advice at 15 minutes after the beginning. Therefore, the authors analyzed only for the initial 10 minutes. The log data of the volume of each participant's utterance is visualized in Figure 2 (right). The data corresponding to the participants A, B, and C are represented in green, red, and black respectively. While it was observed that their prior skills were ranked as C>A>B, the result in Figure 2 (right) shows that the utterance of B was most dominant in the process of communication.

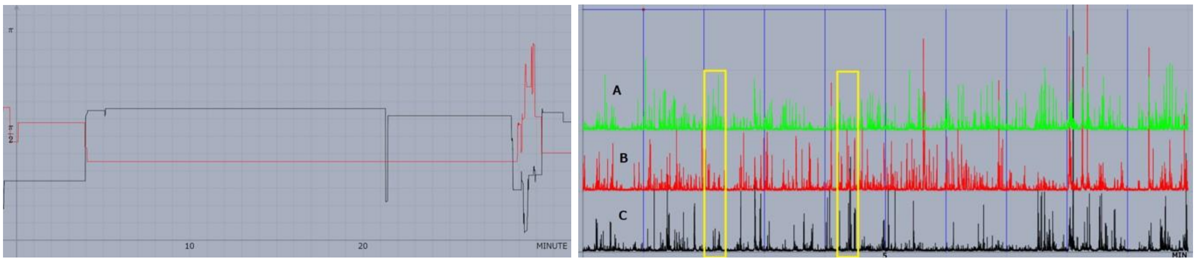


*Figure 2.* Visualization of manipulation (left) and the volume of each participant's utterance (right)

Their initial concern was to identify △BOH on the iPad screen. Then their attention moved to △HOP and △BHQ. In the discussion concerning △HOP, they had argued that the length of the leg OH was equal to cos$\beta$ as in the rudimentary case. During this discussion, B often repeated the statements of A or C. Just after this discussion, A and C said "the leg is …" in unison when they argued about △BHQ. This first synchronous utterance occurred at 131 seconds after the beginning and it is specified with one of the two yellow rectangles on the left in Figure 2 (right). Following this communication, A said, "All right, also it (the fact that the length of the leg BH of △BHQ is equal to sin$\beta$) is not different (from the rudimentary case)." Immediately after that, B repeated it. Judging from the annotated text, all participants could certify that the workflow on the worksheet was applicable to the relevant case until 210 seconds. The next trial (at 240 seconds) to prove the addition formula in case when the angle $\beta$ is negative caused their struggle to identify the angles $\beta$ and $\alpha+\beta$which made it difficult to recognize the counterpart of the side OH of △HOP. The second synchronous utterance occurred in this situation (at 260 seconds) which is specified by the yellow rectangle on the right in Figure 2 (right). After B said, "the length of OH is…", B and C said "cosine beta (cos$\beta$)" in unison. Figure 2 (right) shows that the volume of this second synchronous utterance was far higher and quicker than the first one.

## 3. Conclusion

The annotated text and videotaped record of the participants' communication show that the participants repeatedly alternated between referring to the drawing of those elements and referring to their symbolic representations. This repetitive pattern reflects the process in which all the components of the proof are organized and the narratives about that organization are reified into an object-like metarule. The remarkable point of the participants' concordant utterances is that they are concerned with △BHQ. Since the positional relationship of it to other geometric elements is highly dependent on the choice of the angles $\alpha$ and $\beta$, it can be assumed that those utterances reflected the participants' commognitive conflict as they try to accommodate their current discussion to that in the rudimentary case and triggered the emergence of convergent conceptual change. These results indicate that high resolution multimodal data of learners' verbal and nonverbal communications can supplement the annotated text and videotaped record to help analyze CSCL.

## Acknowledgements

This work was supported by JSPS KAKENHI 18K02872, 19K03175, and 21K02752.

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