When Calculus Learning Collides with The Metaverse

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Abstract: Spatial's multiple virtual platforms with interoperable portals, avatars, shared spaces and audio/text messages help teachers enhance the online learning experience and activities for students. In this paper, we propose the following pedagogical strategies using the metaverse: First, using multiple shared-screen modes from Spatial's virtual environment, students use our real time C3 (competition, cooperation, and communication)-based learning platform to learn from each other while solving synthetic (man-made) and real-life first year calculus problems. Second, teachers, students and peers create their own lifelike avatars to interact with each other in different shared spaces. The role of the C3 platform is to act as a teacher-led learning delivery system to build a reciprocal relationship between students and teachers, where students can learn from each other using online cooperation and/or competition learning modes and peers freely walk through different shared spaces to learn how these students solve math problems interactively and see what kinds of mathematical tools and techniques they use. Third, using interoperable portals, based on six designated Calculus problem sets, teachers can simultaneously place different groups of students in different shared spaces according to their teaching pace and have more opportunities to learn about and better understand what students and peers learning needs are. This paper also presents the pedagogical methods, for example, educational game strategies, the initiation/response/evaluation/feedback/follow-up communication pattern, and conceptual and procedural approaches that were used in this work, and finally demonstrates a few worked examples.

Keywords: E-learning platform, mathematical thinking processes, metaverse, questioning types

1. Introduction

Metaverse, the combination of the prefix "meta" (implying transcending) with the word "verse" that is same as "universe," describes a hypothetical synthetic environment linked to the physical world (Joshua (2017)). The metaverse also offers hands-on experience and activities linking the virtual and physical worlds, as well as multiple platforms that benefit students and teachers who can partake in them right from their own homes. There are numerous well-developed metaverse platforms in the commercial market, e.g., Decentraland (https://decentraland.org/), Gather Town (https://www.gather.town/), Virbela (https://www.virbela.com/), Sandbox (https://www.sandbox.game), and Spatial (https://spatial.io/). Typically, Spatial's multiple virtual platforms with interoperable portals, avatars, shared spaces and audio/text messages help teachers enhance the online learning experience and activities for students. In this paper, we propose the following pedagogical strategies for learning Calculus (Hodgen & Wiliam (2006), Ingram (2021)) using the metaverse:

1. Using multiple shared-screen modes from Spatial's virtual environment, students use our real time C3 (competition, cooperation, and communication) based learning platform (Wong & Li (2021)) to learn from each other while solving synthetic and real-life first year calculus problems.
2. Teachers, students and peers create their own lifelike avatars from a selfie to interact with each other in different shared spaces. The role of the C3 platform is to act as a teacher-led learning delivery system to build a reciprocal relationship between students and teachers, where students can learn

from each other using online cooperation and/or competition learning modes and peers freely walk through different shared spaces to learn how these students solve math problems interactively and see what kinds of tools and techniques they use. Through these activities, the peers can learn MATH and freely voice their ideas and problems without any pressure.

3. Using interoperable portals, based on seven designated Calculus problem sets, teachers can simultaneously place different groups of students in different shared spaces according to their teaching pace and have more opportunities to learn about and better understand what students and peers learning needs are.
4. Through the use of audio and text message exchanges and many-to-many, many-to-one, and oneto-one interaction modes, students actively participate in the learning process through talking, asking questions and giving answers.

The rest of the paper is summarized as follows. In Section 2, we describe how an e-learning platform in Calculus using an educational game in the metaverse is created. In Section 3, we explain how we used mathematical thinking processes in the calculus problem design and describe the different questioning types embedded in the platform. The applications of an iterative model of the initiation- response-evaluation/feedback/follow-up sequences in online Calculus teaching and learning are also emphasized. In Section 4, we demonstrate a few examples of how the metaverse and the C3-based learning platform can be used in face-to-face and online teaching. Conclusions and future works are presented in Section 5.

