Preliminary Use of a Learning Game for Arithmetic Word Problems with Elementary School Students

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Abstract: Although word problems can be challenging for many learners, they are an important part of mathematics instruction. This is because word problems often require students to transfer abstract mathematical concepts to real-world situations, which can be difficult for some individuals. In this paper, we propose a learning game names "Tri-prop-scrabble" where learners can try to create arithmetic stories related to addition and subtraction from among many choices and can obtain feedback. To prove the research questions: (1) Can elementary first grade students create and chain arithmetic stories? and (2) Are they able to enjoy this game?, we conduct a preliminary use of the game on 44 first grade elementary school students who are the target of learning of arithmetic word problems of addition and subtraction. they can make arithmetic stories and some of them have made complex structures of stories. The results shows that this game is feasible for first-grade elementary school students.

Keywords: arithmetic word problems, problem-posing, augmented-reality

1. Introduction

Word problems are often considered challenging for mathematics learners and a significant amount of research has been dedicated to exploring this topic over the past 50 years. Word problems are verbal descriptions of problem situations in a scholastic setting that raise one or more questions. The answer can be obtained by applying mathematical operations to numerical data found in the problem statement or derived from it (Verschaffel et al. 2000). Solving an arithmetic word problem involves reading sentences, extracting the quantity relationships within them, representing them mathematically, and deriving unknown numbers (Mayer, 1992). Researchers have investigated the process of solving word problems and divided it into two sub-processes: (1) comprehension phase and (2) solution phase (Cummins et al., 1988) (Hegarty et al., 1995) (Heffernan and Koedinger, 1998) (Riley et al., 1983). They have also noted that the comprehension phase plays a major role in the difficulty of the word problem. During this phase, a learner must interpret representations expressed in words and create quantitative relationships. This can also be related to mathematical modeling, one of the important aspects in mathematics education, is mathematical modeling, which involves modeling real-world problems as "mathematical models" in the world of mathematics and solving them (Kaiser 2017). Although word problems are in a scholastic setting, if appropriately designed and handled in a mathematics educational context, they can act as accurate and valuable "simulacra" of authentic mathematical modeling problems (Verschaffel et al. 2020). This requires well-defined domain and instructional model (Mitrovic and Weerasinghe, 2009) of arithmetic word problems.

"Monsa-kun" (Hirashima et al., 2007) is a learning environment for arithmetic word problem learning based on the triplet structure model. This is based on the triplet structure model (Hirashima et al, 2014), which is a domain model for describing the relationship between the text of arithmetic word problems and mathematical equations. The characteristic of this model is that it breaks down the verbal description of arithmetic word problems into simple sentences as propositions of each quantity element in the described situation. In

"Monsa-kun" learners can carry out problem-posing as the integration of given simple sentences and receive immediate feedback. Sentence-integration is an instructional model of problem-posing in which learner can concentrate on the structure of arithmetic story not on generating story content. Through this approach, learners can concentrate on thinking about the relation between quantities in the situation and can also get feedback immediately.

The goal of this study is to transform the approach to thinking about the relationship between quantities in given situations, making it a more "open-ended" task. Monsa-kun tackles unit arithmetic word problems that can be solved with a single arithmetic operation, requiring learners to think in a "closed-ended" way where only one correct answer exists within the given constraints. In contrast, this study aims to introduce tasks that are "more open-ended" and have "enlarged structures." This is based on the continuum between well-defined and ill-defined problems (Le et al., 2013). Triplet structure model and problem-posing as sentence-integration can be well-defined domain and instructional models because they clarify the constraints in the domain of arithmetic problems/stories and guidelines to pose problems/stories as well as can provide immediate feedback to learners. This study aims at proposing a learning task as pro-ill-defined educational problem from Monsa-kun.

