Grounding Embodied Learning using Online Motion-Detection in The Hidden Village

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Abstract: In the learning science community, technology-based interventions afford educators and learners opportunities to augment their classrooms through engaging, interactive simulations that ground concepts in embodied experiences. The current study developed and deployed a visual novel platform called *The Hidden Village-Online* (THV-O) that used emulated 3D-motion-capture to offer students opportunities to use their bodies as a means for thinking geometrically. This paper discusses the design process for developing THV-O (e.g., narrative, UX, scaffolds) and data from a small within-subjects randomized design experiment using THV-O. Researchers administered a geometry unit using THV-O in which players performed game-directed actions prior to evaluating geometry conjectures. Students' game performance data, audio (transcripts), and video (gestures) data were processed and analyzed. Results indicated that students whose spoken responses included representational (depictive) and dynamic (enactive) gestures showed greater accuracy for intuitions, insights, and formations of transformational proofs.

Keywords: Design-Based Research, Directed Actions, Embodied Visual Novel, Action-Cognition Transduction, Embodied Cognition, Remote Learning, Inclusion & Access

1. Introduction

Grounded and embodied learning leverages bodily interactions as crucial processes in conceptual understanding and abstraction (Nathan, 2021). Fortunately, digital platforms are affording teachers and students with opportunities to embody concepts in learning environments (Georgiou & Ioannou, 2019) that provide students with grounded experiences onto which they can map mathematical ideas (Skulmowski & Rey, 2018). Nathan and colleagues (Kirankumar et al., 2021; Nathan & Swart, 2020; Nathan & Walkington, 2017; Swart et al., 2020; Walkington, Nathan, Wang, & Schenck, 2022) have developed an embodied learning environment, *The Hidden Village* (THV), that uses 3D-motion capture to allow participants to mimic movements (i.e., directed actions) of in-game avatars before responding to veracity of geometry conjectures. To date, a number of studies using THV have demonstrated when players produce game-directed actions and explanations that include mathematically relevant gestures (often re-invoking game-directed actions) they exhibit more accurate mathematical intuitions, superior insights about the relevant concepts (gist), and more mathematically valid proofs (Swart et al., 2020; Walkington et al., 2022). While this motion-based technology formerly required special hardware like infrared sensors (e.g., Microsoft Kinect) to detect body movement, advances in the focal acuity of digital cameras combined with open-source computer vision packages have enabled the research lab to develop an easily accessible web-based tool, THV-O, that tracks and records players' movements without additional hardware or client-side software. We discuss the design, development, and deployment of THV-O from proof-of-concept (PoC; Fogel et al., 2021) to an alpha version, as well as data from a pilot study addressing research questions investigating students' geometric thinking.

2. Theory

A growing body of literature in mathematics education research suggests that students working with mathematical conjectures struggle to construct viable and convincing proofs (Dreyfus, 1999; Martin et al., 2005; Nathan et al, 2021). Creating valid geometric proofs requires making universal statements about space and shapes. To adequately communicate the abstract and generalized mathematical ideas of a valid proof, students cannot rely on algorithms or procedures, such as long division, that could lead to a correct answer without a deeper understanding of the underlying mathematics concepts. While some students' struggles manifest as overgeneralizations of what can be concluded from specific examples (e.g., Healy & Hoyles, 2000; Knuth, Choppin, & Bieda, 2009), others find valid deductive proofs to be unconvincing (Chazan, 1993) or do not appreciate the essential role of deductive reasoning in establishing generalized propositions (Harel & Sowder, 1998).

People's gestures reflect their thinking; proofs constructed by expert mathematicians are often "fundamentally embodied" and often include gesture as a component of the practice of constructing the proof (Marghetis et al., 2014, p. 228). By definition, gestures are spontaneously generated hand and arm movements that are co-articulated with speech and thought (Kendon, 1972; McNeill, 1992). In effect, *gestures simulate actions* (GSA) (Hostetter & Alibali, 2008; Hostetter & Alibali, 2019), engaging perceptual-motor processes as a means for enacting thought. Reciprocally, performance of goal-directed actions—including gestures—will induce the cognitive states related to those actions. The process underlying this bi-directional relationship between thinking and acting is what Nathan (2017) calls *Action Cognition Transduction* (ACT; Fig. 1). Furthermore, *cognitively relevant* movements, which map a concept onto a set of body poses and movements and conform to gestural congruency (Johnson-Glenberg et al., 2014; Walkington et al., 2022), can be beneficial for reasoning.

