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Exploring Graph Slopes Through a Series of Embodied Learning Experiences

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Abstract: This study presents a sequence of embodied learning experiences designed to help students understand the concepts of rate of change and slope. The sequence begins with an embodied exploration of proportion using the Mathematics Imagery Trainer. Building on this experience, two other learning systems were developed. These generate graphs when learners walk in proportions. Through the second task learners are expected to get a sense of rate of change, by blending the walking experience with the graphs. In the third task, learners generate movements corresponding to given graphs. This sequence of embodied experiences is designed to help learners integrate embodied and qualitative understandings of rate of change with the formal notion of slope, through the process of conceptual blending.

Keywords: Embodied Learning, Embodied Mathematics, Slope of a graph, Rate of change, Conceptual Blending

1. Introduction

Graphs, in particular rate of change graphs, are critical to the learning of various high school topics, in mathematics, physics and chemistry. Graphical representations in textbooks seek to help students understand the relationships between different variables, through the values of x and y components. Science students are expected to be proficient in plotting and interpreting graphs. Our focus here is to support the understanding and prediction of qualitative relationships from graphs. We designed a series of embodied learning experiences, to help students comprehend and analyze graphs, particularly those that capture dynamic real-life scenarios, such as rate of change graphs. Our previous studies demonstrated that students could conceptually blend the elements presented through an embodied learning system, *Touchy Pinchy Integers (TPI)*, and thus understand and internalize the mathematical idea of integers and integer operations. Our analysis showed that the TPI intervention led to students talking about integers using gestures and language that embedded the actions and changes in the embodied system (TPI) they interacted with. Based on empirical work (Hanakawa, Honda, Okada, Fukuyama, & Shibasaki, 2003) on the way abacus-based learning of arithmetic changes brain activity patterns (compared to learning arithmetic using text-based media), we consider this result as indicating a change in student cognition, where they ‘think with’ the interactive system (TPI) post-intervention. This design and analysis, based on the conceptual blending framework from cognitive science (Fauconnier & Turner, 1998), may help extend the investigation of how embodied learning systems allow students to blend various conceptual elements.

To present the rationale of our embodied design for learning slope, we first outline conceptual blending theory (Section 2.1), which analyzes how new conceptual structures form from existing concepts. This is followed by a review of embodied learning systems for mathematics (Section 2.2) and the kind of learning these systems facilitate in children. Building on these two existing literatures, We discuss our design for teaching/learning slope, based on a series of embodied learning experiences (Section 3). We conclude the paper with a discussion of the possible learning outcomes promoted by the design.

2. Typographical Style and Layout

2.1 Conceptual Blending

Conceptual blending (Fauconnier & Turner, 1998) is a dynamic and active cognitive process, present at the moment of thinking and even a part of everyday thought processes such as language generation, particularly metaphors. We organize different aspects of our experience and knowledge of the world by representing it in the form of conceptual packets, and these conceptual packets together are termed the mental space. These mental spaces are constructed for our local understanding and action as we think and communicate. During the process of blending, selected structures from various input mental spaces are projected onto the blended mental space, and cross-space mapping between the elements from the input spaces are formed. On completing and elaborating the blend, new conceptual structures emerge, which were not part of the individual inputs. Composition, completion and elaboration are the processes involved in conceptual blending. Composition, also known as fusion, gives rise to new relationships, which might not be possible to see in the input spaces alone, but comes into existence when the selected structures from two input spaces are brought to the new space, the blend. Completion inherits the background knowledge and nature of the input spaces, which thus becomes part of the blend, but in a new way. In the last stage, extension, the blend is simulated or run, according to the logic and principles of the new space.

The authors extend this theoretical framework, originally developed to account for the generation of new metaphor-based concepts in language, to the formation of new mathematical concepts historically, such as the emergence of complex numbers. They also extend the conceptual blending (CB) framework to new design and interaction metaphors, such as the desktop metaphor, whose generation was central to the development and success of WIMP (Windows, Icons, Mouse, Pointing) interfaces and the direct manipulation interaction paradigm. This design led to the mass transition from the then dominant text-based interaction with computers, and computing becoming social.