This study develops a learning game called "Tri-Prop-Scrabble" (Yamaguchi, 2023) in which learners can construct a chain of stories with the rule like Scrabble (Scrabble, n.d). This environment allows learners to create various arithmetic stories in a chain made up of single sentences and to consider the relationship between the quantitative elements. Posing problems is an effective way to learn a domain and posing arithmetic word problems is beneficial to learn the structure of arithmetic word problems. Although the target of Tri-Prop-Scrabble is not word problem but story, the basic goal is the same as Monsa-kun, that is, to consider the relationship between the quantitative elements in stories. In addition to that, Tri-Prop-Scrabble supports the understanding of the relationship between stories. We call this exercise "chained arithmetic story creation" and it involves creating compound arithmetic stories by predicting patterns for various stories with less conditions than Monsa-kun. This activity encourages learners to think the relationship between the verbal description and the mathematical formulas in a "more open-ended" manner in "more enlarged structures" than Monsa-kun. The results of experimental use by high school students shows that the exercise using the system is available for learners older than the main target of elementary school students (Yamaguchi, 2023).

This paper reports the result of the use of Tri-prop-scrabble by first grade elementary school students. The structure of this paper is as follows. The second chapter describe a relationship understanding support system based on the triplet structure model. The third chapter shows the design and the develop of Tri-Prop-Scrabble that allows learners to explore the possibilities of arithmetic stories as the combination of single sentences and a chain of them. The fourth chapter analyze the result of the use of Tri-prop-scrabble by elementary school students and consider the feasibility of the exercise for learning the relation between quantities in the situation. Finally, the last chapter present the conclusion and future tasks.

2. Modeling Arithmetic Stories with Triplet structure model

The triplet structure model is a domain model of arithmetic word problems by describing the relationship between verbal description and the mathematical formula in a situation. This model can be a well-defined model that breaks down the problem description into simple sentences for each quantity element and associates their semantic quantity relationship with the corresponding mathematical formula.

Monsa-kun is a “sentence-integration” typed problem-posing learning environment based on the triplet structure model, where learners can create problems as a combination of simple sentences and receive immediate feedback. Sentence-integration typed problem-posing is a well-defined instructional model. This defines problem-posing as the task: integrating sentences representing numerical propositions and satisfying the constraints defined in the triplet structure model. Nakano (2002), Kurayama (2010) and Yamamoto (2012) show the effectiveness of Monsa-kun in learning arithmetic word problems even if learners pose problems with provided sentences. Figure 1 shows the interface of the problem creation screen of Monsa-kun. Learning by problem-posing is a method of learning by posing problems rather than solving them. This can be used to support the understanding of the relationship between elements in stories, which is important in understanding the relation between the problem statements and mathematical formulas. Although learning by problem-posing is considered an effective learning method for understanding the relation, it has not been widely used in educational situations due to the difficulty for both of learners and teachers. Monsa-kun has been developed to solve such problems and can execute sentence-integrated problem creation on a tablet device.

Monsa-kun deals with unit arithmetic stories that can be expressed by only one arithmetic operation and allows learners to understand the relation between stories and mathematical formulas through sentence integration. In the triplet structure model, a unit arithmetic story is formulated as a composition of two types of sentences: an existence sentence that represents the existence of a quantity and a relational sentence that represents the relationship between two quantities. A unit arithmetic word problem is a story in which one value is set as unknown and the unknown value can be derived from known values in the story by only one arithmetic operation. Although Monsa-kun only deals with stories that can express quantity relationships in one operation, the triplet structure model can express complex arithmetic stories as a chain of unit arithmetic stories. Monsa-kun encourages learners to be aware of the quantity relationship expressed in the text.

While the task setting in Monsa-kun is "closed-ended" and "a unit structure" problem-posing, this study aims to handle more "open-ended" and "expanded structures" than Monsa-kun. Closed-ended environments are beneficial for learning basic structure, whereas open-ended environments are valuable for developing creativity based on the basic structure. Monsa-kun deals with unit arithmetic word problems and can help learners understand the basic structure of them as the relationship of problems that meet the constraints by presenting the conditions for creating arithmetic word problems. This system requires learners to think in a "closed" manner about the "minimum unit structure" by creating problems that meet the conditions.

