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Chaining embodied learning: a system to help develop an integrated understanding of proportion, slope and rate of change

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Abstract: This paper presents the design of a sequenced chain of embodied learning systems, to support students' transition from basic concepts (such as proportion) to more advanced ideas like slope and rate of change, and thus integrate the three concepts. Drawing on Tall's (2003) notion of embodiment as a cognitive basis for formal mathematics, and informed by enactivist pedagogy (Abrahamson et al., 2022), the approach highlights how bodily actions can ground and enrich complex reasoning. In this design, learners progress through a series of embodied experiences that focus on 3 different kinds of understanding - spatial, numerical, and graphical. By sequencing interconnected systems – rather than providing isolated systems – we aim to examine how such chained embodied learning designs influence students' meaning-making across mathematical ideas.

Keywords: proportion, slope, rate of change, embodied learning systems, chaining

1. Introduction

Recent interactive designs open up novel ways to understand and learn mathematics, by allowing students to directly interact with mathematical entities, across topics such as natural numbers (De Freitas & Sinclair, 2014) algebra (Weitnauer et al,. 2016), proportion (Abrahamson & Sánchez-García, 2016), geometry (Nathan et al., 2022), and integers (Elangaivendan et al., 2023). These learning systems also vary in the extent of bodily involvement they invite—ranging from localized touch-screen actions with fingers, to movements of the upper body, to full-body enactments. Such embodied designs allow learners to perform physical actions that parallel mathematical operations. Examples include combining objects to represent addition (De Freitas & Sinclair, 2014), imagining body parts as mathematical entities (e.g. front part of hand till elbows as a line segment; Nathan et al., 2022), and using the whole body as a resource (e.g., walking at varying speeds to experience changing rates; Swanson & Trninic, 2021)). Such enactments provide opportunities for students to visualize, feel, and manipulate abstract concepts, making the meanings of formal mathematics more transparent and accessible, compared to the opaque use of algorithms promoted by dominant text-based modes.

Most embodied learning environments are based on stand-alone activities focusing on a single concept, or various aspects of the single concept. Our work seeks to extend this design approach, by exploring the nature of learning when different embodied systems are connected in a sequence. Specifically, we investigate how learners could transition from a foundational concept like proportion to more complex ideas such as slope and rate of change, through a chain of embodied activities. To traverse this progression, students start with embodied experiences that serve as initial experiences of the formal mathematical understanding. These experiences are then formalised using symbolic structures such as graphs, numbers and equations. Here we present the design of such a sequenced system and illustrate its potential using a vignette from a pilot study.

2. Theoretical Background

Tall (2003) describes three worlds of mathematics—embodied, proceptual, and formal/axiomatic. He argues that while the embodied mode cannot serve as a basis for proof, it is essential as the foundation of human meaning-making. For instance, rather than introducing the limit concept in calculus purely in formal terms to students, Tall (2003) suggests developing meanings of math topics like limits, differentiability, and continuity by allowing learners to interact with the mathematical topics in enactive way before moving to formal mode. Recent embodied technologies enable engaging with mathematical concepts in ways not possible through static, print-based resources. They help overcome cognitive barriers in transitioning across topics (for example, from natural numbers to integers) through interaction with tangible interfaces. In his approach, software tools supporting enactive controls such as zooming and sliding allow students to perceive differentiability through local straightness and continuity through local flatness. These embodied perceptions of local straightness and local flatness serve as cognitive roots for later development of formal ideas of continuity and differentiability respectively (as illustrated in Figures 1 and 2, using Desmos). Such enactments—sliding and zooming along a graph—enable learners to *feel* continuity and differentiability. This allows learners to develop a more meaningful understanding of functions, such as cases where a function is continuous everywhere but not differentiable everywhere (like modulus function in Figure 1).

