Technological Tools for the Teaching and Learning of Statistics

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Abstract: Statistics and its applications form an integral part of STEM education. In the literature, it is shown that technology is valuable in supporting the teaching and learning of statistics. This paper discusses some technological tools that have been developed to support statistics education in the grade school, junior high school, and senior high school levels. It describes the design and pedagogical basis of these tools, and how these may be integrated in the classroom.

Keywords: Mathematical apps, statistics, STEM, technological tools in statistics

1. Introduction

Statistics continues to play an important role in STEM education as it can be applied to all STEM fields (Adams, 2017). In fact, Ben-Zvi and Garfield (2008) have stated how statistics is becoming a necessary area of study that teaches students how to react intelligently to quantitative information. Mastery in statistics allows students to apply different statistics concepts and techniques across STEM projects or within their disciplines when analyzing data generated from investigations (Watson et al., 2020). Thus, statistics is usually featured in the mathematics curricula of different countries and is often included in multiple grade levels (https://timssandpirls.bc.edu/timss2015/encyclopedia/). In the Philippines, for example, the K to 12 Basic Education Curriculum (Department of Education, 2016) includes *Probability and Statistics* as one of the five main content strands.

Given the importance of statistics education, efforts and resources to enhance or to support teaching and learning statistics remain valuable. The use of technology is usually emphasized in this regard; for instance, the Pre-K-12 Guidelines for Assessment and Instruction in Statistics Education II (GAISE II) highlights "the recognition that modern statistical practice is intertwined with technology, and the importance of incorporating technology as feasible" (Bargagliotti et al., 2020, p.2). There are several methods on how technology is used in teaching and learning statistics, and how these impact student learning (Chance et al., 2007). Studying statistics in an interactive environment using technology tools supports the understanding of statistical concepts (Prodromou, 2015).

While there are already various technological tools available for statistics education, most existing software (e.g., spreadsheet software, programming language) can be too sophisticated or have irrelevant functionalities for beginner learners. In the context of developing countries such as the Philippines, many of these software may also not be affordable or may be more demanding in terms of device requirements. Thus, we have developed a number of technological tools to support statistics education. In this paper, we discuss pedagogical bases and various features of these tools and how these tools can be concretely integrated into statistics classes.

2. Pedagogical Basis

Statistics is often seen to be a branch of mathematics, but it is a distinct field of study requiring both mathematical and non-mathematical skills (Wild et al., 2018). It covers five interrelated themes—variation, expectation, distribution, randomness and informal inference (Watson et al., 2020). Moore (1998, p. 1257) argued that statistical thinking "is a general, fundamental, and independent mode of reasoning about data, variation, and chance." To promote statistical thinking and develop the notion of statistical inference, it is important to provide simulation and randomization-based activities in the classroom.

Rossman and Chance (2014) compiled a list of textbooks that include randomization in the early parts of statistical courses, as well as technological tools that allow students to perform simulations. They also described their own introductory statistics course where students initially perform simulations of random processes using coin flipping, compare their simulations with those of their classmates, and later perform the same simulations using technology. In their curriculum, simulations not only illustrate statistical inference but become the principal mechanism through which they learn to perform statistical inference themselves.

Another important pedagogical consideration, as emphasized in GAISE II (Bargagliotti et al., 2020), is the role of context in statistics. Aside from dealing with the variability of data sets, students should also recognize the context(s) of these data sets so that they can craft interpretations and conclusions appropriately. Hence, to develop such competencies, it is vital that students have access to authentic data sets that are contextually clear and relatable to them. Using such data sets, students can practice procedural skills in statistical analysis and experience contextual learning (Samo et al., 2017).

In designing and developing technological tools, as far as possible, we were guided by Rossman and Chances's (2014) recommended list of features—ease of use, mimicking of by-hand simulations, and consistency of user interface. While the first and third features are doable, we had to do away with the second feature because we wanted the tools to run even on modest devices. To manage this limitation, the tools are accompanied by teachers' guides and student worksheets to enable teachers to use the tools more readily. These guides and worksheets are aligned with the most essential learning competencies (MELCs) specified in the official Philippine mathematics curriculum (Department of Education, 2020).