Here is the illustration of conceptual blending through desktop metaphor, this development allowed for a new way of interacting with computers. Desktop interface includes elements such as folders, calendars, clocks, and music players, which are based on real-world objects. Likewise, the actions that can be carried out on these desktop icons—like storing files, marking dates, or playing music—are derived from tasks that are typically done with real-world counterparts. However, when the aspects of these two inputs—real-world objects and actions—are combined in the blended space - desktop, new structures emerge that don't exist in the original input spaces of real life objects and actions performed on them. For example, cutting and pasting can be done repeatedly in the real world, but doing so could damage the paper. On a computer desktop, this action can be repeated without causing any harm to the words. Thus, the blended desktop space allows for actions that are not possible in the original real-world contexts.

2.2 Embodied Learning Systems for Mathematics

The theory of embodied cognition asserts the inseparable link between the mind, body, and environment, emphasizing the vital role played by the body in thinking and acting in the world (Kosmas & Zaphiris, 2018). With this as the background researchers and education designers have developed embodied learning systems for mathematical topics like arithmetic (Sinclair & de Freitas, 2014), algebra (Weitnauer, Landy, & Ottmar, 2016), geometry (Nathan, Walkington, & Swart, 2022), proportion (Abrahamson,Dutton & Bakker, 2021) and vectors (Karnam, Agrawal, Parte, Ranjan, Borar, Kurup, Joel, Srinivasan, Suryawanshi, Sule, & Chandrasekharan, 2021).

Touch technologies offer direct and immediate manipulations on the screen, corresponding to intentional touch screen gestures. Sinclair and de Freitas (2014) presents their novel interactive systems rooted in multi touch functionality - Touch Counts and Touch Times - which were developed to support basic numeracy in young children. They encompass activities like counting, addition, and subtraction of natural numbers. This design aims for an embodied learning of arithmetic operations, where gestures common to touch media – such as tapping, pinching, and flicking – are given mathematical meanings, allowing learners to explore the intricacies of numbers while engaging with touch media and receiving real-time feedback.

Inspired by Touch counts, and extending the potential of touch interfaces to integers, we designed Touchy Pinchy Integers (TPI, Elangaivendan, Ramaswamy, Albuquerque & Chandrasekharan, 2023), to help students explore the concept of integers through embodied interaction. TPI serves as a dynamic external representation, allowing learners to visualize integers and perform operations on them through gestures. TPI plays the roles of both tool and external representation. As an external representation, TPI allows students to think with it during problem solving related to integer operations. With continued exposure to TPI, students eventually use TPI as an imaginative tool, to visualize and perform integer operations, when confronted with static media like text and figures. This echoes the gestures and visualization observed in experienced abacus users who generate and manipulate a 'mental abacus' to verbally solve arithmetic problems. (Hanakawa, Honda, Okada, Fukuyama, & Shibasaki, 2003). The TPI design study showed that students were able to conceptually blend the interaction elements from an embodied learning system and the mathematical ideas embedded in the system, to develop a comprehensive and coherent understanding of the mathematical topic.

In the Mathematics Imagery Trainer for Proportion (MIT), students perform goal-oriented hand movements to manipulate the space between their hands and make a screen green when the right proportion exists in the space between the hands. Through this manipulation, students assimilate the mathematical idea of proportion, in an embodied manner. This movement-based understanding of proportion is later augmented using symbolic elements such as cursors, grid lines, and numerical representations on screen. These elements facilitate formalizing the embodied understanding, by grounding it in symbolic and mathematical elements. These elements serve as both tools for action and points of reference. Through interaction with these symbolic artifacts using movement, learners' understanding of proportion shifts closer to mathematical reasoning and visualization, which, together with the embodied understanding, deepens their understanding of proportion.

In the upcoming study this system serves as a starting embodied experience, following which the students will engage in a sequence of embodied activities that allow them to build on the initial experience of proportion, to move towards the complex idea of slope of a graph (ratio of rise over run) and rate of change.

3. Sequencing Embodied Learning Experiences

Based on these designs, and our CB analysis of TPI design and results, we hypothesize that complex mathematical ideas can be supported by sequencing a series of embodied interaction systems. Here we report the development of such an embodied learning sequence to test this proposal. The design interconnects three different embodied interaction systems, through which we would like to study if and how such a sequence of embodied interactions could facilitate the learning of the complex mathematical topics of rate of change and slope.

We present the prototype sequence of the design to support the learning of rate of change and slope concepts. It is crucial to note that the system alone may not lead students to attaining the concept. Rather the system, integrated with appropriate pedagogical instructions and intervention tasks, may facilitate the assimilation of mathematical topics. However, these pedagogical instructions and tasks would be novel and different from existing text-based ones, so it would not be possible to design these without building the embodied learning system in the first place. Therefore, we begin by introducing the system, which will serve as the foundation for developing instructional tasks and strategies.