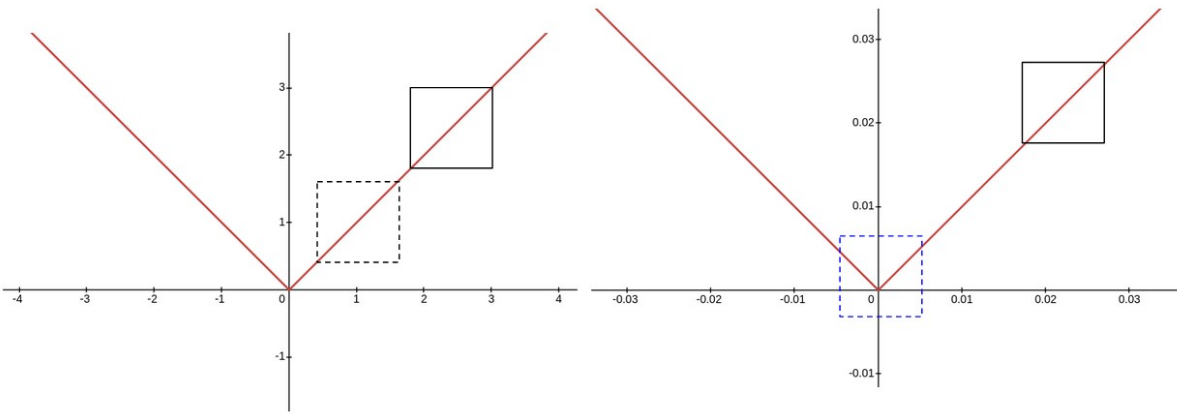

Figure 1. The modulus function y = ∣x∣ appears locally straight everywhere when a square box is slid along its graph (left). However, at the sharp corner, no amount of zooming reveals local flatness. Thus, the function is locally straight everywhere but not locally flat at all points.

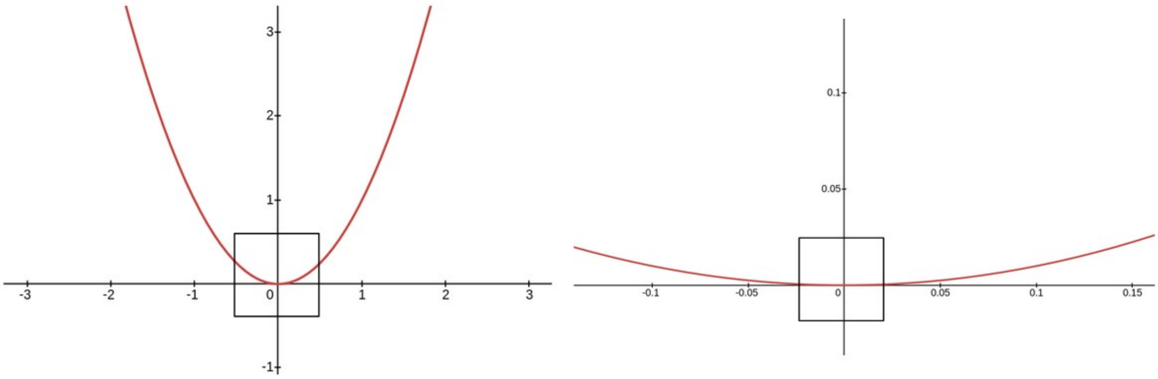

Figure 2. The parabola function on the other hand is locally flat at every point on zooming and sliding along the graph of the parabola function

Recent studies have sought to extend this embodied research perspective to classrooms, through enactivist mathematics pedagogy (Abrahamson et al., 2022). Here, learners begin by developing perceptual strategies in response to a problem, which – through reflection, guidance from teacher and peer discussion – evolve into mathematical concepts. For instance, in a study of learning proportions using an embodied interaction system, students first engaged in embodied understanding, which was then gradually transitioned into spatial and numerical reasoning, through the introduction of formal elements such as cursors, grids, and axes. These helped reframe the initially qualitative discussions in quantitative terms. This approach illustrates how embodied learning systems can be designed to foster the three interconnected modes of understanding mathematics topics: embodied, spatial, and numerical.

Building on this existing work, our project focused on the learning of slope and rate of change. Learners often experience slope physically—for example, when walking up or down a ramp as a measure of steepness—while in functional graphs slope represents rate of change. These two meanings of slope may cause difficulties in understanding the concept of slope. Hoban (2021) emphasizes that deep understanding of slope requires integrating multiple external representations (MERs), graphical reasoning, concepts of ratio and rate, and proportional reasoning (multiplicative thinking).