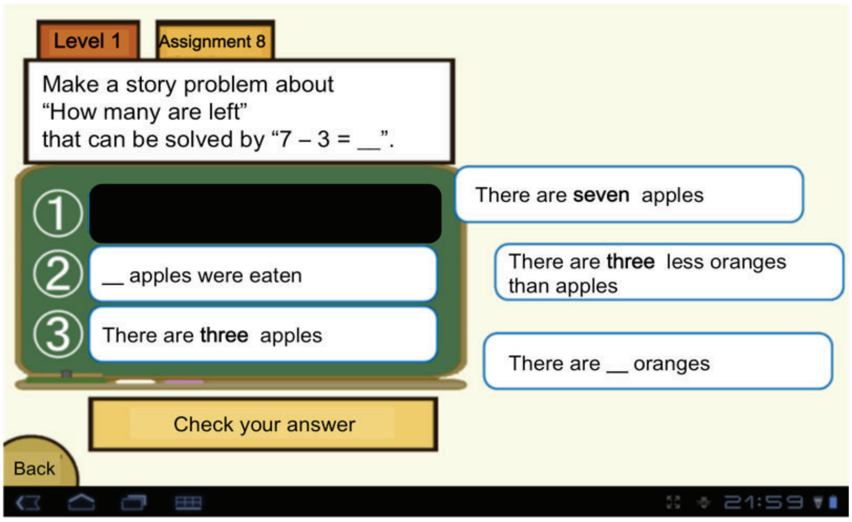

Figure 1 Monsa-kun.

To support the understanding of the relationship between the verbal descriptions and the mathematical formulas in a variety of situations, we design and development an exercise environment in which learners can try to create arithmetic stories from among many choices and can obtain feedback. In the triplet structure model used in Monsa-kun, it is possible to create multiple stories from a single sentence as shown in Figure 2 when considered without conditions (open end), and it is possible to predict various stories. In this study, we call the activity of assembling stories from three quantities and creating a complex arithmetic story by continuously (compounding) creating it without conditions (open end) using some of those quantities as "chain story creation." This activity encourages learners to think in a "more open" manner and prompts them to think about "expanded structures" by allowing them to predict patterns for various stories without conditions. The purpose of this study is to create an exercise environment that allows learners to create chain stories and consider various arithmetic stories in a chain from one sentence that is "diagnosable by the system" and supports the understanding of the relationship between the "real world" and the "world of mathematics" in mathematical modeling.

3. Tri-Prop-Scrabble

We propose the learning game "Tri-Prop-Scrabble," in which, like the English word game "Scrabble," players connect single-sentence cards to create and chain arithmetic stories as shown in Figure 3. Players are expected to learn the structure of arithmetic stories through exploring the potential combinations of single sentences in a chain of stories. The winner of this game is the player who creates the greatest number of arithmetic stories. Currently, as the first step in the design of this game, Tri-Prop-Scrabble only refers the basic rules of Scrabble. Tri-Prop-Scrabble is designed by the fusion method for game design (Umetsu et al., 2003). In the design of Tri-Prop-Scrabble based on Scrabble, letters are replaced with single-sentences, and English words are replaced with arithmetic stories. The thinking required to win this game is to consider the relation between the combination of single-sentences and mathematical formula and it is also required learning activity for arithmetic word problems.