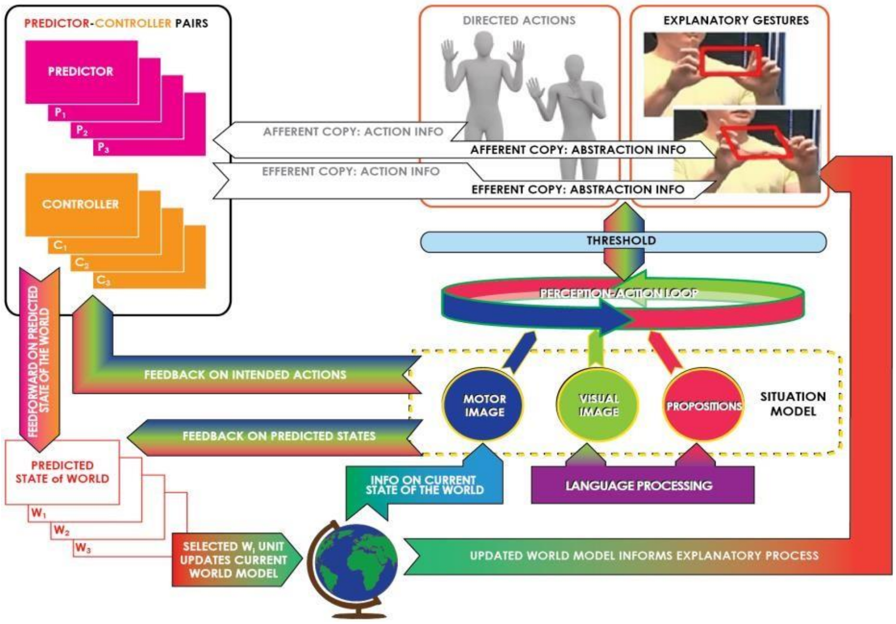


Figure 1. Action-Cognition Transduction (ACT) posits a reciprocal relationship between the environment and the learner, perception and action, and formalized through language.

Building on the theories of GSA and ACT, Nathan and Walkington (2017) developed the Grounded and Embodied Cognition framework to describe how action-based interventions can complement learners' verbal modes of reasoning as a way to influence one's mathematical reasoning. Additionally, recent work by Nathan and Swart (2020) suggests that people who produced *dynamic depictive gestures*, which are gestures that explore the generalized properties of geometric objects and their simulated transformations, demonstrated superior performance in reasoning about geometric conjectures. There is empirical evidence showing that cognitively relevant directed actions have been found to generate dynamic depictive gestures that complement mathematical reasoning (Walkington et al., 2022). As an example, students explaining the concept of parallel lines may hold up their arms or hands next to each other to demonstrate, through body movement, geometric concepts and mathematical reasoning and learning.

In educational technology development, a number of scholars have begun to identify emerging design principles for building embodied educational interventions that purposefully leverage gestures as intentional movements to ground learners' understandings (Abrahamson et al., 2020; Lindgren & Johnson-Glenberg, 2013; Malinverni & Pares, 2014). Recent research corroborates this role of the body as a promising source of educational interventions (Skulmowski & Rey, 2018), including mathematics (Abrahamson et al., 2020; Goldstone et al., 2017; Walkington et al., 2019), reading and literacy and science (Glenberg et al., 2004) and science (Lindgren et al., 2016). For design, Johnson-Glenberg et al., (2014) proposed attending to three aspects of embodiment: motoric engagement, gestural congruency, and perceived immersion.

Thus, the current study investigates the design and delivery of a geometry curriculum using THV-O (operating over the web without specialized hardware) that has participants performing cognitively relevant movements to enact key mathematical relationships. In this work, our research question asks: How do game-directed actions impact students' reasoning about geometric conjectures? We hypothesize that directed actions act as embodied scaffolds that helps structure students' conceptualizations, thus enhancing their mathematical intuitions and insights about space and shape.

