The Effect of Learning Spatial Geometry By Mobile Devices

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Abstract: The aim of this research is to develop a hands-on spatial geometry learning system to facilitate students' geometry learning. The system is developed with Duval's four critical elements of geometry apprehension including perceptual apprehension, sequential apprehension, operational apprehension and discursive apprehension. The system supports senior high school students in the process of spatial geometry problem-solving, allowing them to hands-on manipulate spatial geometry graphics and develop their visualization and mental imagery. In total, 58 participants from different classes were recruited. The experimental group used the hands-on learning system, whereas the control group followed the traditional paper-based approach. The study investigates the effects of the hands-on geometry learning system on students' perceptual apprehension, sequential apprehension, operational apprehension, overall spatial geometry scores and learning attitude. The results revealed more learning attitude, and higher perceptual apprehension, sequential apprehension, operational apprehension in the experimental group.

Keywords: spatial geometry, mathematics, mobile devices, mental imagery, imagery and visualization.

Introduction

The visual dimensions of mathematical learning and the value of visual-spatial thinking increasingly have been acknowledged as essential to mathematics education (Clements & Battista, 1992; Zimmermann & Cunningham, 1991). Conceptualization of geometrics is the basis to develop the mental ability to think in abstract terms by using real-world objects as examples to learn to think in terms of abstract objects and finally postulate and prove formally defined objects (Zhou, 1999; Do & Lee, 2009). Spatial ability is a critical skill in geometric learning. Several studies investigate how to improve spatial abilities (Hannafin, 2004; Hannafin, Truxaw, Vermillion, & Liu, 2008; Kaufmann & Schmalstieg, 2003, Yang & Chen, 2010).

Actions of dissection, rearrangement and recomposing promote the imagination and logical thinking through observation and analysis (Clements & Battista, 1992). However, in real class interactions, teachers often are not able to holistically present visualized views to students because of the lack of props as useful supplements, creating misinformation for students. They are often restricted by the formal definitions as well as the logical order of operations putting them in a position that focuses less on the meaning of geometry that begins with observation, operation, categorization, and organization (Battista, 1994). A mathematical concept or problem presented only in textual data are often overtly abstract and cannot be easily understood (Guttiérez, 1996). When teachers focus strictly on mathematical logical signs and describing them by their definitions, concepts, or properties, it might not be effective to help students build a thinking system to construct a spatial image and to manipulate it when trying to solve a problem in 3-D geometry (Garrity, 1998; Gurny, 2003). Skemp (1987) pointed out that, by directly introducing definitions and formula to teach mathematical concepts might be the most concise way for the teachers; however, it is often at the detriment of their students. When acquiring a brand new concept, most students start their first step from the actual scenario of situations (Anna Sfard, 1991).

Krutetskii (1976) had pointed out that reasoning based on visualization was not unique. Different students can visualize the same problem in different logical reasoning ways. Focusing on just one single point of view will make a stalemate (geometrical rigidity) (Hoz, 1981), a state where students dwell around one single point of the visualized geometric image, unable to reason from different viewpoints.

Presmeg (1986) had suggested the following points for the visualization:

i. Single sample of the illustration was often misleading.
ii. Standardized images would easily limit students' understanding of the non-standardized geometric shapes
iii. Fixed images that cannot be manipulated could limit students' ability to reason

Without focused and detail-oriented reasoning analysis, it was often ineffective for students to understand the reasoning process. Even through, it made misunderstanding for students.

Previous studies on spatial geometry emphasized geometric learning activities should focus on the actual observations and operations (Bishop, 1989; Grand, 1990). Duval (1995) stressed the complex or abstract mathematical concepts specific to visually present (for example: space coordinate system), or the entity manipulation aids, inspiring students to develop their spatial visualization. Bishop (1980) believed there are two particular abilities suitable for dealing with geometric objects. The first ability is to interpret the image information. It is involved in visualizing process, one can manipulate and converse the visual representation with the corresponding mental image. The second one is the ability to manage the visualization process. This is an ability to understand how to use shapes, diagrams, and descriptive narratives to present geometry concepts. Clements (1979) emphasized that learners with good visualization and mental imagery would be able to observe different perspectives of objects, judge and manipulate the mental images of those objects.

