# Variation Based Discovery Learning Design in 1 to 1 Mathematics Classroom

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**Abstract:** Learning retention is an important issue for instruction. However, traditional mathematics teachers tend to use direct instruction to tell students a formula, and ask them to keep in mind. But students may forget it very soon. This study adopts a variation based discovery learning strategy, which controls the similarity and difference among examples and problems to guide students to discover the critical features of mathematical concepts through personal observation and inference. A one-to-one technology-enhanced learning system is therefore designed to provide cognitive tools and scaffolding mechanisms for the guidance of discovery. To investigate the learning effect, a pre-test and a post test were conducted. The result of post test showed that the average score of the experiment group were significantly higher than that of the control group. The interview data revealed that the perspectives of some students showed higher self-confidence and learning motivation.

**Keywords:** Guided discovery learning, critical feature, one-to-one technology-enhanced learning, elementary mathematics

## 1. Introduction

The traditional mathematics education emphasizes on revealing and applying mathematical theorems. In other words, teachers tell students a formula and assign them exercises to make them keep in mind. But telling students a correct mathematical operator might hamper their ability of recall the operator [1], and also makes math a boring or daunting course.

Comparing with traditional direct instruction, education experts widely believed that discovery learning, which is different from providing complete explanation of concept in direct instruction, was an effective way of profound and lasting understanding for students' learning [9] [15] [10].Bruner [2] proposed four benefits of discovery learning better than direct instruction: (1) Growing intellectual. (2) Rewarding from the initiative discovered process and getting satisfaction. (3) Learning the "discovery" method, the capacity of analogy and independent learning. (4) Memorizing knowledge longer. McDaniel and Schlager [16] also pointed out that students in discovery learning were more able to utilize and expand knowledge. Students must discover and induct themselves.

However, only in discovery learning with proper guidance, students may learn better than in direct instruction [15]. The reason perhaps is that students tend to try aimlessly if discovery learning activities have no or insufficient guidance or students lack of precise objectives and discovery skills. Even if students are engaged in learning tasks, there are not much knowledge constructions. Furthermore, lacking clear instruction, students who have less prior knowledge are hard to get basic information and become frustration.

Besides, inappropriate guidance may force students followed the instruction one by one to complete the task; guiding too much also reduced the discovery ingredients. Therefore, guiding instance design and choices were extremely important. Related literature have discussed that discovery learning needed guidance or not (e.g. [10][15]), and what kinds of guidance were effective [5][19][7][17][18]. However, how to design and present learning materials in the discovery learning environment were limited, and the using of computer-aided discovery learning in mathematics was only sporadic [6].

This study proposes a discovery learning environment design which guide properly to promote students to find and organize the critical concepts through summarizing the mathematics text description. Therefore, this study has two research questions:

1) How is the learning effectiveness of variation-based-discovery learning?
2) Could the variation based discovery learning facilitate students' learning motivation?

## 2. Methods

### *2.1 Participants*

This study held an experiment on a formal mathematics class for one year. Each week, the experiment was conducted three times, and each time lasted 40 minutes. The participants were the third grade primary students in north Taiwan. There were two groups. One was experiment group (n = 26), which used the guided discovery learning approach supported with one to one devices to learn mathematics [4]. The control group (n = 26) used traditional direct instruction.

### *2.2 Design Structure of Learning Activity*

Many researches put into the development of guided tools in guided discovery learning [20]. Comparing with those guided tools, this study designs cognitive scaffolding tools to help students dealing with the task and go beyond their original extent [8] [11]. More specifically, the tools had the following characteristics:

1. Using “focus” to guide students discovering the critical features from examples for reducing the cognitive loading and establish the important attributes of concept [13].
2. Using learning content itself as the guidance to reduce the extra guidance and controlling the similarities and differences between examples to naturally highlight the critical features. In other words, “seeking common ground in diversity” and “seeking difference in similarities” may be easier noticed.

The design structure of learning material was divided into three parts: observation, identification, and generalization. In the observation step, the students focused on the relationship between examples and questions. The design of observation highlighted the critical point of concepts. In the identification step, students might check their possible assumption after completing the related questions. If students answer incorrect, they have to go back and observe the differences and similarities between examples and questions again; otherwise the next question would come out for students to make sure the possible manner and answer the next question on the correct base. In the generalization part, our design used short sentences and symbols as the algebra to help students describe the critical feature in a summarized statement for ensuring their mathematical concepts.