3. Methods

3.1 Participants

A convenience sample of university students ($N=27$, $\bar{\mathbf{x}}=21$, $n_{male}=15$ (56%)) was recruited for this study. Participants were recruited through an educational psychology course and necessitated completing a high school geometry course as a prerequisite to participation. None of the students had participated in prior studies on mathematical reasoning conducted by the lab. All identifying information has been removed to protect student privacy.

3.2 Materials: An Overview of THV-O

The Hidden Village-Online (THV-O alpha version; Fig. 2) is structured as a *visual novel*, a video game sub-genre under interactive fiction. THV is designed to deliver a grounded embodied curriculum while capturing evidence of mathematical thinking by recording players' movements and speech. As a tool for research, THV-O investigates: (1) implications of design decisions for narrative, instruction, scaffolds, curriculum, feedback, reward, agency, engagement and immersion, (2) how mathematically-relevant directed actions influence students' intuitions and insights in the processes of proof production practices; (3) how pedagogical language, such as instructions, hints or narrative contexts connect players' movements to the mathematics and influences students' proof practices; (4) how learning differs when players enact or observe directed actions; (5) how collaborative embodied mathematical reasoning benefits both students and teachers.

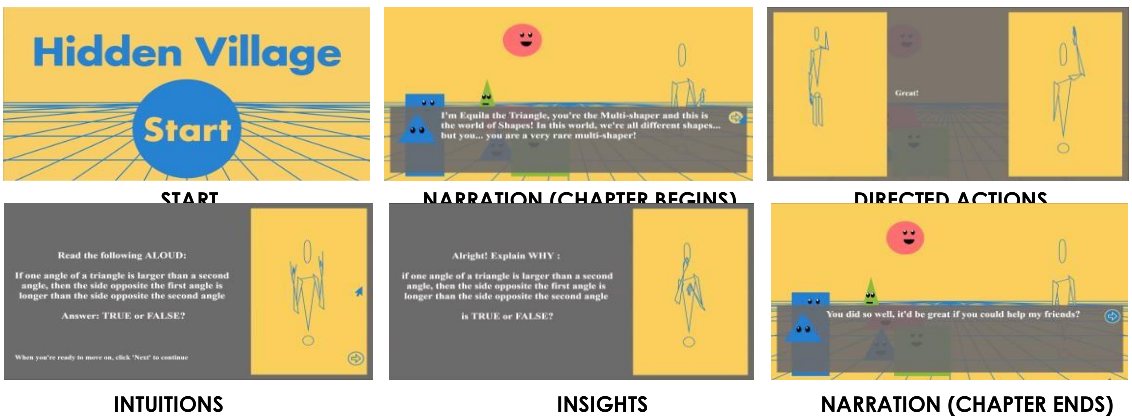


Figure 2. Top. Start Screen, the narrative begins and players match poses of the directed actions. *Bottom*. Players provide intuition, followed by insights and then the narrative progresses.

3.3 Materials: THV-O Gameplay

THV-O (Fig. 2) takes players through an eight-chapter story in a two-dimensional world populated by different shapes. Using a computer's webcam, THV-O detects participants' bodies in real-time by identifying landmarks and calculating positional data to animate the player's avatar, *The Multishaper*, on the right side of the screen (i.e., a mirror-imaged depiction in the fictional world of THV). For each chapter, participants conversed with the shape characters through captioned text; each shape introduced themselves and then asked the player (i.e., The Multishaper) to perform a movement (i.e., directed actions) after which players read a geometry conjecture and provided their intuition and insight on its veracity.

