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Understanding areas of parallelograms through virtual geometrical representations: A Pilot Study

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Abstract: Do virtual representations, intended for teaching and learning, afford different constraints and affordances compared to their physical twins, and may this lead to different ways of interaction and understanding? In this line of inquiry, and with the perspective on learning as an extended activity, where learning can be mediated through the interaction with artefacts, the present study has translated the concrete material from a 30-year-old study on geometry to a digital application with virtual geometrical manipulatives. 26 students from the 6th grade worked with the software for two lessons and thereafter completed a test. Learning outcomes are presented and compared to those from the original study, revealing not only substantially less learning gains, but also other unexpected differences.

Keywords: Concrete and virtual representations, digital and physical interaction, geometry, extended cognition, constraints, affordances

1. Introduction

The digitalization of learning environments is one of the major pedagogical shifts in modern education, and today more than 90% of the elementary schools in Sweden use digital devices as an integrated part of their curriculum (Skolverket, 2022). The transition from traditional hands-on experiences to digital and screen-based activities has, during the last decade, led to a series of studies where physical and virtual representations have been evaluated in relation to learning outcomes, revealing very mixed results (Erickson, 2015; Rau, 2020; Zierer, 2019). Here, a variety of theoretical paradigms has been applied, such as theories about learning and motivation/engagement, embodied cognition, cognitive load theory, and theories on visual perception and attention (Rau, 2020). However, another way of contrasting virtual and physical representations would be to focus on their intrinsic qualities, and how these – through interaction – may help reshaping the learner's cognitive state. We might, for instance, evaluate if and how a student actually makes use of a representation, and how different operations (due to constraints and affordances in the material) might mediate the student's understanding of a specific concept. Additionally, we could investigate whether the students' interpretation of a representation is adequate or not. Such an inquiry, borrowing theories and concepts from situated and extended cognition (Clark, 2014; Sawyer & Greeno, 2009), would place the properties of the teaching material – together with the learner's observable actions – in the center, instead of focusing on the cognitive and/or neural processes within the learner's head. Given the difficulty with separating such inner processes from one another in a rich naturalistic learning situation, this approach may be a valuable starting point for future research on cognition, interaction, and learning.

Subsequently, the goal of the present study was to evaluate if two versions (one digital and one physical) of a teaching material – intended to mediate the understanding of areas of parallelograms – would lead to different interactions and insights, and subsequently, to different learning outcomes.

2. Method

As a baseline and control, a thirty-year-old study, investigating the effects of a geometrical teaching material, was utilized (Sayeki, Ueno & Nagasaka, 1991). In this study, students worked with two manipulatives: a deck of cards and a paper frame, reshaping them into different parallel figures (see Figure 1, left). The cards were also put inside the frame, and by tilting the frame and cards together the students experienced how the frontal area of the frame decreased until it reached the frontal area of the deck of cards, learning that the formula for the area of a parallelogram was the same as for the area of a rectangle (that is, the base times the height). This teaching method was concluded to be very effective, leading to a deep understanding of areas of parallelograms with different shapes (ibid.).

Since no existing software was found to be appropriate for these exercises, we decided to translate the physical manipulatives described above into virtual ones and put them into a learning application, containing a two-dimensional virtual deck of cards (where the cards were moved sideways), and a virtual frame with fixed sides but movable corners that could be tilted (see Figure 1, middle and right). The application now and then posed questions about the base, height, and area of the different shapes, making the students aware of constant and variable properties of the material.

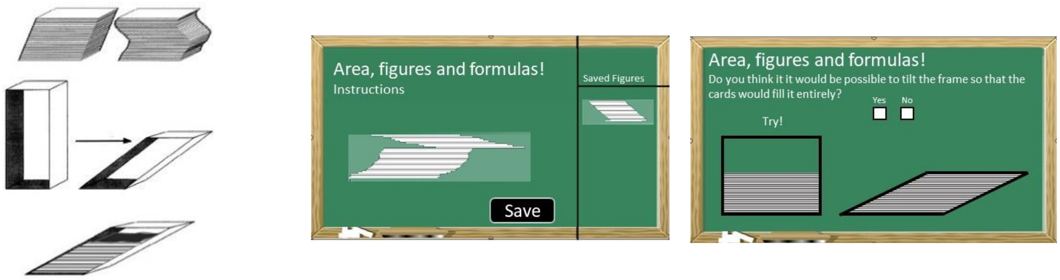


Figure 1. Sayeki and colleagues' (1991) material for learning about areas of parallelograms (left) and the interface for the digital version created for this particular study (middle and right)