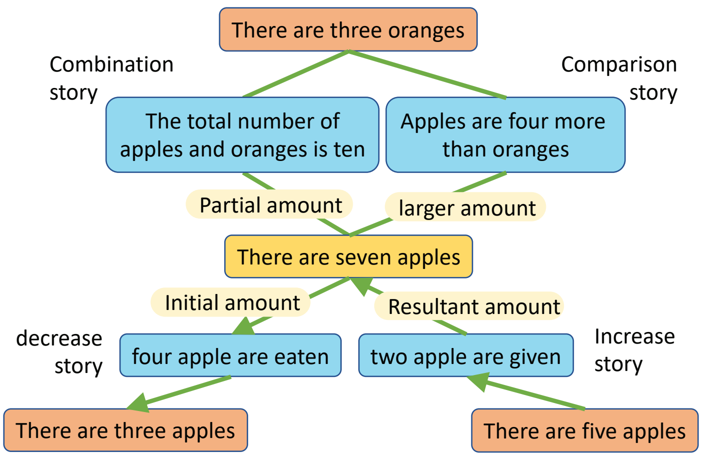

Figure 2 *an example of Possibilities of combining simple sentences.*

Figure 3 *Tri-Prop-Scrabble.*

For example, when playing with multiple people, at the start of the game each player draws six single-sentence cards from the deck. In each turn, players try to create arithmetic stories and then, if a player create a story, the player check the validity of created story using the application named Story-AR-Checker shown in Figure 4. If a player cannot create an arithmetic story using their hand, they can exchange one or two cards from the deck or skip their turn. The game ends when all players cannot create an arithmetic story.

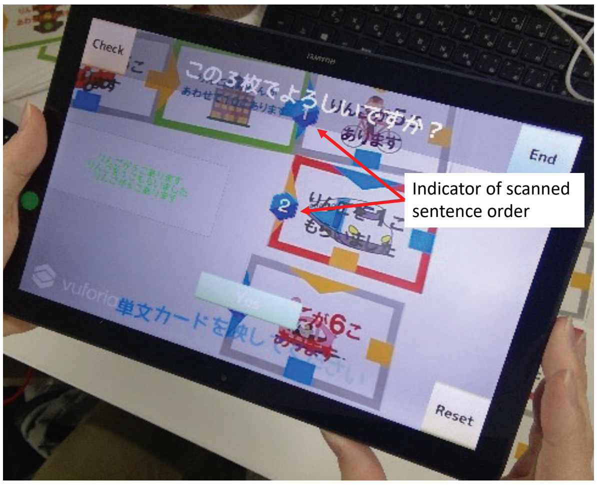

Figure 4 *Story-AR-Checker.*

In Tri-Prop-Scrabble, learners are expected to generate as many stories connected to the cards on the board as possible. Figure 5 shows an example of possibility for connectable stories in the situation. The edge of the cards placed on the board can be connected to other single-sentence cards, and players can find a place to create arithmetic stories and take out two cards to create a story. The left side of Figure 5 shows an example situation of the field and the hand. There are cards "There are five oranges" and "There are five apples" as the edge on the board, and six cards are in the player's hand. Learners are asked to generate possible stories that can be connected to the edges on the board. In this case, an arithmetic story "There are six apples" can be created by connecting "There are five apples" → "I received one apple" → "There are six apples," as shown in the photo on the right, and learners are asked to create a story by connecting the cards. The player who creates the most stories using their hand and the cards on the board wins. Through this activity, the aim is to encourage learners to consider various possibilities of stories without any conditions (open-ended) by combining them in a chain, promoting "more open" thinking that deals with "expanded structures."

Each card represents a simple sentence and the color represent the types of sentences. Existence sentence has a gray frame, and relational sentence has a frame with another color. In this version, cards must be arranged in the order of existence sentences (gray frame), relational sentences (another colored frame), and the other existence sentence (gray frame). The difference in color only indicates whether the card is an existence sentence (gray) or a relational sentence (other colors). Also, each card has an illustration that characterizes the card for the Story-AR-Checker to recognize. This illustration is merely a technique to improve the recognition accuracy of the Story-AR-Checker and does not represent the content of the single sentence written on the card.