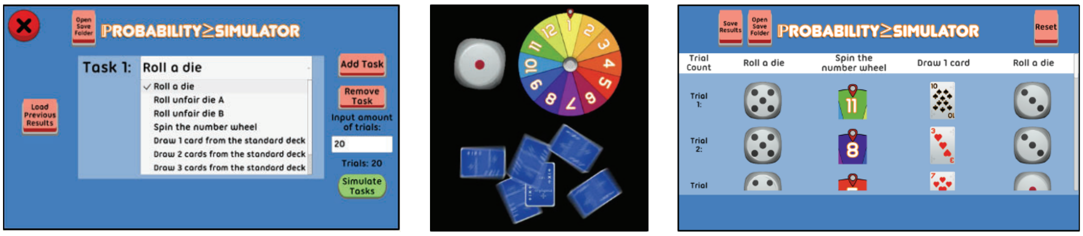
3. Technological Tools for Statistics

The technological tools we have developed include apps, applets, a web-based platform, and videos. The apps have been developed using Unity Editor Version 2021.3.25f1 and are available both for Windows and Android devices. The Windows versions can run on 32- or 64-bit systems and have modest requirements: Windows 7 or later, 1 GHz processor or higher, 4 GB RAM, and approximately 300 MB available storage. On the other hand, the Android versions can run on devices with Android 5.1 until 13, 2 GB RAM, and approximately 120 MB available storage. The applets and web-based platform do not require installations and can run on most internet browsers, with only the platform requiring internet connectivity.

3.1 *Tools for Simulation and Data Generation*

GAISE II emphasizes the importance of the context and variability of data sets that are used in statistics education (Bargagliotti et al., 2020). One aspect of this involves giving students opportunities or experiences of dealing with uncertainty and randomness. In the classroom, it is possible to achieve this by having activities in which students perform repeated trials of actual physical probabilistic experiments (e.g., rolling dice, drawing cards). However, in some cases (e.g., remote learning, due to time or resource constraints), these physical experiments may not be feasible. Thus, we have developed the app *Probability Simulator* that allows students to simulate different probabilistic experiments using only their electronic devices (e.g., computer, tablet, smartphone).

In *Probability Simulator*, the following probabilistic experiments are available for simulation (Figure 1(a)): rolling a fair die; rolling unfair dice; spinning a number wheel; and drawing 1 to 5 cards from a standard deck. In addition, multi-stage probabilistic experiments can also be simulated by setting up a sequence of different tasks. The student can then specify the number of times that the task(s) must be repeated.



(a) (b) (b)

Figure 1. Screenshots from *Probability Simulator*: (a) task selection, (b) animated depictions of tasks simulation, (c) outcomes window.

When the student taps/clicks the Simulate Tasks button, a short animation (Figure 1(b)) is played to depict the simulation of the selected tasks. After the animation, the outcomes of the tasks are displayed in a scrollable window (Figure 1(c)). If the number of trials is too many, the scrollable window may become inconvenient for viewing all the outcomes; thus, there is also an option to save the results in a spreadsheet file (csv) that can then be processed or analyzed further using other software. In the following section, we will illustrate how *Probability Simulator* can be used for learning activities across different grade levels.

Another tool that we have developed is the web-based platform *Senso Eskwela Pilipinas* (*SEP*) (De Las Peñas et al., 2020; Tolentino et al., 2022). In *SEP*, students themselves become part of the data collection process by answering the *SEP* survey and having their anonymous responses become part of the *SEP* database. In turn, the database becomes a source of readily available, authentic, and relatable data sets. Based on initial integration into two Grade 11 classes, *SEP* has shown positive benefits in students' learning (Tolentino et al., 2022).

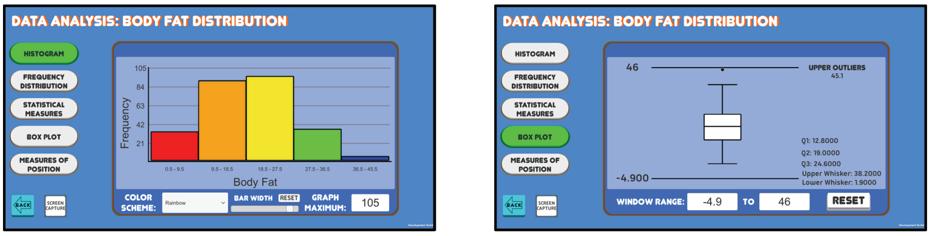
3.2 *Tools for Data Analysis*

In real-world contexts, analyzing different data sets, especially large ones, is usually done using different statistics software. Thus, it is also important to give students opportunities to perform different data analysis activities using technology.