3.1 Mathematics Imagery Trainer (Recreated)

The first system in the sequence is a replication of the Mathematics Imagery Trainer for Proportion (Abrahamson, Dutton & Bakker, 2021) . In our version, this system uses sensors and a screen, and the students must engage in a bimanual activity, moving their hands to turn the screen green when the right hand is twice as far from the bottom as the left. This experience helps students grasp the concept of proportion in an embodied fashion. This relationship is then formalized by executing the same task using coordinate grids, and then adding numbers to the grid. We have recreated this system in the lab using motion sensors (see Figure 1). This pilot version of the design is currently undergoing extensive testing.

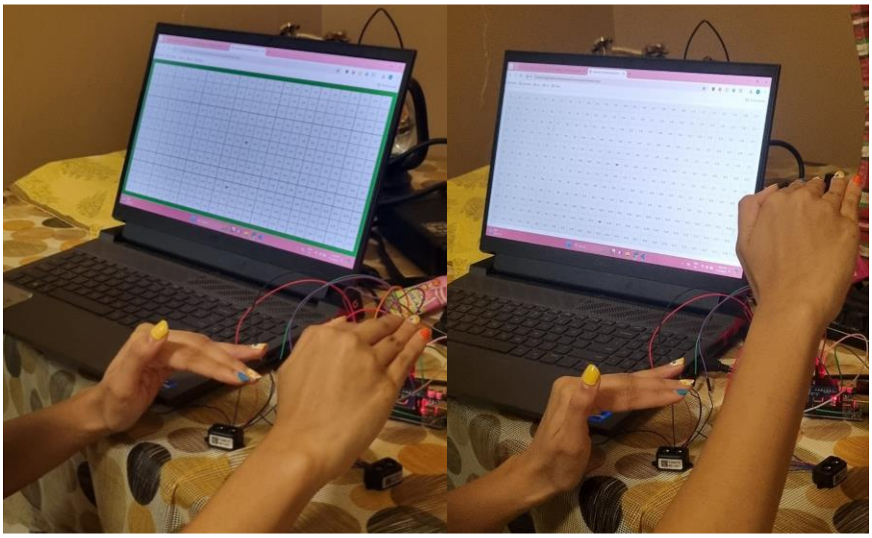

Figure 1. Mathematics Imagery trainer (MIT) for proportion recreated using motion sensors

The experience of this system presents students with 3 different understandings of proportion - Embodied understanding, Spatial understanding and Numerical understanding (see Figure 2). We seek to incorporate and extend these understandings in the next learning experiences in the sequence.

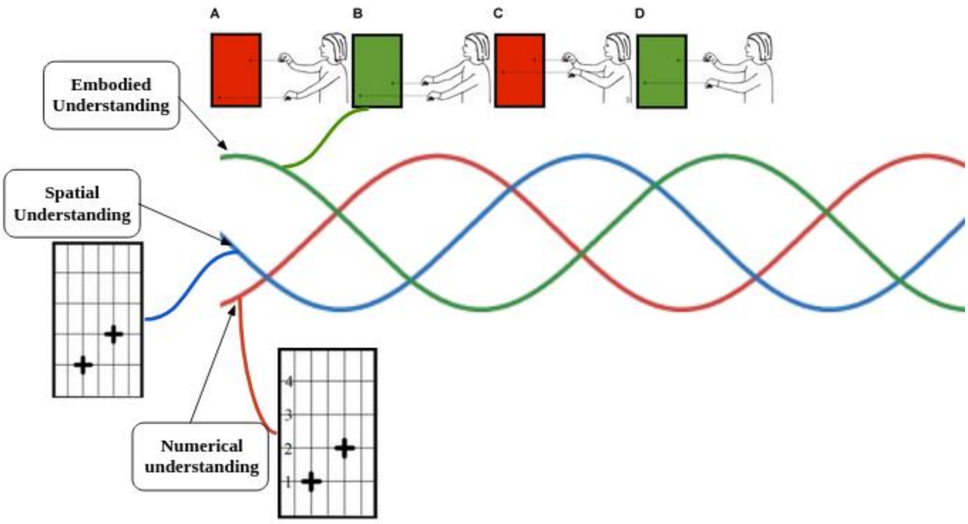

Figure 2. A conceptual blending view of learning based on MIT: the system helps Integrate embodied, spatial, and numerical understandings of proportion represented through three distinct waveforms