Most existing embodied systems for learning mathematics topics are standalone systems. We hypothesize that students experiencing a connected trajectory of related mathematical topics—in our case, proportion, slope, and rate of change—through a chain of embodied learning systems, could lead to deeper mathematical understanding. Such a design could enable students to integrate elements from their prior embodied experiences with new mathematical contexts. The exploratory experiences offered by existing standalone systems often vary across topics. By maintaining a consistent design and pedagogy thread across related topics, we expect our design to help students integrate ideas across the systems. Our central research question, therefore, is: Does engaging with a chain of embodied learning systems support students in developing an integrated understanding of related mathematical topics, and how does student discourse reflect such integration?

Drawing on this perspective, our embodied learning design begins with bodily experience, to ground students' initial understanding in qualitative experience (Tall, 2003), rather than opaque formalism. The design also incorporates principles of enactivist pedagogy (Abrahamson et al., 2022), linking embodied, spatial, and numerical forms of reasoning. While current embodied systems/ environments for proportion (Abrahamson & Sánchez-García, 2016), slope (Abrahamson et al., 2021), and rate of change (Boaler et al., 2016,Swanson, H., & Trninic, D. (2021)) exist as stand-alone environments, we hypothesize that sequencing them into a coherent chain will enable students to transition across these topics smoothly. We propose that such chaining, rooted in embodied experience, will support learners in making sense of formal mathematics when it is later introduced, thus providing 'cognitive roots' (Tall, 2003).

3. Methods

In the pilot study reported here, participants were a group of Class 7 students from an ICSE school in Maharashtra. The initial sample consisted of five students (n = 5) for the first two days. However, due to a flu outbreak at the school, participation later reduced to three students (n = 3). We selected participants based on their interest in the study and parental consent. The intervention spanned 5 days (with 4 days of intervention), with daily sessions of about one hour each. Across these sessions, students participated in a purposefully designed sequence of embodied learning activities, intended to help students integrate proportional reasoning with an emerging understanding of slope and rate of change. Prior to the sessions, students were given a demonstration and instructions on how to carry out the activity. Students were encouraged to think aloud, expressing their thoughts and strategies. Peers were expected to examine their reasoning for correctness and offer challenges. Students' reflections led to discussions and strategies related to concepts such as ratio, slope, and rate.

3.1 System and study design

On Day 1, students engaged with our lab-based version of the Mathematics Imagery Trainer (Abrahamson & Sánchez-García, 2016) using motion sensors. The screen turned green when the correct 1:2 proportion was maintained. Students explored how to coordinate their hand movements, identified multiple positions where the condition held, and practiced sustaining proportional motion, laying a bodily foundation for proportional reasoning. On Day 2, students used the touchscreen version of the same system, moving two fingers along vertical columns in fixed proportions. Unlike Day 1, they encountered varying ratios and repeated the same activity across different proportional conditions. On Day 3, the activity was *Graph the Walk*. Our system recorded 30 seconds of walking data and displayed movement count–time graphs for discussion in both student and teacher systems. With the support of a 60 BPM metronome, students walked at different speeds (1–3 steps per second), then matched graphs to walkers, justifying their reasoning through speed and graph shape. The next activity was *Walk the Graph*, where they were shown a custom target graph to recreate through walking. They planned and executed their walks, compared the resulting graph with the custom graph, and reflected on strategies and challenges in aligning embodied action with expected graphical representations, supported by temporal cues. On Day 4, Graphing Metronomes was introduced. Students adjusted beats per minute (BPM) using a slider while the system displayed a real-time cumulative beat graph. After free exploration of how BPM affected the graph, they attempted to reproduce custom graphs and discussed how their manipulations shaped the resulting representations.