With Story-AR-Checker shown in エラー! 参照元が見つかりません。, players can diagnose the validity of the arithmetic stories they have made. Learners can connect single-sentence cards they have in hand to create arithmetic stories and compete with the number of stories created. Learners can determine the validity of the created story by scanning the cards with the Story-AR-Checker. The system recognizes the story created by arranging the cards in physical space by reading the marked single-sentence cards with a tablet camera. The validity of the story is judged based on three criteria: "problem structure," "object," and "formula." The problem structure confirms that the created problem consists of two existence sentences and one relational sentence, the object confirms that the object corresponding to the relational sentence "one unit of number" is used, and the formula confirms that the created problem is valid as an equation. The system judges the validity based on whether the created story meets these three criteria. In this game, players take a picture of the arrangement of cards on the board using the camera of a tablet and Story-AR-Checker scans the cards to identify the story created by the cards. If the order of the cards that make up the story is identified, an indicator representing the order of the sentence is displayed on the scanned card image on the tablet screen, as shown in Figure 4. Vuforia (PTC, Inc., n.d) Augmented Reality SDK and Unity (Unity Technologies, n.d) are used as the development environment for this system.

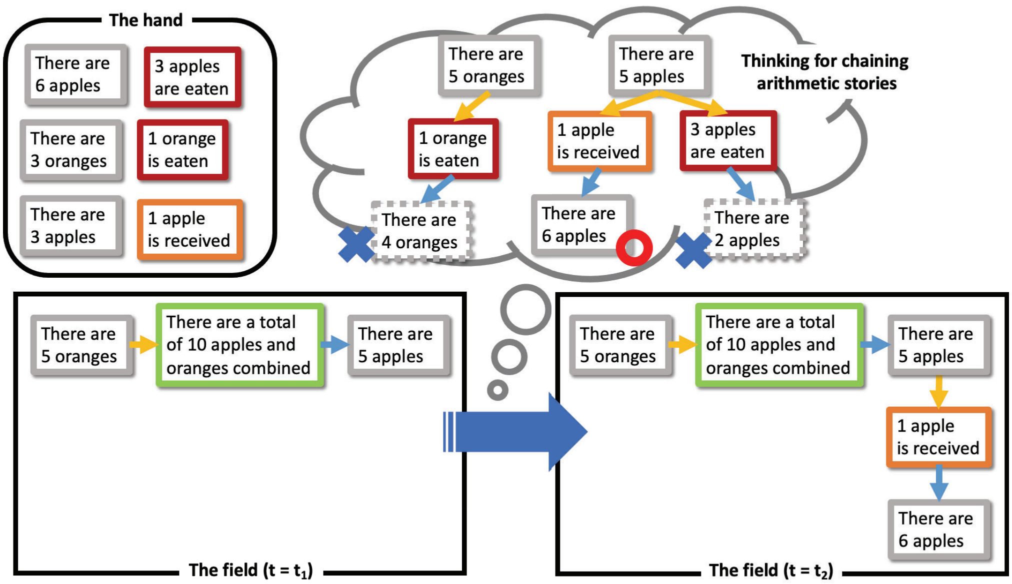

***Figure 5** an example of Thinking for chaining arithmetic stories*

4. Analysis of the Preliminary Use by Elementary School Students

To verify whether exercises with Tri-prop-scrabble for elementary school students can be feasible, 44 first graders of an elementary school in Japan played Tri-prop-scrabble with Story-AR-Checker. Due to the influence of the COVID-19 pandemic, the students played not in group but individually. Before playing Tri-Prop-Scrabble, they had the exercises Monsa-kun and Monsa-kun Tape Block (Hayashi, 2021). Through the exercise they had time to understand problem posing by sentence-integration based on Triplet structure model. In addition to that, before and after the exercises they also took the tests to solve arithmetic word problems.

The exercise including the tutorial was conducted about 30 minutes. At the beginning of the use of Tri-Prop-Scrabble, the author explained the rules of Tri-Prop-Scrabble and how to use Story-AR-Checker in the game. However, since this exercise was conducted individually, there were no rules such as rotation of turns and winner determination, and participants were simply asked to create as many stories as possible using the provided cards. The participants were allowed to confirm the rules of the game and how to use the system by asking questions to the authors during the game.