2. C3-based learning platform

Our aim here is to develop a social learning network and sphere using the C3-based learning platform to link the Spatial metaverse. To ensure more measurable competitive and cooperative learning and keep an (turn-taking) activity going, we use educational games. Different simultaneous/sequential games for two students (players) (or two groups of students) from the applications of game theory (see e.g., Straffin (1993)) in strategic thinking are first embedded in the platform. After they play a game, the winner has the first choice of which of the two Calculus problems to solve, that is, a choice between solving the easy or the hard problem. There are six sets of problems addressing first year Calculus topics: limits, continuity and differentiability, differentiation, indefinite integrals, definite integrals, applications of differentiation and integration. For each problem set, we use various types of two person games, for example, the Three Boxes Game (Gardner (1959)), the Spoof Game (Schwartz (1959)), the Odd Even Game (Cohon (1979)), the Bluffing Game (Graham (1997)), the Fibonacci Nim Game (Whinihan (1963)) and Game of Dice (Prisner (2014)). After five problems are solved, the two players play the game again, and the winner gets to choose one of two problems. Every fifth problem they are required to play another game. Detailed descriptions of each two-person game with the rules of the game can be found via the html link: https://www.math.cuhk.edu.hk/~mathcal/mathgame/

3. Mathematical thinking processes in the calculus problem designs

3.1. Questioning types in the platform

Our aim is to use different types of mathematical thinking processes in the calculus problem designs – Mason's (Watson & Mason (1998)), Walsh and Satters's (Walsh & Sattes (2011)), and Sahin and Kulm's (Sahin & Kulm (2008)) questioning types are also embedded in the platform to encourage students to interact and share experience, improve mathematical thinking and orchestrate students' conservations before they respond to the questions.

For example, as proposed by Sahin and Kulm's questioning types (Sahin & Kulm (2008)), we use the criteria shown in Table 1.

Table 1. Criteria for describing the question types

For designing probing questions, we:	For designing guiding questions, we:	For designing factual questions, we:
• Ask students to explain or elaborate on their thinking. • Ask students to use prior knowledge and apply it to a current problem or idea. • Ask students to justify or prove their ideas.	• Ask for a specific answer or ask for the next step of a solution when students are confused or stuck. • Ask students to think about or recall a general heuristic or strategy (Pólya (1947)). • Ask a sequence of factual questions that provides ideas or hints that scaffold or lead toward understanding a concept or completing a procedure.	• Ask students for a specific fact or definition (Vacc (1993)). • Ask students for an answer to an exercise. • Ask students to provide the next step in a procedure.

As shown in Figure 1, the questions we construct sometimes involve the intersection of two or even three different question types, i.e.,

- Probing and Guiding Questions (in purple)
- Guiding and Factual Questions (in green)
- Factual and Probing Questions (in orange)
- Probing and Guiding and Factual Questions (in pink)

For example, when solving each problem, students are required to solve a few sub-problems that may contain any combination of these question types. Details on each questioning type model with their criteria can be found via the html link: https://www.math.cuhk.edu.hk/~mathcal/mathgame/

Figure 1. Venn diagram for the combination of question types.

3.2. An iterative model of the initiation-response-evaluation/feedback/follow-up sequences in online Calculus teaching and learning

In order to help students develop higher order thinking skills via the platform, we use an iterative model of the student-teacher-student communication interaction pattern sequences in online Calculus teaching and learning.

In terms of an iterative model, students are allowed/encouraged to revisit the same problem set whenever they like. In terms of a student-teacher-student communication pattern (see e.g., Sinclair & Coulthard (1975), Swann et. al. (2004), Walsh (2011), Ingram (2021)), we have a two-move sequence, i.e., Initiation-Response (IR) (see e.g., Mehan (1979)) or three-move sequences, i.e., Initiation-Response-Evaluation/Feedback (IRE/IRF) (see e.g., Rustandi & Mubarok (2017)) and Initiation-Response-Follow-up (IRFo) (see e.g., Miao & Heining-Boynton (2011), Park et. al. (2020)). Table 2 summarizes these two types of move sequences.

Table 2. *List of communication interaction patterns*

Communication Interaction Mode		Communication Interaction Pattern	
Dyadic	Person-Computer	Initiation-Response (IR)	✦ Initiation – Teacher asks a question ✦ Response – Students answer the question
Triadic	Person-Computer-Person	Initiation-Response (IR)	✦ Initiation – Teacher asks a question ✦ Response – Students answer the question
		Initiation-Response-Evaluation/Feedback (IRE/IRF)	✦ Initiation – Teacher asks a question ✦ Response – Students answer the question ✦ Evaluation/Feedback – Teacher evaluates the answer
		Initiation-Response-Follow-up (IRFo)	✦ Initiation – Teacher asks a question ✦ Response – Students answer the question ✦ Follow-up – Teacher asks another question

