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Conceptual Metaphor in Teaching Logic

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Abstract: In this paper, we discuss the concept of *conceptual metaphor* and its role in the teaching of logic. Starting from our experience in the teaching of logic to students in human sciences at University of West Britanny and students who take general education courses at various universities in Taiwan and from our research in the field of logic, we defend the point of view according to which the teaching and learning of logic adopts metaphoric devices to adapted students' previous knowledge.

Keywords: Conceptual Metaphor, Logic, Logic of Determination of Objects (LDO), Teaching Logic.

1. Introduction

The notion of conceptual metaphor was first introduced by George Lakoff and Mark Johnson in their works, Conceptual Metaphor in Everyday Language (Lakoff and Johnson, 1980) and Metaphor We Live By (Lakoff and Johnson, 1980). This notion has been studied in related to some philosophical notions such as the the nature of meaning, truth, rationality, logic and knowledge. By its later developments, it is further analyzed by postulating the concept of conceptual blending by Gilles Fauconnier and Mark Turner in (Fauconnier and Turner, 2003). A systematic study from the conceptual metaphor to conceptual blending as a new framework of integrating two conceptual spaces has been studied in theoretical computer science by Kutz et al. (Kutz et al., 2010). Starting from the classical optimality principles of blending (Fauconnier and Turner, 2003), Goguen and Harrell explore more conditions for applying these principles in (Goguen and Harrell, 2004). This framework "as a basic mental operation will lead to new meaning, global insight, and conceptual compressions useful for memory and manipulation of otherwise diffuse ranges of meaning" (Fauconnier and Turner, 2003, p. 57). Classical examples for the notion of conceptual blending are new emergent conceptual spaces of boathouses and houseboats. As a process of analyzing two conceptual spaces, the transfer operations, which lead from concepts in source space to new concepts in target space, has been modeled in the framework of the Logic of Determination of Objects (LDO) in (Pascu et al., 2014) in order to build a computational system for its analysis. All of these works are cognitive analysis of conceptual translation, which includes either to interpret old concepts by new ones or to build a new concept by appealing to the combinations of old ones.

In this paper we present conceptual metaphor for teaching logic. From the viewpoint of knowledge acquisition, we investigate the role of conceptual metaphor by understanding logic both as a language and a system of reasoning. Logic as a scientific discipline impacts other scientific education will be highlighted in this paper. Finally, the role of conceptual metaphor in logic education is noted. This paper is organized as follows: The introduction reviews some notions introduced in the literature around conceptual metaphor and conceptual blending. In section 2, we roughly explain the model of conceptual metaphor in Institution Theory. In section 3, based on the LDO model, we analyze some logical notions in natural deduction that has been taught to the students in humanities. In section 4, the conceptual metaphor in inferences of a deductive system is discussed. Finally, a conclusion about the importance of teaching logic will be given.

2. Conceptual Metaphor in Institution Theory

Institution theory is a very general algebraic description by using category theory. In (Diaconescu, *IEP*), a very good and brief characterization of Institution Theory is given.

An *institution* is a *mathematical structure* built initially to model mathematically logical systems. The concept of institution is built from the concepts of algebraic theory of categories. An institution consists of four kinds of entities: *signatures*, *sentences*, *models*, and the *satisfaction* between models and sentences. All these are considered fully abstractly and axiomatically. This means the focus is on their external properties, how they relate to the other entities, rather than what they actually are or may be.

Institution Theory

Formally, according to (Goguen and Burstall, 1984), an institution is a quadruple (Sign, sen, Mod, rel) where:

- Sign is a set of *signatures* with a set *N* of sorts partially ordered by a sub-sort relation.
- sen, sen : Sign $\rightarrow$ Set is a functor building for each signature Σ, the set of its sentences, sen (Σ) and for each morphism $\sigma : \Sigma \rightarrow \Sigma'$, the *sentence translation map,* sen (σ) : sen (Σ) $\rightarrow$ sen (Σ').
- Mod is a functor, Mod : $\text{Sign}^{op} \rightarrow$ Cat that builds for each signature Σ, the category of models, Mod (Σ) and for each signature morphism $\sigma : \Sigma \rightarrow \Sigma'$, the *reduct functor* Mod(σ) : Mod (Σ') $\rightarrow$ Mod (Σ).
- A *satisfaction relation* $\models_\Sigma \ \subseteq |\text{Mod}(\Sigma)| \times \text{sen}(\Sigma)$ for each $\Sigma \in$ Sign, such that, for each morphism σ, $\sigma : \Sigma \rightarrow \Sigma'$in Sign, the following satisfaction condition holds: M' $\models_{\Sigma'}$ $\sigma(\varphi)$ if and only if Mod(σ)(M') $\models_\Sigma$ φ.