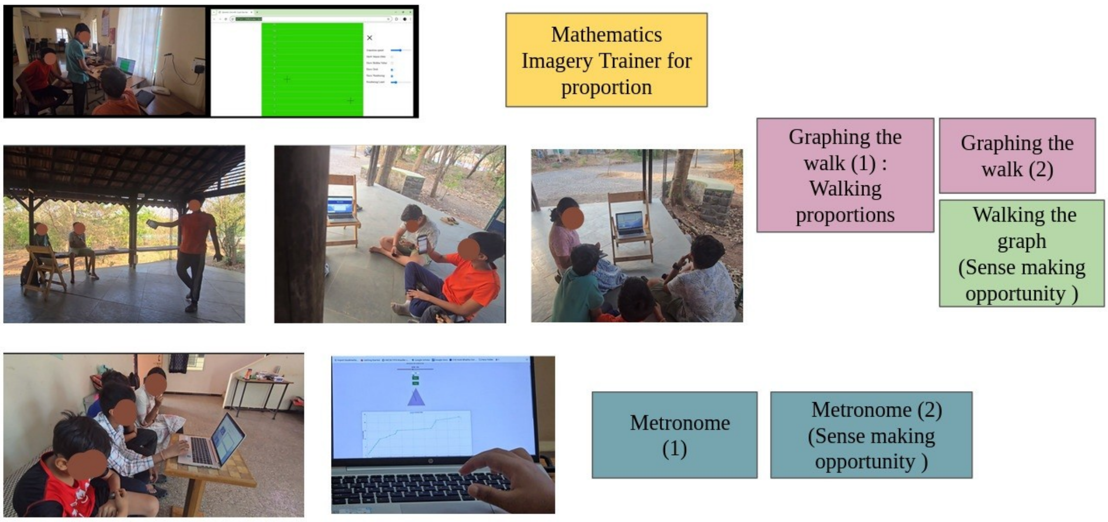

Figure 3. Day wise intervention task

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4. Discussions

On Day 1 and Day 2, students worked with ratios using Mathematics Imagery Trainer, comparing two quantities of the same kind, namely the distance from the bottom of the sensor. Studies have shown that students develop proportional reasoning through goal-oriented body movements, which can then be transitioned to formal mathematical discourse by introducing elements such as cursors, grids, and numbers (Abrahamson & Sánchez-García, 2016). For instance, students might articulate that "for every one box on the left, two boxes must be taken on the right." The phrase '*for every*' plays a key role in fostering multiplicative comparison between the two sides (Hoban, 2021).

Building on this foundation of ratio, the Day 3 activity introduced students to a special case of ratio, namely *rate*, where one type of quantity (movement count) is compared to another (time in seconds). In the first activity of Day 3 activity, students were asked to walk at varying speeds and then match the resulting graphs to the individuals who had generated them, justifying their reasoning. The graph below (Figure 4) was produced by Inu, Avi, and Karthik (names anonymized). Selected phrases from the episode, along with the corresponding analysis, are presented in table 1 (link to transcript)

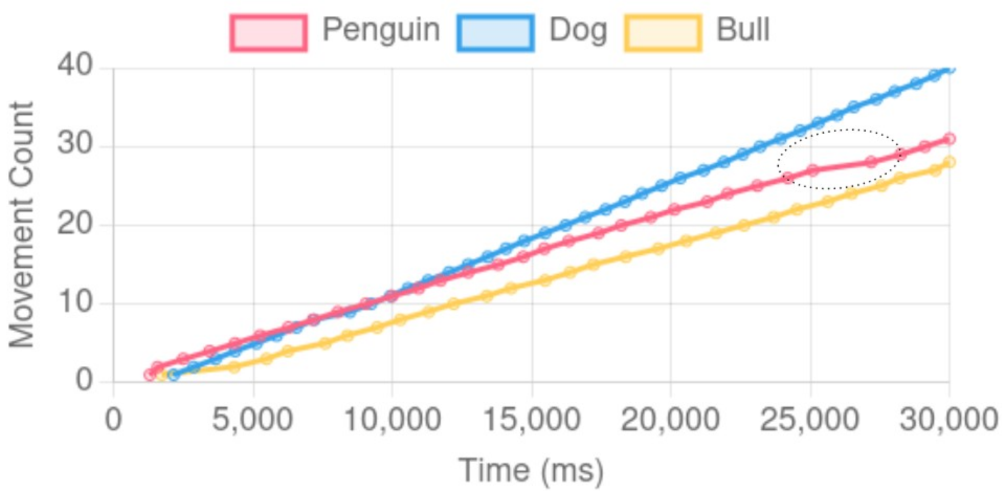

Figure 4. Graphs generated by Avi, Inu and karthik through their movement s at varying speed