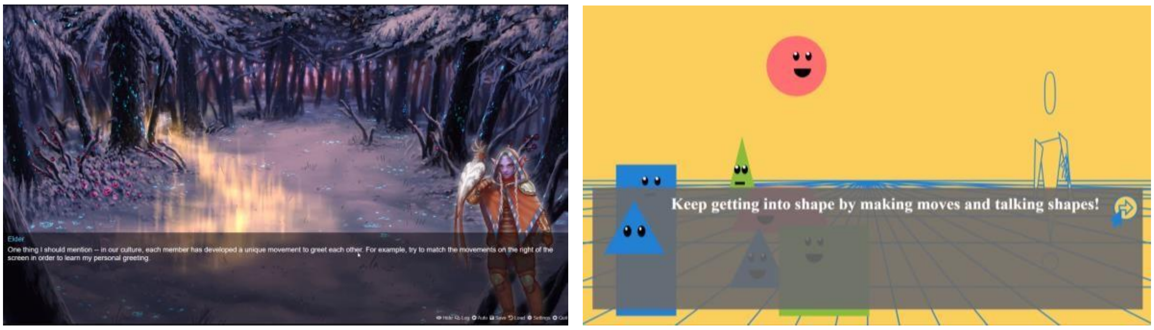
3.4 Materials: THV-O Technology

The Hidden Village-Online leverages players' movements for both navigation as well as core gameplay using consumer grade webcams and algorithmic-based software packages. Both THV-O PoC and alpha versions use Mediapipe Holistic Pose Detection Algorithm (Lugaresi et al., 2019). The Mediapipe algorithm identifies key landmarks on a player's body (e.g., wrists, elbows, shoulders, and fingers), which are used by THV-O to determine whether players have positioned their bodies in the correct poses (i.e., the directed actions). Graphics and interface for THV-O (alpha version) was implemented using ReactPixi (Brouwer, 2021) for graphics and XState.js (Khourshid, 2021) for state logic.

3.5 Materials: Improvements in THV-O

THV-O alpha version (https://www.github.com/UW-MAGIC-lab/hidden _village) created a gaming shell that allowed developers to manage and deploy custom assets, backgrounds, and characters, as well as tailor the data collection, experimental design, and game delivery modules (Latin square factorialization, conjectures, chapters, instructions, narration, navigation, pose recognition, segmentation, videos, transitions, tutorials) to the research.

3.6 Materials: THV-O Interface Redesign



THV-O PoC **THV-O alpha**

Figure 3. Interface designs for THV-O PoC and THV-O alpha.

Narrative. In THV-O PoC, the narrative situated players on a distant planet where their spaceship had crashed and in order to return home, members of *The Hidden Village* ask for assistance, in which players perform in-game movements (directed actions) before considering geometry conjectures. The revised narrative in THV-O alpha situated players in a digital world as the Multi-Shaper, there to help geometric shapes "get in shape" by performing in-game movements (directed actions) before considering geometry conjectures and returning home.

Navigation. THV-O PoC required users to click the mouse or use buttons on the keyboard to navigate through the game or to choose menu options (Figure 3, left). THV-O alpha allowed users to move an arrowed cursor using motion capture to activate hotspots displayed as buttons on-screen (Figure 3, right). These HCI improvements streamline the experience of players for an integrated and highly embodied gameplay (*ref.* Skulmowski & Rey, 2018).

Avatar, Calibration & Pose Feedback. THV-O PoC rendered players' avatars using the motion-detection landmarks, including the face, upper body, arms, hands and fingers. Calibration was delivered in real-time throughout the game as a text-based warning to position players correctly within frame. Pose feedback used a traffic light color scheme (red/yellow/green) to indicate when players directed actions ranged from incorrect to correct, respectively. Like THV-O PoC, the alpha version also used motion-detection landmarks to render players' avatars but now without facial details (the research team decided to remove the display of the facial tessellations). Moreover, to fit the new narrative, the alpha version avatars were rendered as a more stylized collection of shapes (i.e., the Multishaper). Calibration for the alpha version was delivered as a separate module at the beginning of the game and varied in two ways: (1) if players were not at the correct distance, their avatar would not render properly; (2) players performed a series of poses to train the players to understand how to use this novel interface. Pose feedback was delivered using the color red that ranged in opacity (high to low) to indicate when players directed actions were incorrect to correct, respectively.