Duval (1995) discovered that figure through the process of manipulation, description, and reasoning can deepen our understanding of the four critical elements of geometry apprehension (perceptual apprehension, sequential apprehension, operational apprehension and discursive apprehension). Perceptual apprehension is that which we recognize and distinguish properties of shapes. Sequential apprehension is through construct figure from properties. In this case the figural units depend not on perception but on technical constraints (e.g. ruler and compass, primitives in computer software) and mathematical constraints. Discursive apprehension is that we compose family of figures and analyze families of figures. In any geometrical figure the perceptual recognition of geometric properties depends on discursive statements, via descriptive words to explain the properties of geometric system and to reason based on these descriptive constructs. Operative apprehension depends on the way in which a given figure is modeled, for example by dividing it into parts, or by transforming it optically or changing its orientation in the plane. These modifications can be performed mentally or physically. Duval (1995) suggested that operating on a figure in certain ways is critical in the heuristic process and that pupils need to be taught how to do this. However, operative apprehension is not independent of the other apprehensions, discursive and perceptual apprehensions very often obscure operative apprehension.

It is suggested that work with computers may support the development of sequential apprehension and it also might encourage the development of operative apprehension if the software has been designed with this in view (Duval, 1995). Therefore, students will have more sufficient experiences in manipulating geometric shapes and visualizing the figures in order to develop the ability to reason from geometric figures. Computers can be the supplemental tool in assisting students to experiment, trial out, and comprehend the properties of geometry (Lin, C. P., Shao, Y. J., Wong, L. H., Li, Y. J., Lin, C.P. et al., 2011). Moreover, when conducting geometry instructions, teachers should emphasize on how to develop students' ability in visualizing geometric spatial relationships as well as presenting the concepts both holistically and independently (Guttiérez, 1996). Using computers to provide multiple angles of the diagrams, in additional to being able to freely manipulate, and observe geometric relationships can facilitate visual geometry learning and teaching.
Following the advances of technologies, multimedia aided learning system has increasingly been the focus in the pedagogical area. Dixon (1997) discovered that GSP (The Geometer's Sketchpad) had helped grade eight students in understanding conceptual constructs of rotation and mirror images of geometric diagrams. Berta Tünde (2002) believes that computers can present different visual forms of geometric structures and characteristics which benefited students in solving geometric problems. Osta (1998) discovered that computers can provide a rich interaction between learners and the diagrams. Chang, et. al. (2007) proposed that multimedia materials have an evidential effect on students' ability to visualize, analyze, describe, reason, and organize geometric information. The National Council of Teachers of Mathematics even suggested to utilize interactive computer software applications for students to learn geometry (NCTM, 2000).

Since information technology allows people to communicate concepts of multiple views via 3-D that are difficult to convey with traditional 2-D illustrations, it can help students develop the ability to think geometrically and sharpen their skills on geometric studies. Moreover, many researchers believe that when students transit to an area of higher mathematics, the ability to visualize spatial relationships becomes even more important since the complexities of visual recognition would have increased dramatically (Smith, 1964; Fennema, 1977). It highlights the importance of spatial geometry teaching and learning.

Compared to personal computers, mobile devices (PDA, Tablets, etc.) are portable and capable of performing calculation tasks which make them ideal as supplementary tools for classroom learning (Trimmel & Bachmann, 2004; Hennessy, 2000; Sung, Y. T., Chang, K. E., & Wu, L. J., 2007). They enable one to interact directly with what is displayed, rather than indirectly with a cursor controlled by a mouse. Secondly, it lets one do so without requiring any intermediate device that would need to be held in the hand.

