A Case Study of Learning by Problem-Posing in Introductory Phase of Arithmetic Word Problems

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Abstract: We have developed an interactive environment for learning by problem-posing targeting arithmetic word problems that can be solved either by addition or subtraction. In our previous work, the environment was used by third or fourth grade students who have already acquired ability to solve the targeted problems. Therefore, problem-posing was an additional practice for the students and the purpose of the learning with the environment was sophistication of their ability. In this paper, practical use of the environment for the first grade students is reported. Just after the classes of problem solving of the arithmetic word problems, the way of problem-posing itself was taught in classes, and then, the environment was used as exercise of the problem-posing. Through this practice, we have confirmed that (1) the first grade students were able to pose problems in the environment, and (2) the practice to pose problems improved their ability not only in problem-posing but also in problem-solving.

Keywords: Problem-posing, Sentence-integration, Teaching method, Problem structure, interactive learning environment

Introduction

Design and practical use of a teaching method in which learners learn problem structures of arithmetic word problems through problem-posing is described in this paper. Learning by problem-posing is well known as an important way to promote learners to master the use of solution methods [1, 2]. Several researchers have already suggested that understanding the problem structure is important to solve arithmetic word problems and poor problem solvers often fail to elicit problem structures from the problems [3-5]. We have continuously investigated technology-enhanced learning by problem-posing in arithmetic word problems and practically used a developed learning environment (we called the environment as "MONSAKUN" (problem-posing boy in Japanese)) for fourth and third grade students in an elementary school [6, 7]. In these projects, although we have defined problem structures of several types of arithmetic word problems, the structures were only used by MONSAKUN in order to diagnose learners' problem-posing or design a series of problem-posing exercise.

Based on the practical uses of MONSAKUN for four years, we (including responsible teach of the elementary school where we have used MONSAKUN) planed to teach the problem structures used in MONSAKUN and to carry out exercises of problem-posing with MONSAKUN. The participants were the first grade students just after they learnt problem-solving of the word problems. Through the practical use, we have confirmed that (1) the first grade students were able to pose problems in the environment, and (2) the

practice to pose problems improved their ability not only problem-posing but also problem-solving.

In previous researches of MONSAKUN, students who have already acquired ability to solve the targeted problems were subjects. Therefore, problem-posing was an advance practice for the students and the purpose of the learning with the environment was sophistication of their ability. In contrast, the subjects of this practice were the first grade students who had classes of problem solving of the arithmetic word problems just before this practice. In this practice, the way of problem-posing itself was taught by a teacher, and then, problem-posing with MONSAKUN was used as exercise to operate the structures.

In this paper, in the next sections, the problem structures and their classification are explained. Teaching method of the problem structures with MONSAKUN used in this practice is explained. MONSAKUN used in this practice were able to be used with a tablet PC and wireless LAN. We call this version as "MONSAKUN Touch". This improvement is indispensable to realize this teaching method in a usual classroom. This improvement is also explained in Section 2. In Section 3, procedure of practical use of MONSAKUN Touch and analysis of the results are described.

1. Problem Structures Used in MONSAKUN

1.1 Problem Structures

We have proposed a model to describe problem structures of arithmetic word problems that are solved by one operation of either addition or subtraction. From viewpoint of calculation, the word problems include two given numbers and one required number. By operating with the two given numbers, the required number is derived. In the model, therefore, a word problem is composed of three sentences, that is, two sentences express two given numbers and one sentence expresses one required number. Then, every word problem has a cover story. In many investigation of arithmetic word problems indicated that there are four types of cover story, (1) increase-change, (2) decrease-change, (3) combine, and (4) compare [8]. These cover stories express one numerical relation between two numbers. The relation corresponds to an operation, that is, addition or subtraction. For example, in increase-change story, there is one number at first, and then, a number that is added to the first number is shown. At last, the number after the addition is shown. For example, in case that "Tom has 3 pencils" is the first sentence, "Tom buys 2 pencils" is the second sentence, and "Tom has 5 pencils" is the last sentence, the three sentences form one increase-change story. Then, the numerical relation in the cover story is 3+2=5.

In MONSAKUN, we have expressed each type of story by using two "existence sentences" (corresponding to the first and the last sentence in the above example) and one relation sentence (to the second sentence). We call the series of sentences as "cover story". A problem is specified by the location of required number. In case of the above example, there are three problems is included in the cover story. For example, if the number included in the first sentence is set to required number, a problem that includes numerical relation expressed as "?+2=5" can be generated. The problem, then, can be solved by "5-2". The equation expressing the numerical relation included in the problem is called "story operation structure", and the calculation that is used to derive the required number is called "calculation operation structure".