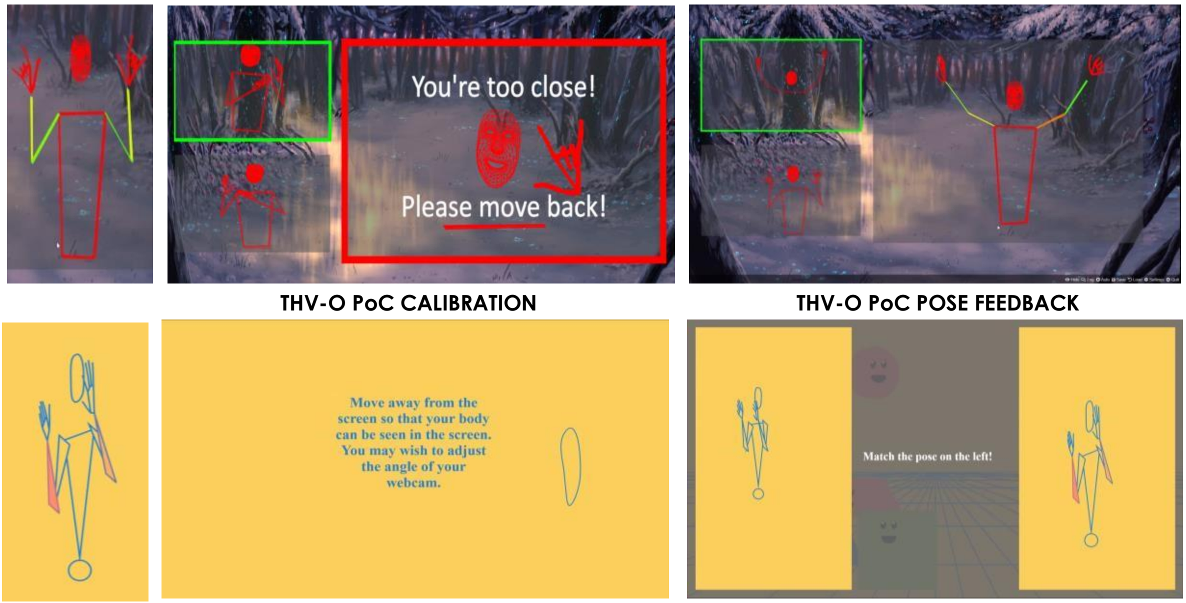


Figure 4. Avatar, pre-game calibration and pose feedback for THV-O PoC *(top)* and alpha *(bottom)*.

Open Shell. THV-O PoC delivered a game shell that was hard coded. THV-O alpha created an open shell that curriculum developers could edit by changing a human-readable reference file. Similarly, character assets could also easily updated or exchanged.

Educational Intervention Experimentation Features. As a research tool, a number of features were added to THV-O alpha, including: (1) implementing a Latin-square factorial that counterbalances the presentation order of the curriculum; (2) experimenter controls that allow customized administration of the curricular intervention; (3) interstitials that allow for variable administration of experimental protocols (i.e., within and between subjects).

3.7 Procedure

The Participants were video-recorded as they played through four chapters of the experimental platform (Figure 3). To avoid ordering effects, a Latin-square factorial design was used to order the geometric conjectures presented to participants. Finally, after completing the entire story, participants participated in a semi-structured exit interview.

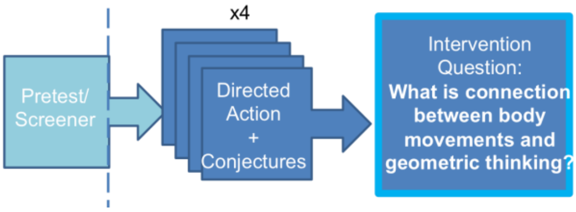


Figure 5. Participants completed a pre-test, 4 game chapters (each with a conjecture) and a post-interview

3.8 Data Analysis

Coding. The audio of the video recordings was transcribed. We then analyzed the video and transcript data qualitatively using the coding scheme described in Table 1. The coding scheme was developed using a grounded approach informed by prior analyses (Garcia & Infante, 2012; Nathan et al., 2021) on similar data. The data were analyzed gesture production on the conjecture level. Participants were video-recorded as they played through all four chapters of the experimental platform (Figure 5).

Table 1: Codebook and Inter-rater Reliability

Code	Kappa	Rho	Definition	Example
FINAL INSIGHT CORRECT	0.98	0.00	The final assessment as to whether a conjecture is true or false is **correct**; if no explicit assessment is made at the end, this will align with the assessment initially made during the intuition phase	Conjecture: Reflecting any point over the x-axis is the same as rotating the point 90 degrees clockwise about the origin. Final answer: "[false] Because if you were reflecting it over, it would be 180, 90 degrees would just be like half the rotation."
FINAL INSIGHT INCORRECT	0.88	0.00	The final assessment as to whether a conjecture is true or false is **incorrect**; if no explicit assessment is made at the end, this will align with the assessment initially made during the intuition phase	Conjecture: If you double the length and the width of a rectangle, then the area is exactly doubled Final answer: "[True] Um, if you're doubling the length and the width, then that means you're doubling all sides of the rectangle, which would in turn double the area of the rectangle."
DYNAMIC GESTURE	0.97	0.00	Gesture that depicts **both** a mathematical object or space **and** a transformation of the object	
NON-DYNAMIC GESTURE	0.95	0.00	Gesture that depicts a mathematical object or space, but **does not** transform the object; includes tracing the object in space and deictic gestures for the traces object	