According to these reasons, this research has utilized mobile devices to develop interactive geometric learning practice system, allowing students to actively perform operations on the live shapes as a method of practicing, guiding students' cooperation and discussion to elevate the students' spatial and geometrical learning results.

1. Hands-on Spatial geometry learning activity and GeoPlay system

GeoPlay (Spatial geometry hands-on learning system) and its spatial geometry learning curriculums are designed based on the four elements of logical reasoning Duval (1995) had proposed. In order to meet the learning needs, the experimental tools developed in this research have been used in a classroom setting helping students to immediately apply and build their logical reasoning roadmaps after the instructor has provided formal instructions. The research focus on students' actual practice to facilitate them to develop the other three apprehensions. The learning activities are as follows:

- Perceptual apprehension vs. questions with text-format: When learners click the sample exercises, the first problem will be presented as text format (Figure 1). As in the general text questions, learners can think and generate answers accordingly from the description of questions. Their perceptual apprehension will form through learners' understanding of the text description. They can solve problems by their mental imagery, however, their mental imagery is not necessarily the same as the given graph, which may have the wrong perception of understanding. If learners choose to answer the question in this stage, the system will immediately give feedback after learners sent the answer. Learners may share the feedbacks with peers or teachers.
- Sequential apprehension vs. questions with figures-format: Younger students might particularly face greater challenges in understanding abstract geometry concepts or problems simply by giving them text descriptions. The process of individuals constructs a figure by using visualization tools (such as paper and pen, computers, or generating image in mind) to aid in analyzing, interpreting, and learning concepts is a kind of sequential apprehension. Constructing figures can be used as a tool to understand abstract geometric concepts (Yakimanskaya, 1991) or to reduce the individual burden of the working memory area. Therefore, learners can choose to click the "Figure" button to transfer text-format questions into figure-format through GeoPlay system (Figure 2). And they can use handwriting function to mark some important key information on the figures. If learners cannot convert the text description to the figures; instead of, they can create their own visual image assisted by given figures.
- Operational apprehension vs. questions with dynamic manipulation: Duval (1995) observed that the figure can be manipulated, and through changing the figures in different ways, learners will get operational understanding to help them solve difficult geometry problems. Operative apprehension is a kind of cognitive process that individuals transfer figures to mental imagery. These modifications can be performed mentally or physically. Thus, if learners still cannot think and reason through the figure-format questions, they can click the "Manipulate" button. System will show the solid geometry components constructed form Cabri 3D which can provide learners manipulate by touch pen (Figure 3). Through manipulating the geometry components (changing the size or its orientation in the plane), they will easily start to think and reason possible answers of the questions. Learners also can improve their skills of mental imagery and visualization through their manipulating to the solid geometry components.
- Discursive apprehension vs. solving questions with dynamic reasoning: Mathematical properties represented in a drawing cannot be determined through perceptual apprehension. A graphic without denomination or hypothesis is an ambiguous representation, so that not everyone will see the same things or the same properties. A graphical presentation of identification will affect the individual perception. That is, although individuals can simply perceived by the figure nature of some of the geometric, but may be wrong. Therefore, some must first be given through speech (denomination and hypothesis) and others can be derived from the given properties. In other words, the discursive apprehension of individual of the figures is not based only on perceptions of figure symbols have yet supplemented by narrative text. Discursive apprehension is the cognitive processes that individual describes a figure through language or narrative text or use the text representation to reason. Therefore, students can click the "Solution" button and show the Flash animation walkthrough reasoning problem-solving process to help learners build their own reasoning abilities.