In MONSAKUN, a problem is composed of three sentences. Then, a problem is categorized by (a) cover story, (b) story operation structure, and (c) calculation operation structure. Figure 1 shows several existence sentences and relation sentences. By using the Sentence-1, Sentence-5 and Sentence-3 in this order, a problem that is "combine" in cover

story, “5+?=8” in story operation structure, and “8-5” in calculation operation structure is generated.

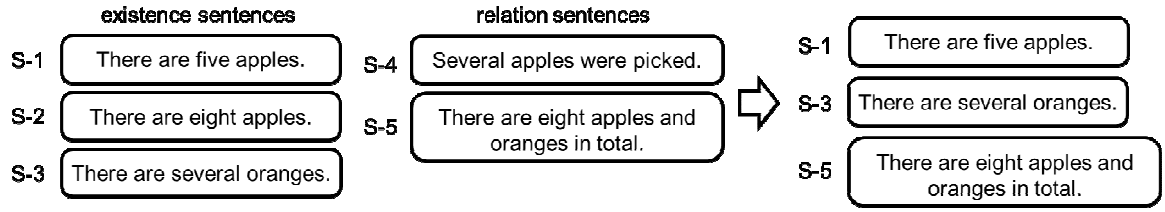

Figure 1. Order of Simple Sentence (Combine Problem)

1.2 Task Model of Problem-Posing

We have already proposed a task model of problem-posing based on the expression of the problem structure [9]. The model is shown in Figure 2. Problem-posing task is divided into four tasks (1) decision of calculation operation structure, (2) decision of story operation structure, (3) decision of cover story and (4) decision of three sentences. A learner should complete these tasks to pose a problem correctly though the execution procedure of the tasks is not decided in the model. So it is important to understand the task model of problem posing for problem solving because this task model means the relations between the problem structures to constitute a problem. In MONSAKKUN, difficulties of problem-posing are interpreted in the task model.

If operator (+ or -) of calculation operation structure is the same one with the calculation operation structure, understanding the cover story is almost same as solving the problem. We call such a problem as “forward-thinking problem”. Then, if operator (+ or -) of calculation operation structure is not the same one with the calculation operation structure, it is necessary to transform the story operation structure to calculation operation structure after understanding the cover story. We call such a problem as “reverse-thinking problem. Because the learner is more required to comprehend the relations between two structures, the reverse thinking problem is more difficult than forward thinking problem.