The research questions of this exercise are as follows:

(1) Can elementary first grade students create and chain arithmetic stories?

(2) Are they able to enjoy this game?

To confirm (1), we analyze the numbers of operations with Story-AR-Checker, stories and chains created by the students. If the students keep trying to create stories and most of students can create and chain stories, we can say the learning exercises with Tri-prop-scrabble is feasible for the main target of it, first and second graders in elementary schools in Japan. To confirm (2) whether participants can enjoy this game, a questionnaire about system usage was conducted.

Figure 6 is the boxplot of the numbers of operations with Story-AR-Checker and shows that students continuously kept playing Tri-prop-scrabble. From 6 minutes to 12 minutes later, this is just after the explanation in the tutorial and the students try to create the example shown by the authors. From 12 minutes to 18 minutes later, the authors made explanation of the rules again and then start the free exercise. From 18 minutes to 30 minutes later, they constantly used Story-AR-Checker.

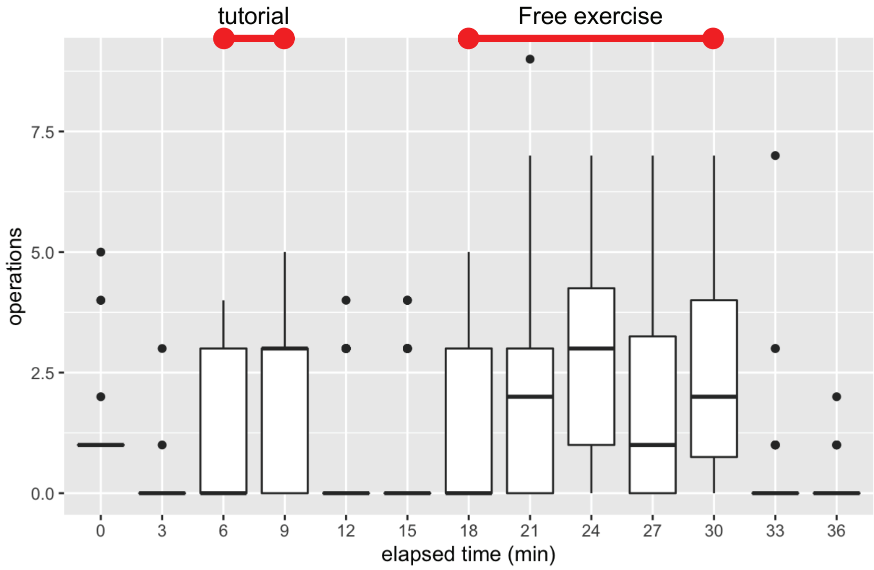

Figure 6 *Number of operations on Story-AR-Checker per 3 minutes.*

Figure 7 and Figure 8 show the number of arithmetic stories and chains created by the learners, respectively. These data show that most of student could create more than one story and it is difficult for them to chain arithmetic stories. The median number of stories created was 1, with a maximum of 10 and a minimum of 1. As for the number of chains, the median number was 1.84, with a maximum of 5 and a minimum of 0. From these data, whereas most of students could create at least one or two arithmetic stories, it is difficult for them to chain stories. It can also be said that this is a valid result because in Japanese standard curriculum children learn arithmetic word problems requiring multiple calculations and a chain of arithmetic stories is the base of such problems. Although chaining arithmetic stories is the out of the curriculum, more than half of students could chain arithmetic stories. This shows the potential of learning for them. On the other hand, a few students created only one story including the example shown at the tutorial. However, even such students try to create arithmetic stories.