Focusing on competencies for Grades 7, 8, and 10, we have developed a data analysis tool subject to two important considerations. First, we want the tool to focus only on selected functionalities that are relevant to the aforementioned grade levels. This is to make the tool relatively simple and easy to use so that students and teachers are not overwhelmed with many functionalities and complicated commands. Secondly, we want to maximize the accessibility of the tool; that is, it must be freely available, uses relatively low storage, and can run offline in a wide range of low-cost electronic devices.

Given the above considerations, we have developed the app *Mathplus Statistics*, a tool for computing descriptive statistics and generating data visualizations, which can be accessed via Windows or Android. Its functionalities include the generation of histograms (Figure 2(a)), frequency distribution tables, and box plots (Figure 2(b)), as well as the calculation of measures of central tendency and variability and measures of position.

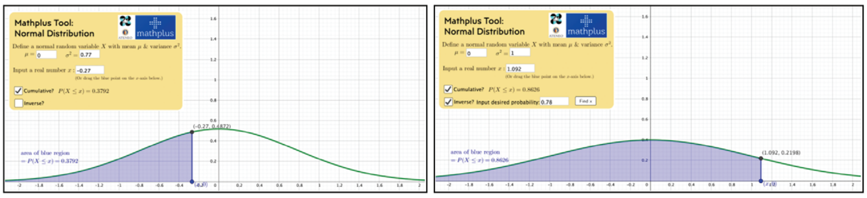
Students may upload their own data (in csv format) or use the app's built-in datasets. The app is also compatible with SEP in the sense that data sets downloaded from the *SEP* platform are already properly formatted and may be uploaded directly to the app. Data filtering is also available and can be applied to data to keep (or exclude) observations based on various variable-dependent criteria.



(a) (b)

Figure 2. Screenshots from *Mathplus Statistics*: (a) histogram, (b) box plot.

In addition to *Mathplus Statistics*, we have also developed two Grade 11 tools or applets using the free software *GeoGebra* (https://www.geogebra.org/): *Mathplus Tool for Normal Distribution* and *Mathplus Tool for t Distribution*. The applet *Mathplus Tool for Normal Distribution* allows students to compute probabilities involving the normal distribution for which the mean and variance are user-inputted (Figure 3(a)). A graph of the normal probability distribution function and the corresponding area under it are also displayed. Lastly, there is also an inverse function (Figure 3(b)) in which a student can input a desired probability then obtain the corresponding value at which the normal cumulative distribution function gives this probability. *Mathplus Tool for t Distribution* has similar functions but for the *t* distribution.



(a) (b)

Figure 3. Screenshots from the *Mathplus Tool for Normal Distribution* applet.

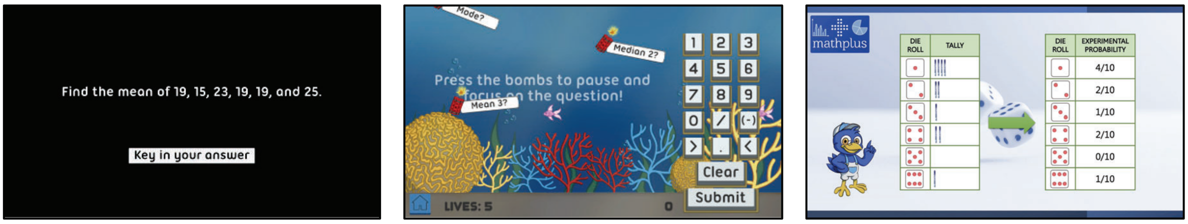
Aside from being an alternative to printed out distribution tables, the applets can also be used for strengthening students' understanding of these probability distributions. Particularly, the visualization of the graphs and probabilities-as-areas can help students master different properties of computing probabilities and of the probability distributions.