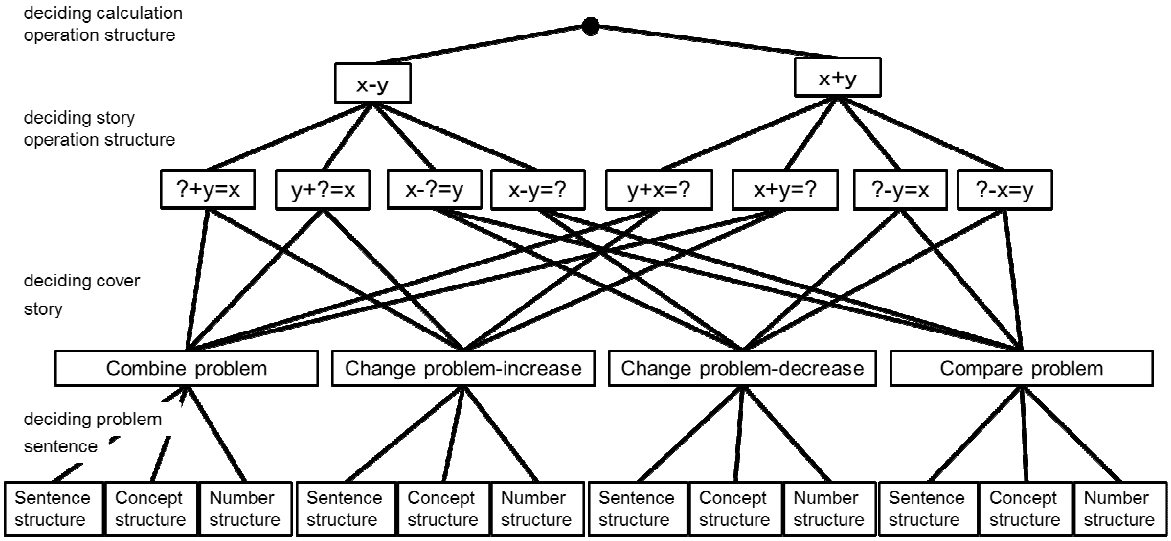

Figure 2. The Model of Problem-posing

2. Teaching Method with MONSAKUN

2.1 Teaching by a Teacher

We have designed a teaching method based on the problem, and we have developed MONSAKUN based on this structure. The teacher teaches the problem structure on the black board by using several sentence cards that are parts of problems. The teacher also prepares several cover story name cards, story operation cards, and calculation operation cards. These cards are provided to the learners as a request of problem posing. For example, the teacher requested learners to pose problems that can be solved by a specific calculation operation. The teacher lets the students pose the problem which will be solved by the prepared calculation expression and story by selecting several sentence cards and arranging them in a proper order.

In this process, the teacher teaches the students the problem structures described in Section 2. This teaching consists of five contents; (1) simple sentence is composed of an object or event, countable attribute and a value of the attribute, (2) problem is composed of two existence sentences and one relation sentence, (3) cover story, (4) calculation operation structure, and (5) story operation structure. (1) and (2) correspond with the problem sentences of the model shown in Figure 2. Firstly, the teacher presents one simple sentence card to students from prepared cards, and he/she teaches the elements of simple sentence. This teaches contents (1). Secondly, the teacher presents the students to one simple sentence card from prepared cards one after another. Then, the learner answers whether presented simple sentence card is necessary to pose a problem or not. They are also made to answer about the reason why a card is necessary. They are also made to answer about the reason why a card is necessary, and the teacher explains a problem structure based on their answer. Through this teaching, the students understand the following; (2) problem is composed of two existence sentences and one relation sentence, (3) a sentence representing each story, a relations among them, and proper order of simple sentences in each story, (4) calculation expression to represent a story directly, and (5) calculation expression to find an answer. The relations among these structures are also taught.

2.2 Interactive Environment for Learning by Problem-posing as Sentence Integration

We have used MONSAKUN at an elementary school for four years. However, MONSAKUN could be used only in a computer room because previous version of MONSAKUN was implemented on the desktop PC platform. In this practice, the responsible teacher hoped to let learners not only to use MONSAKUN as exercise but also to receive lectures of problem structure as usual classes. Therefore, we have implemented MONSAKUN on tablet PC platform so that the teacher was able to use it in the usual classroom. We named it MONSAKUN Touch. In this learning environment, the learner selects the difficulty of problem-posing task before carrying out a problem-posing exercise. After that, a learner is presented the problem-posing area shown in Figure 3 to a learning environment. The area on the left side is problem composition area. At the top, a calculation expression and story is given. Several sentence cards are presented at the right side of the interface. The learner poses the problem by moving a simple sentence card with a finger and putting a card into blank. When a learner finished posing problem, he/she can push a diagnosis button under the problem composition area. Then the system diagnoses the combination of sentences, and shows the results of the diagnosis and message to help the learner's problem-posing on another window. Then the system diagnoses the combination of sentences, and gives messages to help the learner's problem-posing on another window. The messages composed of two kinds of indications, one is indication of correct or incorrect of

the posed problem and the other is indication of wrong cards. Former indication is called Flag Feedback and the latter one is called Pointing Hint [10].