The platform is treated as a teacher. The first move is the initiation, where the teacher (the platform) asks a question to initiate student interaction in a teacher-led online platform instead of face-to-face in a classroom. The second move is the response, where students interact in response to the teacher's stimuli. The last move is the evaluation/feedback, where the students' answers are evaluated by the online platform, and it gives a reply such as right or wrong. It means that students get the correction or evaluation for their response immediately. Or the last move is the follow-up, where the teacher invites students to answer extra problems after students have made the second move. Our main contribution is the combination of evaluation, feedback and follow-up together as a single move, namely the Initiation-Response-Evaluation/Feedback/Follow-up (IREFFo) pattern but we also contribute the embedding of these two-move and three-move sequences in six problem sets.

When students are solving each problem in these six sets, they are required to analyze/synthesize all given choices and infer/obtain an answer. As illustrated in Figure 2, to enhance the interactions and responses, they are required to:

1. Insert the following types of answers inside the box using a MATH calculator mode:
    a. A mathematical expression
    b. A numeric number
2. Complete multiple choice tests
3. Match the items
4. Visualize the graph of the given function using GeoGebra as a graphical visualization aid
5. Select either TRUE or FALSE for a mathematical statement
6. Reorder/Reshuffle mathematical statements using a dragging button mode
7. Perform reciprocal marking using a checklist clicking mode

In what follows, the implementation of the reciprocal marking activities are presented. For the pilot study, all illustrated examples were taken from MATH0001 at CUHK, where 17 students participated in the in-class activities.

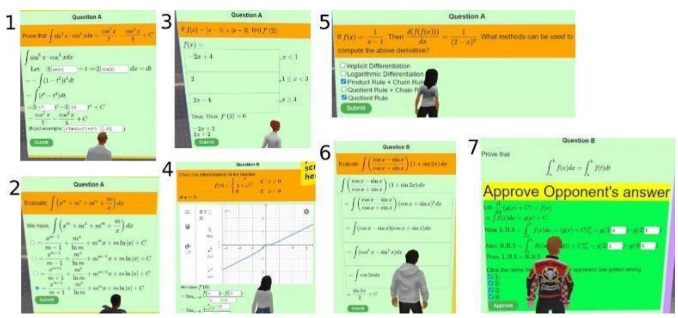

Figure 2. Seven types of interactions and responses.

4. Interplay between the metaverse and the C3 platform

Before solving any math problems, students are required to log in at Spatial.com to peer into the world of avatars, join to participate through avatars and get to know their classmates in three-dimensional fictional platforms, create a portal and discuss any math problems with them. Two solution designs are built for bridging an interplay between the metaverse and the C3 platform:

4.1 Solution Design I

In Figure 3(a), two avatars who represent two students stand in a virtual reality world. The C3-based learning platform is embedded in a virtual backboard. The users are allowed to type/click/select all the Calculus problem answers using their real world computer.

The question presented in Figure 3(b), is provided to assist students in verifying the limiting value of the given problem in a step-by-step manner and matching all information in a correct sequential order by recalling important prior knowledge of mathematical results such as quadratic equations by completing the square/minimum element of a set/trigonometric identities/a well-known limit formulae/sum of the series with two variables and giving a reason why the answer is obtained, for example, the sum of a geometric sequence formula. When a student hits the submit button, the platform gives an immediate reply! The red colour indicates an incorrect answer while the green colour indicates a correct answer.

(a) (b)

Figure 3. (a) Two players with peers who represented by avatars stand in a virtual reality world; (b) The C3-based platform provides an automatic feedback system using a computer-and-student interaction.

4.2. Solution Design II

As shown in Figure 4, where a red arrow indicates the travel of interactions from one space to another space, in a virtual reality world the users can travel different interoperable portals to join different Calculus activities. Typically, peers watch the other students to see how they interact and solve problems. When the peers have any questions, they can post their messages and voice their ideas and thoughts through typing and audio devices from their real world computers. Hence, multi social networks of avatars are created.

Figure 4. Different interoperable portals that join different Calculus activities.

4.2.1. Example 1

An example of using an initiation-response communication pattern is given in Figure 5. After students finish a two-move sequence, GeoGebra, which is embedded in the platform, allows students to plot the given function and the derivative of the given function to assist them to find their answers by, for example, inserting corresponding mathematical expressions and answers inside the box using a MATH calculator mode.