Inter-rater reliability. A second coder evaluated 111 randomly selected utterances of the coded data, representing 20% of the coded utterances. A second coder, not involved in code generation or the initial coding process, coded the 111 segments. Cohen's κ was estimated for each code using the irr R package (Gamer et al., n.d.), with each code's agreement exceeding the conventional $\kappa = 0.65$ threshold for inter-rater reliability (Cohen, 1960; see Table 1). To test the how well the inter-rater reliability on a subset of the data generalizes to the rest of the coded dataset, we calculated Shaffer's ρ (Shaffer, 2017) for each code using the *rho*R R package (Eagan et al., n.d.). The Shaffer's ρ value for each code was below 0.05, indicating an acceptable Type I error rate (<0.05). These results suggest acceptable reliability for the codes used in this study.

4. Results

This randomized within-subjects pilot study employed a mixed-methodology that provided quantifiable data for the impact of a grounded embodied curriculum and further qualified that data by presenting case-study evidence that helps researchers better understand how learning is being impacted and why students' gestures reveal their conceptualizations.

4.1 Quantitative Analysis

How do game-directed actions impact students' reasoning about geometric conjectures**?** To address this research question, we analyzed the incidence of gestures through the pre-intervention tasks by evaluation performance on the conjecture (Table 1). In particular, we examined the incidence of NON-DYNAMIC GESTURES, or gestures that represent mathematical objects or spaces, and DYNAMIC GESTURES, or gestures that represent mathematical objects or spaces *and a motion-based manipulation* of the represented objects or spaces (Garcia & Infante, 2012). For example, tracing the shape of a triangle in the air would be considered a NON-DYNAMIC GESTURE as only the triangle was being represented through the gesture. However, if the gesture represented both the triangle and the area of the triangle growing or shrinking over time, the gesture would be a dynamic gesture.

Table 2 presents the mutually exclusive gesture categories that were generated by collapsing the coded and segmented discourse data to the conjecture level. If a participant made both distinct DYNAMIC GESTURES and NON-DYNAMIC GESTURES while reasoning about the conjecture, we attributed the gesture occurrence to DYNAMIC GESTURES rather than NON-DYNAMIC GESTURES. Though participants made more NON-DYNAMIC GESTURES (43.5% of the four conjectures compared with 30.6% DYNAMIC GESTURES), participants gestured more frequently (74.1%) than not (25.9%) while using THV-O. Additionally, when participants made DYNAMIC GESTURES during the four experimental tasks, they evaluated the conjecture correctly (22.2%) more often than incorrectly (8.3%). These results support findings from Nathan and colleagues (2021), in which they found that performing dynamic depictive gestures was strongly predictive of producing mathematically valid proofs.

Table 2: Gesture Occurrence During Conjectures by Correct Evaluation

	DYNAMIC GESTURES	NON-DYNAMIC GESTURES.	No Gestures	Total
Correct	*24 (22.2%)*	*29 (26.9%)*	*21 (19.4%)*	*74 (68.5%)*
Incorrect	*9 (8.3%)*	*18 (16.7%)*	*7 (6.5%)*	*34 (31.5%)*
Total	33 (30.6%)	47 (43.5%)	28 (28.9%)	108 (100%)

4.2 Qualitative Analysis

In addition to seeing a relatively high occurrence of representational gestures when participants correctly reasoned about these geometric conjectures, qualitative analysis of the video and discourse data also suggests evidence students making dynamic depictive gestures, providing evidence for ACT while playing THV-O. In these first experimental tasks, some participants evaluated the conjecture aloud before interrupting their own explanation to perform gestures. In the example below, when the student was asked to evaluate whether a conjecture was true or false, she trailed off, spoke under her breath while gesturing, and then continued with her reasoning.

(a) (b) (c)

[1] Reflecting any point over the X-axis is the same as rotating the point 90 degrees clockwise about the origin true or false. I have to think about that...