The satisfaction condition expresses that truth is invariant under change of notation (and also under enlargement or quotienting of context).

For two logical systems $L_1 = (\Sigma_1, \models_1)$ and $L_2 = (\Sigma_2, \models_2)$ having the set Σ_1 and Σ_2 (of propositional symbols) as signatures, and a function $\rho: \Sigma_1 \rightarrow \Sigma_2$ between such sets as a signature morphism, the functor Mod is a *model translation*.

Model translation, which is interaction between semiotic systems in general, can be seen as the underlying *logical* behavior of *conceptual metaphor* (Pascu et al., 2014), where source domain and target domain are taken as two semiotic systems. As Diaconescu states in (Diaconescu, *IEP*), "the key step was the definition of the concept of institution in (Goguen and Burstall, 1984) intended to capture formally the structural essence of logical systems beyond specific details. Since semantics plays the primary role in formal specification, institutions lean towards the semantics side of logic, known as model theory."

3. Conceptual Metaphor in Logic of Determination of Objects (LDO)

Institution Theory is a very general framework. It can be applied to build ontologies. However, if we model the conceptual metaphor process, then taking the framework of logic of Determination of Objects (LDO) will be suitable and not be too laborious. The LDO was proposed by Jean-Pierre Desclés et al. LDO is a logical system, which take into account a definition of "concept" and "object" by generalizing Frege's (Frege, 1971) notions by providing a formal distinction between "concept" and "object". Moreover, LDO was used to account, in particular, for the distinction between typical and atypical instances of a concept in (Desclés et al., 2011). The primitives of this logic are the concepts and the objects. Inspired from Frege's definitions (Frege 1971), it extends and formalized them in the framework of applicative systems of Curry (Curry and Fey, 1958). The concepts are operators in the sense of Frege (Frege, 1971) and the objects are operands. The whole language of the LDO is an applicative system in Curry's sense (Curry and Fey, 1958). The differences between LDO and the classical logic are: (1) objects in LDO are of two kinds: fully (totally, completely) determinate objects and more or less determinate objects; (2) objects in LDO are typical and atypical; (3) the duality between extension and intension of a concept is not kept.

In (Pascu et al. 2014), based on LDO, we analyze the construction operated by "conceptual metaphor". It is a *complex transfer-operator* pairing from the *source concept—object* space to *the target concept- object* space. This operator is applied to concepts, more or less determinate objects or determinations. It is not only a simple transfer; it can change the category of the operand (i.e. a concept from essence from the source space can become the determination of a more or less determinate object in the target space). In (Pascu et al. 2014), among other examples, the analysis of the example of "boat people" is presented. In this current paper we are interesting only in the structural part of the LDO, that is, the definitions of concepts and objects.

In the mathematical model of LDO (Desclés and Pascu 2019), a concept is a quadruple < f, Ess f, Int f >, where f is a property, Int f a set of properties defining f and Ess f a subset of Int f, a set of properties necessarily defining f. The concepts of Int f are organized in a network as represented in Figure 1. This network ends with the concept f to which is associated τf the typical object totally indeterminate associated to f. Following this, we can obtain the *objects more or less determinate* represented in the part $\mathcal{O}$ of the Figure 1 by *determination* operations. In Figure 1, $\mathcal{F}$ represents the sub-network of concepts and $\mathcal{O}$ the subset of objects. In what follows, we present the mathematical model associated to LDO following (Desclés and Pascu 2019), in Figure 1. To apply this model to conceptual metaphor, we take as semiotic spaces a space ($\mathcal{F}_1$, $\mathcal{O}_1$) as source space and ($\mathcal{F}_2$, $\mathcal{O}_2$) as target space. Between them must define a *translation operator* as in Figure 2. The model of metaphor in LDO consists in considering the two spaces, source space and target space as a LDO spaces of concepts f1 and f2 respectively. The translation of a feature g0 of f1 as feature g'0 of f2 can be done in the following way