3.2 Graphing the walk

In the second system, the MIT idea is extended, by replacing the 2 hand movements to match a given proportion with 2 people walking at a given proportion (Abrahamson, Dutton & Bakker, 2021). In this activity, both students move in sync with a set rhythm, but their steps are proportional. One student takes a single step per beat, while the other takes two steps per beat. This system uses a mobile phone's accelerometer to track movements over 30 seconds, generating two types of graphs in a single screen: movement versus time (with time on the X-axis and total movements on the Y-axis) (see Figure 3 and Figure 4, below and accelerometer data versus time (with time on the X-axis and accelerometer readings on the Y-axis) (see Figure 3 and Figure 4). The individual graphs from the students system can be sent to the teacher’s system, which works as a shared screen for the class. By walking proportionately, learners gain an embodied understanding of the rate of change and its graphical representation presented on the learner screen as well as the teacher's screen (see Figure 5). The teacher’s screen presents the graphical representation of movements from all the learners, using the same classroom code. This structure allows students, along with the teacher’s facilitation, to discuss their embodied experience of rate of change concept, connecting it to the spatial understanding of each student’s relative position and their numerical understanding, by comparing the graphs on the teacher’s screen.

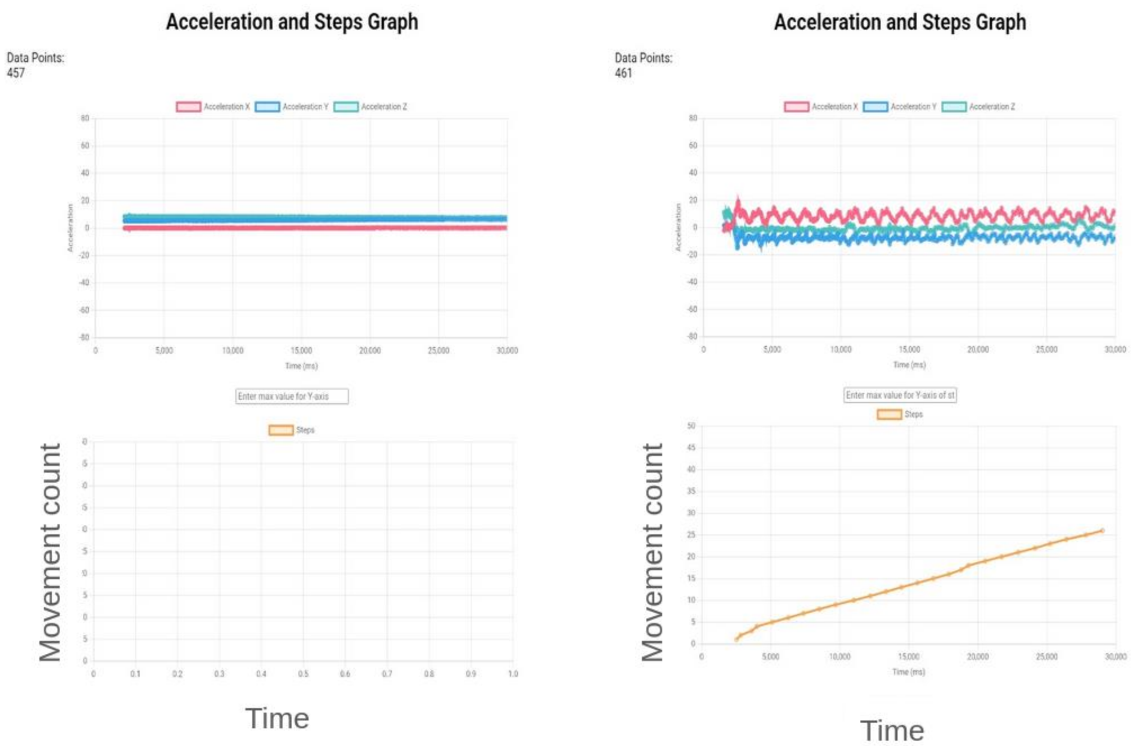

Figure 3. (Left) Graph generated while at rest; (Right) Graph generated during an attempt to create uniform movement at 1 step per beat.