3.3 *Instructional Resources*

In addition to the data generation and analysis tools discussed above, we have also created resources that can contribute to statistics instruction. These have been made according to the list of most essential learning competencies of the Department of Education (DepEd, 2020).

One such resource is the app *Just Keep Solving 2 (JKS 2)*, a game that helps students develop skills in carrying out computations under chosen topics in probability and statistics. As the name implies, *JKS 2* is an expansion of an earlier app *Just Keep Solving!* (Garciano et al., 2023). The *Statistics* topic in *JKS 2* has three levels covering measures of central tendency, measures of variability, and the identification of correct statistical measures in data analysis and interpretation. In the game, questions (Figure 4(a)) are shown as bombs that will explode upon reaching the bottom of the screen (Figure 4(b)). Players tap on these bombs to pause their movement and see the questions they are challenged to answer. Players must answer a set number of questions before losing all lives or answer enough questions to survive within a specific time limit. Garciano et al. (2023) provides additional details of *JKS 2*.

Some videos have also been produced to complement some of the apps discussed above. The videos have been developed to include a demonstration of an app and a concise discussion of the relevant math lesson(s). The project's mascot *Banoy Bilang* plays the role of a teacher's assistant in most of the videos. A particular example is the *Experimental Probability with Banoy Bilang* video that is intended for Grade 8 students. In the video, the *Probability Simulator* app is used to review the computation of experimental probabilities (Figure 4(c)).



(a) (b) (c)

Figure 4. Screenshots from *Just Keep Solving 2*: (a) sample question, (b) main level screen; (c) screenshot from the video *Experimental Probability with Banoy Bilang.*

4. Integration of the Technological Tools

A possible obstacle in using any technological tool for statistics education is teachers' "lack of awareness and comfort with new technologies" (Chance et al., 2007, pp. 18-19). Additionally, teachers are often busy enough with their usual teaching responsibilities and other ancillary activities (Batilantes, 2021) to have sufficient time for learning or designing new activities that use technology. At the same time, students also need direction or guidance when it comes to learning activities that they can do independently.

Given the above considerations, we have also developed teachers' guides and student worksheets detailing learning activities that make use of the discussed technological tools. The teachers' guides contain complete information (e.g., competencies covered, materials needed, instructions) related to implementing concrete learning activities. On the other hand, the student worksheets contain instructions for doing the activities; the worksheets are ready-to-use and can also serve as students' answer sheets, if applicable.

In Grades 1-3, for example, one set of learning competencies under the *Probability and Statistics* strand (Department of Education, 2020) is focused on introducing and developing students' notion of uncertainty and likelihood. In the teacher's guide *Introduction to Uncertainty* and its corresponding student worksheet, we present a learning activity that makes use of the *Probability Simulator* app. This allows the students to perform probabilistic experiments more practically, compared to using physical objects (e.g., dice, cards) that may be too time-consuming. Other examples of teacher's guides and student worksheets can be accessed for free from mathplusresources.wordpress.com.

5. Conclusion and Future Direction

In this paper, we recognize the significance of statistics in students' STEM education and how the teaching and learning of statistics can be improved by using technology. We have developed a number of technological tools that can support different aspects of statistics education. The tools can be used for simulation and data generation (*Probability Simulator* app and *Senso Eskwela Pilipinas*) or for data analysis (*Mathplus Statistics* and *GeoGebra* applets for normal and *t* distributions). There are also resources (the game *Just Keep Solving 2* app and instructional videos) that can support instruction. These resources can easily be downloaded and used on low-cost mobile devices, making them accessible and suitable for the use of most students in the Philippines. We have also developed teachers' guides and student worksheets that present concrete learning activities that use the different technological tools or resources.

Among the activities for future work is Investigating the effectiveness of the tools presented in this paper in terms of increasing student engagement, interest in learning and strengthening statistical knowledge and skills. Another future activity is gathering data on usage and acceptance of the technological tools on the part of students and teachers. Moreover, other ways of integrating the use of these tools in the classroom can be further explored in collaboration with Filipino high school teachers. A greater knowledge of the requirements for statistics education for both students and teachers will result from such collaboration.

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