Table 1. *Table of analysis*

Discourse	Analysis
Karthik: Because Inu walked faster. So, that's why that blue line is more like up (upward slanting gesture). So, he has taken more steps.	Here, the words *faster* and *up* are connected by the student, meaning that the faster the movement, the steeper the graph will be. The learners are thus able to link the bodily experience of *faster* with the graphical representation of *steeper*
Avi: Inu took 40 steps in 30 seconds, i took 30 steps in 30 seconds and Karthik took 28.. or 29 steps	The presence of the x- and y-axes in the system enabled students to summarize and compare each participant's movement counts, adding a quantitative dimension to the discussion.
Avi: I remember in my graph, when I was looking at the orange (colour of the graph in the student system), in the end, there was a little down. *Instructor 1: Why do you think that happened?* *Avi: Because in the end, I decreased speed.*	Avi interprets the graph in relation to his own walk. This is evident when he refers to the dip at the end (the dotted circle in the pink graph in figure 4) as the representation of a reduction in speed.

Apart from the slope of the graph (in the presence of the coordinate axes) as a spatial understanding, this idea was also evident in the actual distances the students walked. However, they did not explicitly discuss this. The researcher could have initiated this conversation, but this was recognized only later during the analysis. For instance, Avi walked twice along the corridor, Inu walked three times, while Karthik walked only once. Bringing attention to this might have led to a different kind of discussion, which can be incorporated into future studies. Based on this instance, we hypothesize that students' conversations and experiences need to be braided together by the instructor, to integrate the embodied, spatial and numerical understanding, as illustrated in Figure 5.

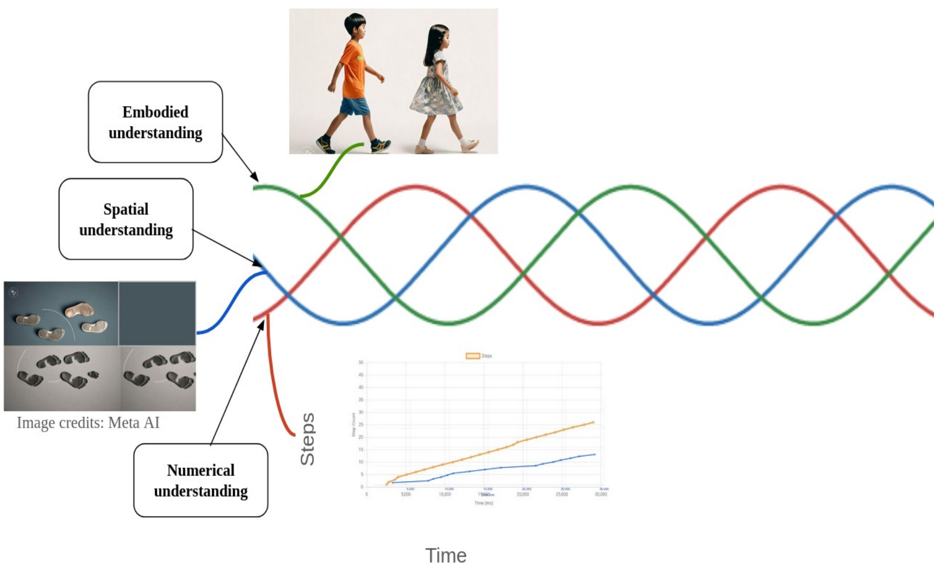

Figure 5. Illustration of braided understanding across embodied, spatial and numerical dimensions in the 'walk the graph' and 'graph the walk' activities.

In the second activity of Day 3, students moved from walking at assigned speeds to free exploration. Finally in the last activity they were challenged to reproduce the shape of the given graph (which we term as custom graph) through their walking. The graph below (Figure 4) presents the custom graph presented to the students (pink) and the graphs produced by Inu, Avi, and Karthik (see appendix 2 for the entire transcript).