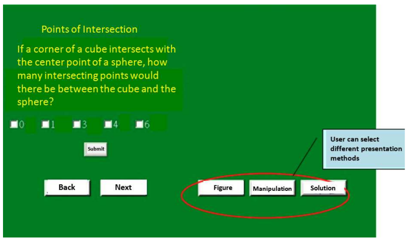

Figure 1. Text presentation of Question

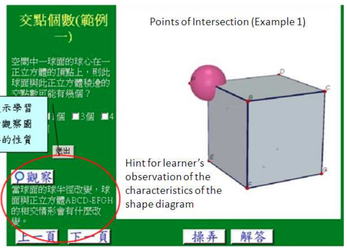

Figure 2. Pictorial Description of the geometric figures

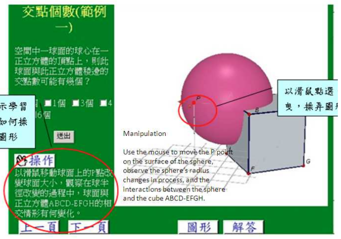

Figure 3. Manipulation of the geometric figures

2. Methodology

2.1 Participants

In order to determine how effectively the Geoplay improve the ability to think geometrically among senior high school students, the study uses the pre- and post-test quasi-experimental design. The study selected two senior high school classes in Taipei City which divided into an experimental group and a control group. There were 31 students in the experimental group (19 boys and 12 girls) and 27 in the control group (13 boys and 14 girls).

2.2 Experiment

A quasi-experimental design was used in which the independent variable was the group (control group or experimental group). The dependent variable was the post-test score for each learning part and the pretest score was the covariance. The experimental group used Geoplay to practice the activities of geometric learning while lecture-based instruction was used for the control group. Moreover, experimental and control group had the same teacher and learning material. After the experimental treatment, the ANCOVA analysis will use to evaluate the significant effects of different learning outcomes.

2.3 Material

2.3.1 Handheld mobile learning software for spatial geometry

The study selected three units from senior high school spatial geometry curriculum, space coordinate system, the spatial relationship between line and plane, spatial intersection between shapes. The students in the experimental group practiced with Geoplay system. Each student was required to complete every activity.

2.3.2 Pre-test and post-test

This experiment involved conducting tests to find out the changes in students' geometry ability after practicing with every activity. The questions were selected from the three units and organized into pretest and post-test. The questions in the pretest and post-test were based on the same concepts and belong to the same question types. There were 7 questions about perceptual apprehension, 11 questions about the sequential apprehension, and 7 questions about operational apprehension, total 25 questions. The questions about perceptual apprehension aimed to assess students' basic space mapping capabilities. This part of the composition is unique and different students will not have much different ideas in the graphics. The questions about sequential apprehension were to assess students' visual ability (visualization) of geometry figure. This part of the composition is not unique, different students may have different ways of composition. The questions about operational apprehension tested students' ability to manipulate graphics in geometry problem solving procedure. Students are required to reason according to the description of questions, based on composition, reasoning and then manipulation of visual graphics, the last inference the answer.

2.4 Procedure

Prior to the experiment, both groups were conducted a 40-min pretest, after which the formal experiment was performed. The students in experimental group practiced geometry learning activities by using GeoPlay, and control group of students involved in a lecture-based instruction. After completing the experiment, each participant was given a post-test.

3. Results

To understand the effect of "Spatial geometry hands-on learning system" (GeoPlay) on students learning of spatial geometry, the single-factor analysis of covariance (ANCOVA) was used to analyze the change of students' pretest and posttest scores. The pretest scores as a covariance to analyze the difference between posttest within the two groups. The results were divided into four part (perceptual apprehension, sequential apprehension, operational apprehension, and overall spatial geometry scores), and were analyzed if posttests in experimental group and control group had significant difference in each part.

The test scored one point for each question, perceptual apprehension part contained 7 points, sequential apprehension part contained 11 points, and operational apprehension part included 7 points, total 25 points. Table 3 lists the pre- and post-test scores of experimental group and control group.