[2] Okay. Reflecting any point over the x-axis is the same as rotating point 90 degrees...clockwise...
[3] X-axis… <*Jane moves her hands in front abdomen, see Fig. 6a*>
[4] <*Keeping her left hand in front of abdomen, Jane moves her right hand in-front of (above) her left hand, see Fig. 6b*>*...*
[5] <*Jane rotates her right hand clockwise, see Fig. 6c*>*...*
[6] Um, I think it's false cuz [because] you have to rotate 180.

Figure 6. Representative (NON-DYNAMIC) (a) and DYNAMIC GESTURES (b, c) discussing geometry conjecture.

After trailing off, the student made a NON-DYNAMIC GESTURE by moving her hands in front of her abdomen (Fig. 6a) and identified it as the x-axis. The student made a DYNAMIC GESTURE by moving her right hand over her left (Fig. 6b), and rotated her right hand clockwise (Fig. 6c), suggesting that the right hand represented the point in the conjecture. After performing this gesture, the student resumed her verbal explanation, providing a clear rationale for why she decided the conjecture was false.

5. Discussion

As a platform, the pilot data collected with THV-O demonstrates its utility as a highly embodied and integrated research tool. Traditionally, geometry concepts are presented in words, symbols, and highly regulated diagrams. In contrast, in THV-O these ideas (such as intersecting lines, reflection) are grounded in players' body movements. Future iterations of THV-O will support data collection modules that record participants' speech (audio), actions/gestures (video), and body position (motion-capture accuracy, time, attempts). While the platform currently utilizes motion-capture data to power the gameplay experience, those data are not yet collected, and thus, did not factor into the final analysis. Thus, further investigations to develop design principles and guidelines for creating effective embodied interventions (i.e., using directed actions) will help make multimodal interventions like THV-O more impactful for learning.

In this work, research question investigated how directed actions impact students reasoning about geometric conjectures, hypothesizing that directed actions serve as embodied scaffolds that help structure student's conceptualization of geometric objects and transformations (H1). The preliminary data from this randomized experiment using the alpha version of THV-O suggests that when people made dynamic gestures, they had more correct insights. Additionally, the qualitative evidence suggests that among some individuals, performing dynamic gestures that represented geometric objects and enacted transformations was an influential intermediary step in reasoning about the geometric conjectures. The data provided evidence consistent with Nathan and colleagues' (2021) finding that producing dynamic gestures is strongly predictive of producing mathematically valid proofs. These findings bolster the growing evidence for one of Action Cognition Transduction's core claims: there exists a bi-directional relationship between cognitive states and movement.

These results are not without limitations. First, the small sample size means that these results do not have enough power to generalize beyond the sample. Future work will continue to test THV-O's viability with a broader population. Additionally, participants played through THV-O in the context of a laboratory experiment. Future work will investigate its deployment as a pedagogical tool in classroom contexts. Finally, the instantiation of THV-O presented in this paper is a proof of principle representing a theory-driven design for specific geometric conjectures. Future work will expand upon the existing instantiation to investigate broader embodied design principles.

Despite these limitations, this design paper suggests that an embodied education platform such as THV-O is a viable platform for both enacting pedagogical interventions as well as continuing to investigate the qualitative nature of directed actions as learning scaffolds. To this, there is a growing corpus of evidence that suggests not only that directed actions can beneficially impact thinking and conceptualization, but that the relevance of the directed actions is crucial for effectively grounding embodied understandings of abstract concepts like geometry and improving mathematical reasoning. In some instances, individuals who may not have even be aware of the game-directed actions relevance when performing them still made deductive leaps (evidenced by their verbal and gestural explanations) while reasoning about geometric conjectures (Walkington et al., 2022). These findings, in combination with the current study, highlight that simply designing directed actions deemed relevant by a domain expert may sometimes require that participants recognize how and why those actions are conceptually salient (Rau, 2020). Consequently, further investigation is needed to understand how learners' perceptions of the relevance and salience help grounded embodied curricula. Nonetheless, we posit that making these connections explicit may make directed actions even more beneficial as pedagogical tools for teachers and students. Fortunately, the continuing development of THV-O will provide educators with the flexibility to make the connections between the movements and target concepts explicit.

Acknowledgements

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