Furthermore, this result suggests the further potential of Tri-Prop-Scrabble, which promotes thinking for chaining arithmetic stories, for learners. Figure 9 shows the structure of arithmetic stories created by a learner who had the highest number of chains. The feature of this structure is that there is an arithmetic story loop in the center. In this structure, not only simply connecting stories but also more complex thinking is required. A loop is the minimum structure to compose four arithmetic stories and it requires to consider the constraints of arithmetic stories in a challenging situation. For example, by providing motivation for creating a loop, Tri-Prop-Scrabble has the potential to offer learners a different perspective. Learners can consider at a various level depending on their understanding on Tri-Prop-Scrabble.

To confirm (2) whether participants can enjoy this game, we conducted the questionnaire about this game. As a result, many positive evaluations were obtained regarding playing this game. The questionnaire results in Figure10 show that participants felt "want to try again," "understand how to use the system," and "the game is fun." However, some participants answered "difficult to use" in the free description section. This suggests that there are points to be improved in Story-AR-Checker.

From these results, we answer yes to the research question (1) and (2). For the research question (1): “can elementary first grade students create and chain arithmetic stories?”, the students continuously kept playing Tri-prop-scrabble and most of students could create more than one arithmetic stories. Even the students could not create any arithmetic stories by themselves kept playing this game. For the research question (2): “are they able to enjoy this game?”, most of students gave positive answers. This means learners could enjoyed Tri-prop-scrabble as a game even though it required thinking about the structure of arithmetic stories.