Table 3 Average and standard deviation of test scores for the experimental group and control group in the pretests and post-tests

Groups	N	Perceptual	Sequential	Operational	Overall
Experimental	31	4.61(1.75) 5.61(1.38)	8.45(1.63) 9.45(1.43)	2.79(1.38) 3.29(1.42)	15.86(3.27) 18.36(3.47)
Control	27	4.26(2.57) 4.82(1.98)	7.96(1.81) 8.48(1.76)	2.09(1.46) 2.17(1.49)	14.32(4.91) 15.46(4.38)

The mean and SD values of the posttest in both experimental group and control group had improved after experiment treatment (see Table 3). Thus further analysis of pre- and posttest in both groups to investigate whether there is a significant difference between the two groups.

One-way ANCOVA was used to identify significant differences between experimental and control group post-test scores for perceptual apprehension, sequential apprehension, operational apprehension, and overall spatial geometry scores after eliminating the effects of the pretest scores. First, the homogeneity of the regression coefficients of the total post-test score and post-test score for each part were tested. The F values for the homogeneity of the regression coefficients on the post-test scores for the four parts match the basic hypothesis of the homogeneity of the regression coefficient ($F (1,54) = 1.77, 0.07, 0.15, 0.01$; $p > .05$). ANCOVA could be used to examine these data. The result of ANCOVA showed the scores in perceptual apprehension were not reach statistical significance ($F (1,55) = 3.71$, $p=.06>0.05$) between experimental group and the control group, but the result in sequential apprehension, operational apprehension and overall spatial geometry scores were significantly higher ($F (1,55) = 4.23$, $p=.05$; $F (1,55) = 4.701$, p=.034<.05; F (1,55) = 8.111, p=.01<.05) in the experimental group than in the control group.

4. Discussion and Conclusions

First, the result showed that the neither experimental group nor the control group showed a significance in their perceptual apprehension. The cause of the inconsistency may be that students may solve problems with visual approach, or even they solve problems directly using memorized formulas. Therefore, there is no significant difference on perceptual apprehension between the experimental group by using system and the control group by using paper-based instruction. Second, there are significant differences in sequential apprehension between two groups. This finding is consistent with the researches of Battista (2002) and Clements (1997). GeoPlay guides students in developing visual thinking by means of presenting illustrations of a variety of objects. This method is equally beneficial for developing a student's visual thinking through the use of physical teaching tools to guide the student's geometric thinking. Third, part of visual manipulation, most are more complicated geometric problems, requiring much higher level of multi-cognitive analysis. Students must reason and solve problems by using mental and visual method (visual imagery). Students must use Duval's (1995) operational understanding of the visual image manipulation to reasoning problem-solving. The results showed that performance in operational apprehension part of the experimental group and control group, significant differences can be inferred. By using the "Spatial geometry hands-on learning system" (GeoPlay), there are positive effects on spatial geometry teaching. It's also helpful for the operation of the visual imagery of students' problem-solving. It is in line with Osta (1998) presented that interactive operating environment is conducive to the development of the visual capabilities, such as mental imagery and visualization. Fourth, the Overall spatial geometry scores showed that the experimental group and control group were significantly different. It can be inferred using the "Spatial geometry hands-on learning system" (GeoPlay) can facilitate spatial geometry teaching. Also, through the dynamic manipulation environment provided by the system, the mechanism helps to improve students' spatial geometry learning. It is in line with Battista (2002) theory of geometry learning, and also consistent with previous research investigated that computer interactive learning environment can assist the development on spatial skills (Berta Tünde , 2002; Dixon, 1997). Sixth, the results of the learning attitude questionnaire showed that students hold a positive attitude about the use of "hand-held mobile learning tool in the spatial geometry learning system" (GeoPlay). Most students showed the positive agreement for the use of computers to more clearly show three-dimensional geometry materials, and the manipulation of graphics also help build spatial concepts. In particular, the ratio of up to 93.6% considered that computer images can be shown more specific in three-dimensional geometry, 80.6% of the proportion thought that "hands-on manipulation" will be the best way to help understand the three-dimensional geometric problems, which show that the system can indeed provide effective support in the three-dimensional geometry learning.

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