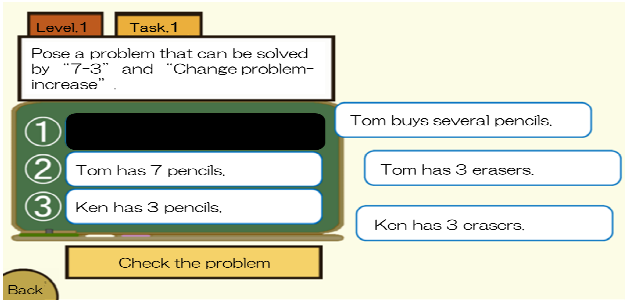

Figure 3. Interface of MONSAKUN

3. Practical Use of MONSAKUN Touch and Teaching Method

3.1 Procedure of Practical Use

The subjects of our experiment are 40 students in the first grade of an elementary school (one student absent from the pre-posttest and questionnaire). The arithmetic word problems are usually taught on the first grade of elementary schools, but the problem structures are not taught explicitly. In this practice, the problem structure of arithmetic word problems used in MONSAKUN Touch were taught explicitly and carried out problem-posing exercise with MONSAKUN Touch as exercise to operate the problem structure. This practice used nine lesson times (45 minutes per lesson, 3 weeks, 9 days). Students took the pretest before the period, and took a posttest and questionnaires after the period. Each test took 45 minutes. Problem-posing exercises divided into 6 levels. Contents of each level are shown in Table 1. The levels categorized by (1) forward-thinking or reverse-thinking, (2) story operation stricture given or calculation operation structure given, and (3) cover story. In a level, students were required to pose problems following provided story operation structure or calculation operation structure and cover story. Cover stories were excerpted from several textbooks. Also, if the student finishes problem-posing exercise in a level in a class, he/she repeats the same level exercise.

In this practical use, students used the MONSAKUN Touch as an introduction of new level problem-posing (5 min) at the beginning of a class. The students, then, are taught the problem structures by the teacher on blackboard (35 min). Finally, they used the MONSAKUN Touch as confirmation of teaching (5 min). The teacher has taught the problem structures and its relations by using the teaching method explained in Section 2.

In pre- and post-test, we used the same problem solving test and problem-posing test. Problem solving test used to assess the students problem solving performance. In problem-posing test, the students are required to pose four problems by composing several sentence cards provided beforehand. This test is used to examine the student's problem-posing performance.

Table 1. Level that Implemented by MONSAKUN

Level	Number of task	Kinds of problem	Kinds of problem-posing task	Story structure
1	12	forward thinking problem	story operation structure	combine· increase· decrease· prepare
2	3	forward thinking problem	story operation structure	combine-increase
3	12	reverse thinking problem	story operation structure	combine· increase· decrease· prepare
4	3	reverse thinking problem	story operation structure	combine-increase
5	12	reverse thinking problem	calculation operation structure	combine· increase· decrease· prepare
6	12	random		

3.2 Analysis of Log Data, Questionnaire and Students Remark

Figure 4 is the rate of correct problems that were posed on MONSAKUN Touch in each class. Vertical axis shows the rate and number of correct problems. Horizontal axis shows the days of practical use and the level that correspond to it. And the number of students that finished posing problems in each level is shown in Table 2. The students performed level 1 and 2 during the 3rd day from the 1st day, level 3 and 4 during the 6th day from the 4th day, and then, level 5 at the 8th day. The teacher has taught the problem structure corresponding to level 5 in detail in the 7th day. The task in level 5 is very difficult for learners, because it requires them to pose reverse-thinking problems from calculation operation structure. Then, problem-posing with MONSAKUN was not carried out in the 7th day and took almost double times for the exercise on the 8th day. In Figure 4, a rate of correct problems increased between 1st and 2nd days and between 4th and 5th days. But a rate of correct problems decreased between 2nd and 3rd days and between 5th and 6th days because the students worked on the new problem-posing task respectively. A rate of correct problems decreased sharply 8th day because the students were required to pose reverse-thinking problems from calculation operation problems as a task in level 5. These results suggested that teaching method about the task to present story operation structure was effective for understanding of forward thinking problem and reverse thinking problem. But it is necessary for teaching method about the task to present calculation operation structure to be improved.

The results of the questionnaire are shown in Table 3. Almost all students agreed that problem-posing exercise by using MONSAKUN and effective to learn, but, we supposed, because of level 5, many students answered the problem-posing is difficult. The teacher agreed that it is easy to teach problem-posing using a tablet PC in the general classroom, and he said that he want to use the MONSAKUN in his class. But, also he suggested that it is necessary to improve the sentence of feedback and to expand the kinds of feedback.

Through this teaching method, the student was required to explain not only by using the block but also by using the problem structures and its relations. For example, the student said that, this problem story is increase so the order of the simple sentence card is decided. Also, they indicated the problem has story operation structure and calculation operation structure. They called story operation structure "story expression" and called calculation operation structure "calculation expression". These results suggested that this teaching method for problem structure using MONSAKUN Touch was effective to learn problem structures and its relation.