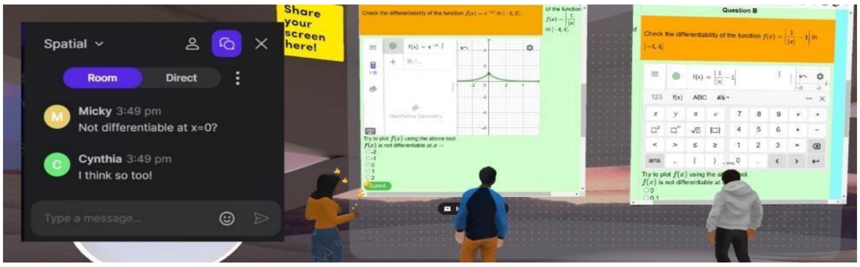

Figure 5. Inserting corresponding mathematical expressions and answers inside the box using a MATH calculator mode.

4.2.2. Example 2

As shown in Figure 6, to provide an effective follow-up response, such as peer reviewing or comparing and contrasting students' activities rather than having a simple evaluation/feedback or follow-up, we use an iterative model of the IREFFo pattern:

1. [Game Play] Students play warm up game activities.
2. [Initiation]
   a) Both students solve the same problem by filling in the blanks.
   b) Click the Submit button
3. [Response] Students mark their opponent’s answers via reciprocal marking,
   a) Opponent’s answer is shown in the darker green region.
4. [Evaluation/Feedback]
   a) Students select items they think their opponent got wrong. If the students think their opponent got it all correct, none of the items has to be selected.
   b) Click the approve button
   c) Each student's answer is shown in the darker green region.
   d) Both students can see what the other thinks of their answer.
   e) Both students can amend the answers that they think they did wrong.
   f) Click the final Submit button.
   g) The system will check the answers.
   Correct answers are indicated in green. Incorrect answers are indicated in red.
5. [Follow-up] Before working on the next problem, the platform will use a recommendation system to suggest other problems as follow-up questions students can use to improve their math drill skills.
   a) Both students will do the same follow-up questions and learn from them.
6. [Open-ended Question] To provide more than a short and fixed response, open-ended questions are also added in the platform. Students get a comprehensive review of the whole question and see how to get the answers and reinterpret the results.

Steps 3 and 4 are for making role changes between student and teacher, and letting peers participate in a learning process without pressure. Steps 5 and 6 provide an opportunity for users to do another learning activity whenever they want to return to the knowledge point and polish their skills.

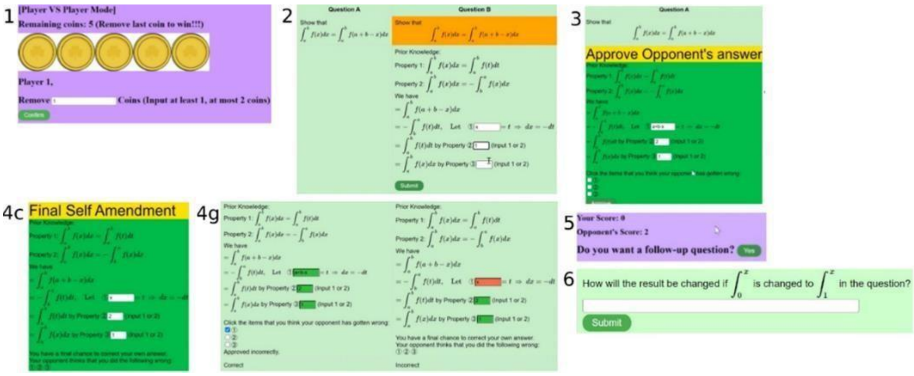

Figure 6. Illustration of an effective follow-up response from Step 1 to Step 6.

5. Conclusions

Using the metaverse together with a C3-based platform based on the frameworks of educational game strategies, and Mason's, Walsh and Satter's, and Sahin and Kulm's questioning types for teaching and learning in Calculus, helps students to diversify their social network sphere and improve their problem solving skills and their mathematical thinking processes. The C3 platform in the world of avatars and the IREFFo pattern are designed to create a more accessible relationship between the users and peers, e.g., how they are beneficial to each other. A few ongoing works are:

- collection of more student feedback data about how they work on each problem, e.g., what kinds of games students chose;
- analysis of their social networks using data mining techniques, e.g., how group members interact with each other within the group using text and voice analytics;
- embedding in the platform any metaverse tools that better fit students' needs and teachers' purposes.

These findings will be published elsewhere in future.

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