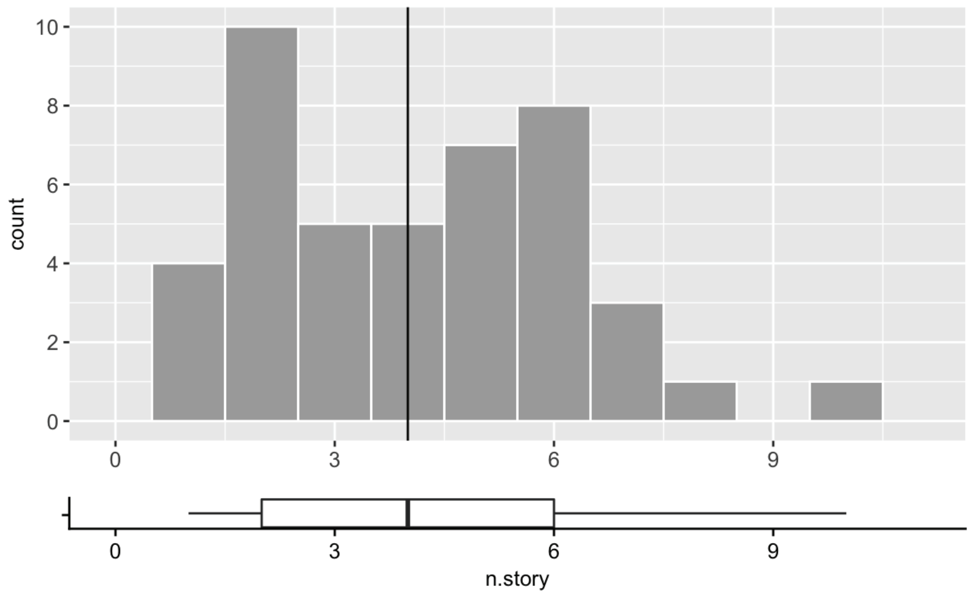

Figure 7 *numbers of stories created by learners.*

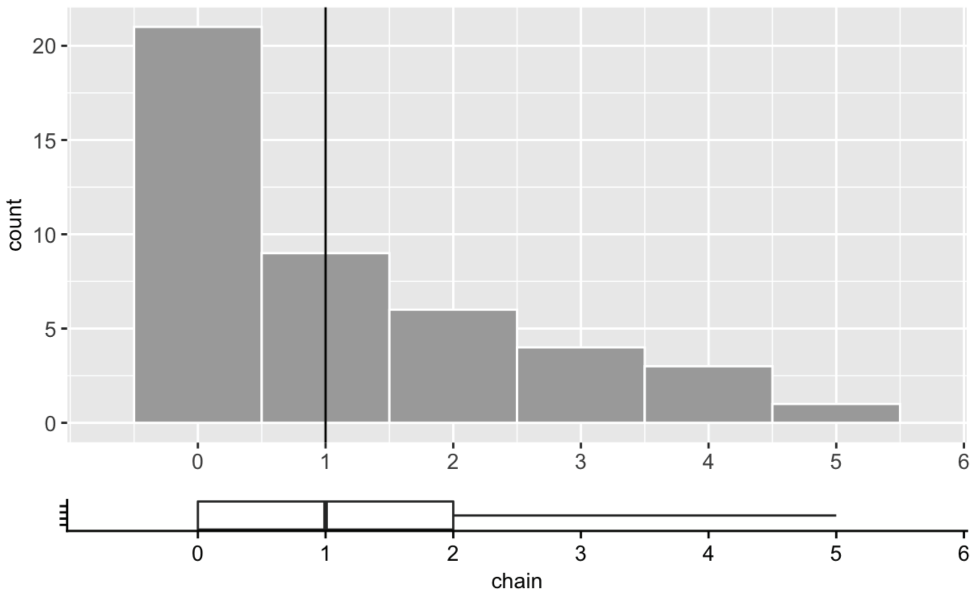

Figure 8 *numbers of chain created by learners.*

5. Summary

In this study, we developed a game called "Tri-Prop-Scrabble" that explores arithmetic stories as an integration of simple sentences, and a diagnostic system called "Story-AR-Checker" that supports it with AR technology. The game allows players to explore arithmetic stories that can be created using the simple sentences on the board and in their hands. Story-AR-Checker uses a camera to recognize cards and judge the validity of the created arithmetic stories. Th use by the elementary school students as the main target of this game shows that the students were able to create arithmetic stories, and some were able to chain arithmetic stories. From these results, it was suggested that exercises with Tri-prop-scrabble are feasible for elementary school students and that this exercise can be one that encourages the consideration of the relationships between the verbal description and mathematical formula in arithmetic stories. Of course, this doesn't show that learning has happened, and we heed to investigate the effectiveness of this game for learning of arithmetic word problems. In the future, we plan to analyze practical data, expand the scoring rules to encourage consideration of relationships, and further verify the learning effects of the proposed game.

Figure 9 *chained arithmetic stories with a loop*

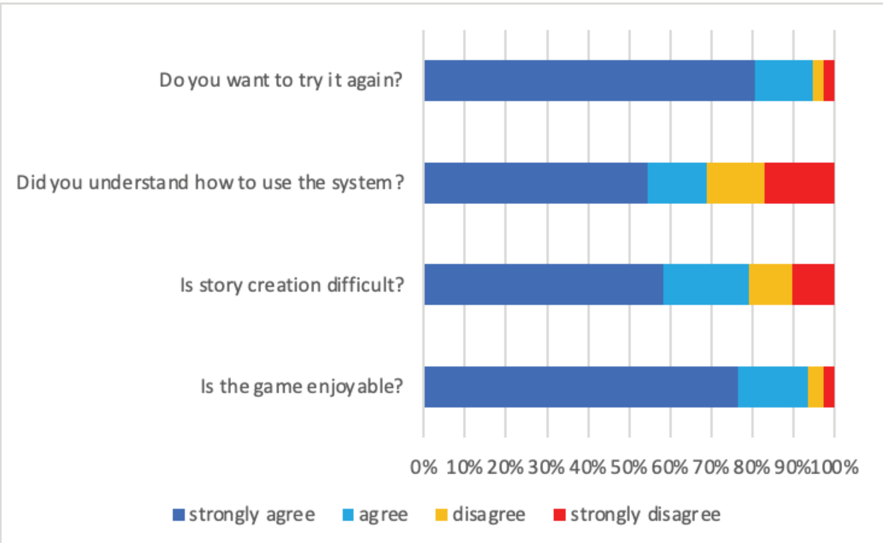

Figure 10 *the questionnaire results*

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