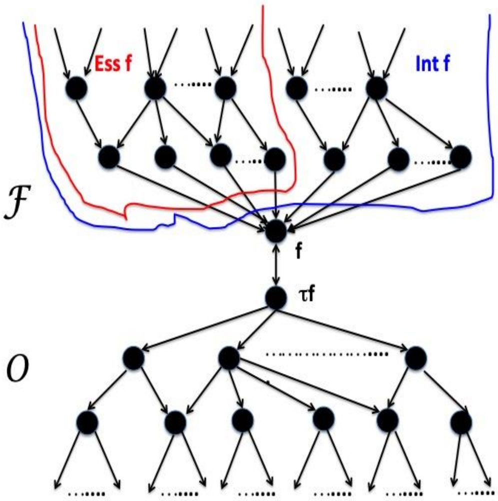

Figure 1. The lattice structure of concept and object in LDO

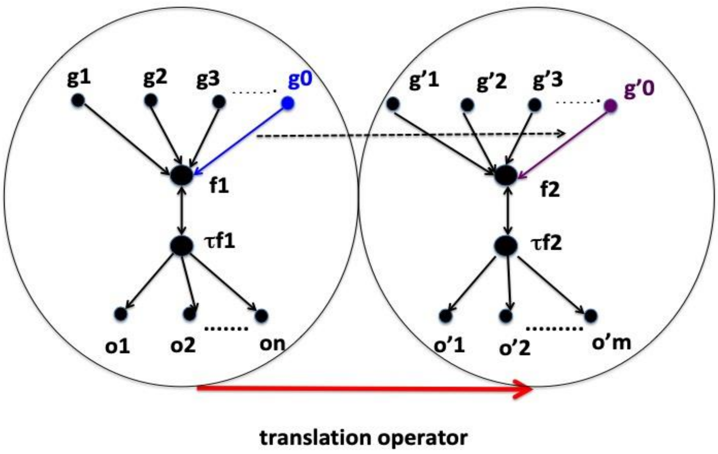

Figure 2. Conceptual metaphor in LDO

- Either g0 belongs to essence Ess f1 of f1;
- Or
- g0 belongs to essence Int f1 de f1 and not to Ess f1 of f1.

Some other possible cases concerning the objects are presented in (Pascu et al. 2014).

4. **Conceptual metaphor in teaching logic**

If we speak about classical logic presented in two forms: *natural deduction* and the *theory of models* we can distinguish the great feature differentiating one from the other. The natural deduction expresses each logic connective by the pair of introduction rule and elimination rule. The theory of models postulates the language, its model and the correspondence between them. Taking our natural reasoning as a logical system, natural deduction can be taken as a conceptual metaphor of our natural reasoning, where natural deduction system can be presented by a class of valid argument forms, e.g. (Tidman and Kahane, 1998). Moreover, the theory of model can be taken as a conceptual metaphor of natural deduction. That is to figure out the meaningful part of natural deduction, called interpretation that is usually used in model theory. The scope of this analysis is the empirical digging of our natural reasoning. By the following example we give an analysis of the two rules, the *rule of deduction (modus ponens)* and the *rule of abduction.* Let us recall the deduction rule (modus ponens) and abduction rule. For deduction rule:

$$\frac{p,\ p \rightarrow q}{q} \qquad (1)$$

Interpretation: for two propositions p and q, if p is true and $p \rightarrow q$, then q is true.
For abduction rule:

$$\frac{q,\ p \rightarrow q}{p} \qquad (2)$$

Interpretation: for two propositions p and q, if q is true and $p \rightarrow q$, then p is *plausible*.

Consider two logical systems L1 and L2 with their corresponding sets of propositions P1 and P2, respectively. $P1 = \{P1_1, \ldots P1_{n1}\}$ and $P2 = \{P2_1, \ldots P2_{n2}\}$, have structures in the form of categorizations. Suppose that the logic L1 is equipped with the rule of deduction (modus ponens as only rule) and L2 is equipped with both deduction rule (modus ponens) and abduction rule. In L1 we can have only the truth values {t(truth), f(false)}, but in L2 we must have several truth values or a mean to express the notion of "plausible".