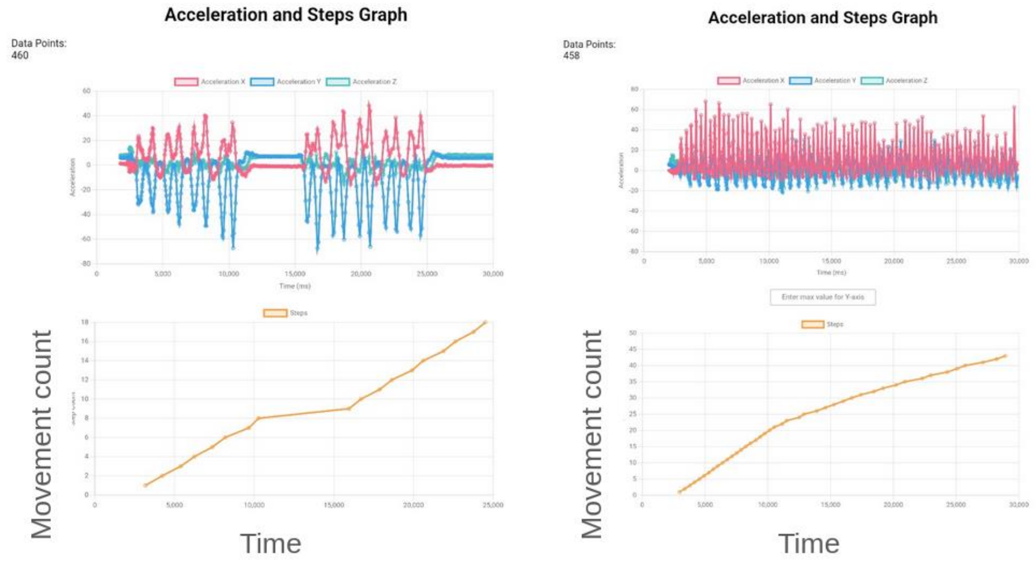

Figure 4. (Left) Graph generated while attempting to create uniform movement for 1 step per beat with pause in between (Right) Graph generated while attempting to capture movement count graph for 3 steps per beat. The graph shows a reduction in movement speed towards the end.

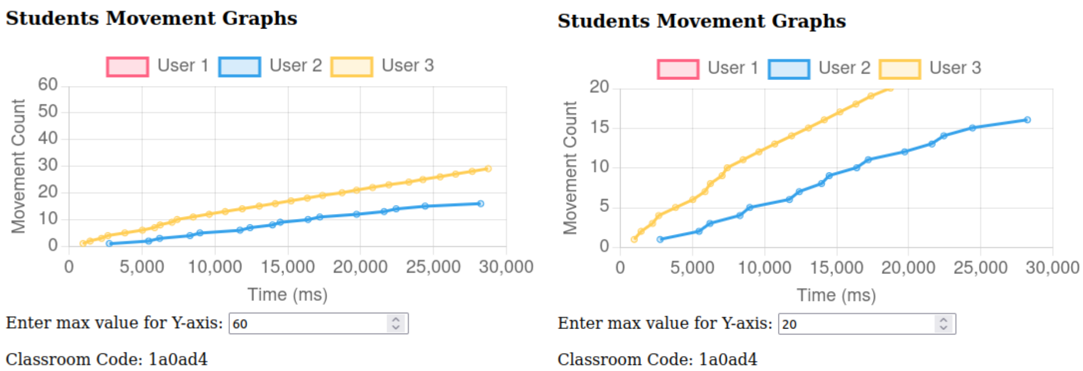

Figure 5. The teacher System for graphing learners' movements

3.3 Walking the graph

In the next learning task in the sequence, students are challenged to generate motions that match given graphs, and compare their results with target graphs. This task seeks to promote student analysis of the given graph, and planning of movements such that the graph that is generated as a result of their movement closely resembles the shape of the graph they are trying to recreate.

4. Expected Learning Outcomes

The following are some of the learning outcomes of the embodied interaction sequence:

- Integration of the concepts of slope and rate of change with proportion, through embodied, spatial, and numerical understanding.
- Drawing systematic linkages between qualitative experiences and quantitative data from the graphs generated through learner actions.
- Making qualitative inferences and identifying potential factors that may have influenced the shape of the graph.

- Anticipating the sequence of one's own movements to align with the given graph in the system.

We are currently running pilot studies of the system with school students, and plan to report results from these studies at the conference.

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