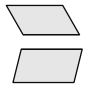
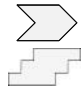
The software was then piloted by letting 26 6th-grade Swedish students (aged 11-12 years) use it for two 20-30-minute lessons. The students worked in pairs, sharing one computer, and the experiment was conducted in small groups with four students at a time. This experimental setup made it possible to gain access to the students' thoughts and perceptions through their dyadic conversations (which were recorded together with screen activities). Half a week after the second lesson, the students took a post-test (consisting of multiple-choice questions with suggestions for area formulas for different shapes) and another half week later, the students' conceptual knowledge were explored through discussions with the experimental leader. To control for students' different levels of prior knowledge in math, their teacher assessed their general mathematical skills, resulting in two student groups: Higher-achieving (N=14), and lower-achieving (N=12).

3. Results and discussion

One central question regarding the virtual manipulatives was whether the students would interact with them in the intended – beneficial – way. When analyzing the shapes created with the virtual deck of cards it turned out that the students often created strange spidery figures, quite different from classic parallelograms or more block-like shapes. This was not at all symptomatic for the Sayeki et al's (1991) study. It was primarily lower-achieving students who made these spidery shapes, although higher-achieving students made more figures in total (see Table 1).

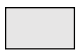
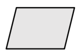
When the students interacted with the frame and cards together and were asked if they thought it would be possible to tilt the frame so that the cards would fill it, 86% of the higher-achieving students, but only 45% of the lower-achieving ones, answered 'yes'. All students completed the tilt, and several of them were surprised when seeing the cards and the frame merging into the same area. When it came to answering questions about the figures' base, height and area, the most difficult questions were those regarding the area and height of the frame, and many students confused the height with the length of the frame's tilted side. The difficulty with grasping the concept of height was also confirmed during the final oral discussion.

Table 1. *Average number of shapes created with the cards by higher- and lower achieving students.*

					Total No of shapes
Higher achieving	0.9	1.9	1.5	1.6	5.9
Lower achieving	1.0	1.0	0.7	1.8	4.5
Sayeki et al (1991)	Yes	Yes	Yes	No	

Results on the post-test reveal that the most difficult area to calculate was the one for a classic parallelogram (Figure 2 in Table 2). Only 50% of the higher-achieving students succeeded on this task – a result quite different from Sayeki et al's (1991) study, where 94% of the students were successful. Areas for other parallel figures (Figure 3 to 5 in Table 2) seemed easier to calculate, although higher-achieving students tended to overthink the formula for the staircase-like shape (Figure 5 in Table 2) by tripling its total area.

Table 2. *Proportion of correct answers on the post-test for higher- and lower- achieving students.*

	Fig 1 	Fig 2 	Fig 3 	Fig 4 	Fig 5 
Higher achieving	1.00	0.50	0.79	0.64	0.57
Lower achieving	1.00	0.27	0.45	0.64	0.73
Sayeki et al (1991)	1.00	0.94		0.91	1.00

There are several potential reasons for the relatively modest learning outcomes in the present study compared to the original one. First of all, the two experiments are not identical, since the students in this study only performed two shorter lessons, while in the original study, they performed 5 lessons and were guided by a teacher. However, this does not explain the differences *within* each experiment, indicating that the classic parallelogram (Figure 2 in Table 2) was the most difficult to calculate in the present study with virtual manipulatives – but not in the original study with physical manipulatives. Notably, parallel figures with non-straight sides (such as spidery ones) were also often created in the virtual, but not in the physical, condition (see Table 1), although the effect of this interaction isn't clear. But the students also had difficulties with interpreting the properties of the virtual frame, possibly due to the level of abstraction. Consequently, one interpretation of the low proportion of correct answers for Figure 2 in Table 2 in the present study is that this figure was interpreted as a frame – ending up with the wrong formula (*area* = *base* x *length of side*), while the other parallel figures were interpreted as decks of cards – and thereby more often associated with the correct formula (*area* = *base* x *height*).

In the current era of educational digitalization, the risk for insufficient or inappropriate interaction with different types of manipulatives, together with the potential difficulty to interpret their intrinsic qualities, have to be considered important to investigate – and especially so if the students are young and unexperienced. This might be done by comparing the use of interchangeable virtual/physical representations in controlled but ecologically valid learning environments.

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