We can rewrite the deduction rule in both L1 and L2 by:

$\forall q, \forall p, q, p \in P1$, if p is true and $p \rightarrow q$, then q is true (3)

But for the abduction rule in L2, we have:

$\forall q, \exists\, pi \in P2$, if q is true and $pi \rightarrow q$, then pi is plausible (4)

Here we can give a LDO model of conceptual metaphor for the *rule of abduction*:

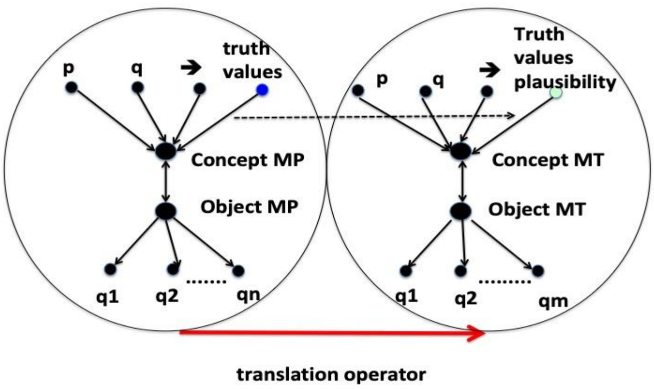

Figure 3. LDO model of conceptual metaphor for the rule of abduction

5. Discussion and Conclusion

We show that the conceptual metaphor is a *logical notion* w.r.t. LDO. What lesson can we learn from this?

From this short study about the notion of conceptual metaphor, we are aware that one of the basic feature of our reasoning in knowledge acquisition is the analogy of features. A kind of analogy operates that can be constructed by a new *logical* system. We can call this type of analogy "conceptual metaphorization". It is a very complex process that can be modeled by a number of mathematical or non-mathematical models.

A child learns a new concept in three forms:

- From a concept already known by a transfer of structural properties;
- From an object already known by transferring functional properties;
- From a known context built with concepts and already known objects by removing and adding properties already known.

We call this cognitive process a *conceptual metaphorization process*. Obviously, the first years of education from preparatory school to junior high school the logic is introduced by the game and a concept is born from the game by a *conceptual metaphorization*. Suppose that logic can be taught at all levels of education, then learning logic can be taken as a conceptual metaphorization process, which means that conceptual metaphorization process can be a framework of learning logic, including learning natural reasoning by natural deduction and learning natural deduction by the concept of interpretation in the theory of model.

In the process of acquisition of a new concept or in the process of understanding a new system of reasoning, learners need to adapt their previous concepts. As stated in (Desclés et al. 2010), *"Logic is the art of reasoning, the discipline of deduction, rigorous demonstrations, the mechanization of proofs ...But logic is also the place of interpretations, of the meaning of utterances, of possible models or worlds. Thus, logic is built in the opposition between syntax and semantics:*
• The syntax is the world of symbols, grammatical operations empty of any content,
• Semantics is the place of interpretations, possible models or worlds, the place of realizations, the place where a meaning is given.
In mathematical logic, we must distinguish between an 'axiomatic' conception of logic, which was that of Frege, Russell, and Hilbert, and a more 'pragmatic' conception in terms of proofs, which we find in systems deduction of Gentzen".

(Our translation) .[1]

For learners, learning this subject is multi-perspective. To understand this learning process is interesting for educators to take suitable metaphoric devices to teach this subject. Nevertheless, to know symbolization as a highly structured device is usually important for learners to understand this subject, where conceptual metaphorization process can be the underlying process from a perspective of cognitive linguistics.

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[1] The original texts:
«La logique est l'art de bien raisonner, la discipline de la déduction, des démonstrations rigoureuses, de la mécanisation des preuves...
Mais la logique est aussi le lieu des interprétations, de la signification des énoncés, celui des modèles ou mondes possibles.
Ainsi, la logique se construit dans l'opposition entre syntaxe et sémantique :
- *- la syntaxe est le monde des symboles, des opérations grammaticales vides de tout contenu,*
- *- la sémantique est le lieu des interprétations, des modèles ou mondes possibles, le lieu des réalisations, le lieu où une signification est donnée.*

En logique mathématique, on doit distinguer entre une conception "axiomatique" de la logique, qui fût celle de Frege, Russel et Hilbert, et une conception plus "pragmatique" en terme d'actes de preuves, que l'on retrouve dans les systèmes de déduction naturelle de Gentzen.

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