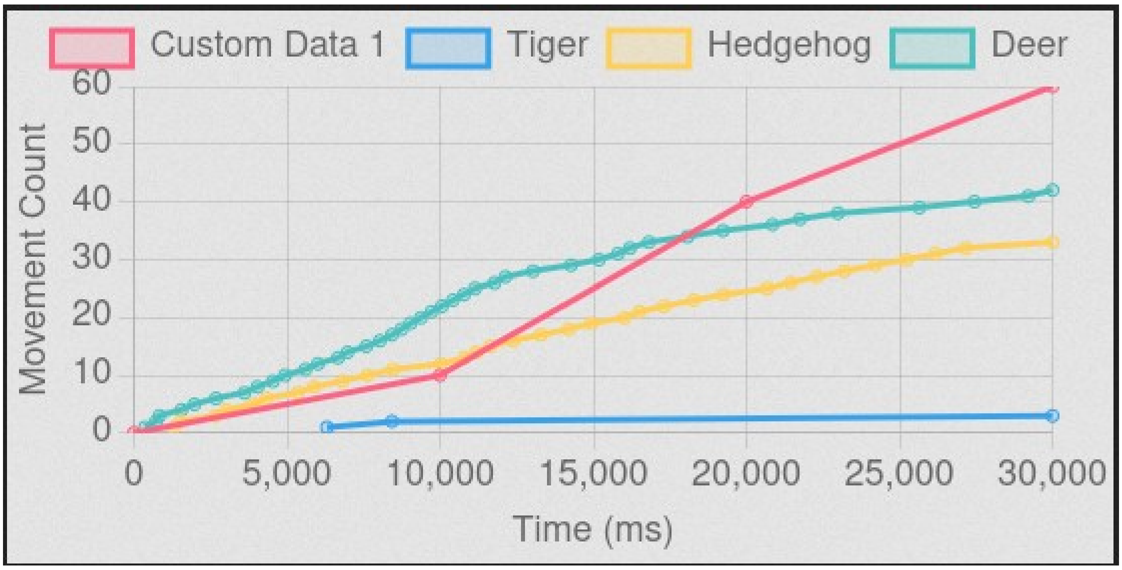

Figure 6. Student generated graphs based on their analysis of the custom graph and corresponding action plan.

As soon as the custom graph was presented, Karthik remarked "*bhaag bhaag*" (run run), while Avi added, "we need to jog and then sprint." Although Avi had not generated this graph, he was immediately able to interpret it in terms of movement. He observed that the first 10 seconds were less steep than the next 10 seconds, leading him to infer that while both segments required continuous motion, the second part demanded a faster pace than the first. By the end, the group identified that in the first 10 seconds they should take 1 step per second, in the next 10 seconds 3 steps per second, and in the last 10 seconds 2 steps per second. This was based on their observation that the graph represented 10 steps in 10 seconds, 30 steps in 10 seconds, and 20 steps in 10 seconds, respectively. In doing so, the children converted these observations into unit ratios to plan their actions. During this activity, students also engaged with phrases such as "1 step per second" and "3 steps per second," where the term *per* served a role similar to "for every," supporting multiplicative reasoning rather than additive strategies. Such proportional reasoning is critical for developing an understanding of slope, and thereby, the concept of rate.

5. Limitation and future work

We summarize below the design limitations revealed by the study, and the planned revisions to address these.

1. The study aimed to explore interconnected understanding across the chain of systems. However, our current prompts and questions did not sufficiently elicit this interconnectedness in students' thinking. We have revised and refined the prompts to better probe such links.
2. This study did not include pre- and post-intervention assessments. The analysis of the pilot data revealed that such assessments could offer valuable insights, particularly related to shifts in students' thinking. We are currently developing such assessments to include in future studies.

6. Conclusion

The pilot study indicates that our chained system of embodied interactions creates opportunities for students to integrate mathematical ideas, using a trajectory that begins with ratio, progresses to graphs where they encounter the notion of slope, and ultimately leads to conversations about rate of change and its relation to proportional reasoning. Ongoing work examines how students move back and forth between embodied experiences, spatial reasoning, and numerical representations, to make sense of problems and develop solutions. We also aim to explore in future studies how learners transition across different activities within this chain of embodied learning experiences as they construct their mathematical understanding. The pilot study indicates that the task and pedagogical aspects of the interventions (the embodied lesson plan) would need to be refined and systematically designed, to better probe and support such dynamic movements. While the system offers valuable affordances for both teaching and learning, its effectiveness relies on thoughtful pedagogical facilitation. We thus view the teacher's participation as central. The design serves as a tool to support – not replace – the teacher in helping students develop an integrated mathematical understanding.

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