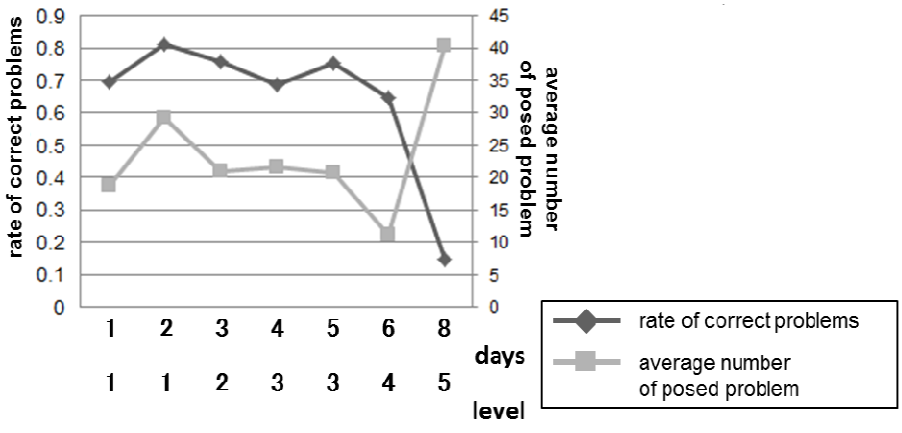

Figure 4. Rate of Correct Problems

Table 2. Number of Students that Finished Posing Problems

Level	1	2	3	4	5	6
Number of students	39	39	39	38	39	23
Number of not finished students	3	1	11	0	17	16

Table 3. Results of Questionnaires

	Strongly Agree	Agree	Disagree	Strongly Disagree
1.Do you enjoy posing problems in arthmetic?	35	3	0	0
2. Are arithmetic problems easy to pose?	8	7	19	4
4.Do you think that posing problems made it easier to solve problems?	20	17	1	0
7.Would you like to attend arithmetic classes where problem posing is used?	36	2	0	0

3.3 Analysis of the Pre- and Post-test

The results of pre- and post-test are shown in Table 4 and Table 5. And the scene of using MONSAKUN Touch is shown in Figure 5. The full marks of problem-posing test are 4. The problem-solving test is composed of 9 forward-thinking problems and 8 reverse-thinking problems. So, the full marks of problem-solving of forward-thinking problems are 9 and the full marks of problem-solving of revers-thinking problems are 8. In the scores of problem-solving test shown in Table 4, there was a significant difference in the scores between pretest and posttest of reverse thinking problems (two sided p-values from Wilcoxon matched-pairs signed-ranks test with correction for ties, p=.009), and effect size is medium ($|r|$=.45). These results suggested that explicit teaching of problem structures was effective to understand the reverse thinking problem. In problem-posing test, there was a significant difference in the between pre-test and post-test as for the number of correct problems at reverse thinking problems (two sided p-values from Wilcoxon matched-pairs signed-ranks test with correction for ties, p=.0006), and effect size is medium ($|r|$=.39). In contrast with this, the number of correct problems at forward thinking problems decreased. These results suggested that the students would be aware of the difference between the reverse thinking problems and forward thinking problems. Based on these results, we have judged that this teaching method with MONSAKUN Touch is a promising way to teach arithmetic word problems.

Table 4. Results of Problems Test (*1% significant)

		forward thinking problem	reverse thinking problem
pre-test	M	8.82	7.13*
	SD	0.6	0.65
post-test	M	8.71	7.66*
	SD	0.39	1.28

Table 5. Results of Problem-posing Test (*1% significant)

	Number of problem-posing	Number of correct problems		Number of wrong problems
		forward thinking problem	reverse thinking problem	
pre-test	3.72	1.54	0.74*	1.44
post-test	3.87	1.44	1.44*	1

Figure 5. Scene of Using MONSAKUN

4. Concluding Remarks

In this paper, we have described the practical use of the learning environment for the first grade students in introductory phase of arithmetic word problems. For using the interactive environment for learning by problem-posing in the general classroom, we have developed a learning environment can use on tablet PC, and designed the teaching method. Then, in this class, the teacher taught problem structures and its relations that were implemented in environment, and the environment was used as exercise of the problem-posing. Through this practice, we have confirmed that the first grade students were able to pose problems in the environment, and the teaching and using learning environment are improved their ability not only problem-posing but also problem-solving in the reverse thinking problem. Also, it is accepted by students and teachers as an effective teaching method. As our future works, monitoring of learners' problem-posing behavior and detection of their errors aiming at remedial feedback for their problem posing is one of the most important issues. Sophistication of the task model of problem-posing and evaluation of learning effect of the teaching method with MONSAKUN is also important future works.

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