### *2.3 Theoretical Structure*

This study based on the variation theory which was originated from Bruner’s [2] discovery learning and proposed by Marton, Runesson, & Tsui [14]. Considering the primary school mathematical concepts of relevant learning theory and the assistive role computer played in the classroom, we controlled the similarity and differences of examples and established theoretical structure to help students learn mathematical concepts. Variation theory highlighted the key point which students needed to notice and identify from the overall phenomena. It was only when students noticed and linked more than one thing change in appearance at the same time, they could experience the similarities and differences [12] [14]. As for how to change was based on four modes of variation theory: (1) Contrast: Provide positive and negative examples in the same type to compare. (2) Generalization: Present different concept facet for students extracting the general rule. (3) Separation: Change the results corresponding to the value of key property, but other properties held constant. (4) Fusion: Multiple properties changes at the same time.

### *2.4 Data Collection*

This study considered two parts to evaluate the learning effect of variation based discovery learning: for the academic ability, learning effectiveness, pre-test in beginning of the first semester before first mathematics instruction and post test after one week of the midterms test were used and to prevent the practice effect, post-test using a parallel test and also changed the questions order and numbers; for the affection conservation, such as students' mathematics motivation. We interviewed teacher and six high, medium, and low performance students of experiment group about their learning perspective.

## 3. Result

### *3.1 Comparison of Learning Performance*

Using variation based discovery learning approach to learn the concept comprehension of mathematics, students needed to observe examples, imitate examples and words summarized to explain to the learning concepts. From the result of a t test to compare the pre-test scores of this approach with the traditional teacher's direct instruction, $t_{(2,\ 51)}$ = 0.447, $p$ =0.657>.05, there was no significant difference. However, after the experiment of half year, the average score of post test in direct instruction group was 63.5 points and that of variation based discovery learning approach group was 75.92 points. The average score difference of the two groups was 12.42 points. There was significant difference between two group, $t_{(2,\ 51)}$=-2.429, $p$=0.019<.05, so the learning performance of variation based discovery learning approach was better than the direct instruction.

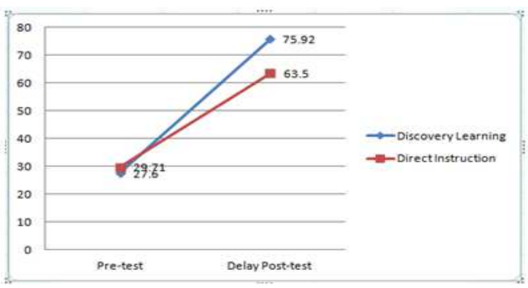


Figure 1. Pre ınd post-test scores of discovery learning and direct instruction

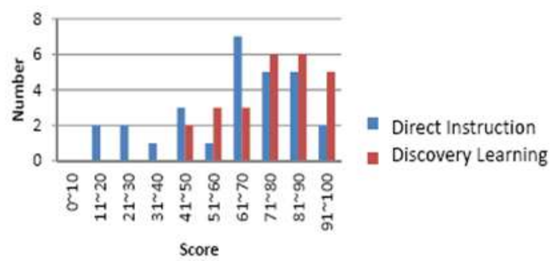


Figure 2. Post-test scores histogram of discovery learning and direct instruction

Furthermore, discovery group not only improved significantly, but their standard deviation also dropped from 18.24 points in the pre-test to 13.95 points in the post test. However, the standard deviation of direct instruction increased from 16.23 points to 21.90 points. It meant that using one to one computer supported discovery learning approach could lessen the students' learning difference.

### *3.2 Learning Interest*

Compared to traditional teachers teaching, students of low and medium achievement preferred variation based discovery learning, and they also cultivated the concentrative and quiet habits of self-directed learning in mathematics. A medium-achieving S1 said: "*I prefer the discovery learning more. I want to think. The teacher always teaches one after another, I feel that it was too fast*". Low-achieving students generally preferred the discovery learning. A low-achieving S11 said: "*I like to see, discover and find out the key points. I do not like the teachers telling me about what is the key point, although sometimes it is difficult to discover on my own*". Students feel more solid and less pressure to control their learning speed and proceed to the next concept after they real understand the critical feature rather than pursuit the teachers' fast instruction, to hurry completing a lesson but did not real know the learning content. So discovery learning enhanced the self-confidence and accomplishment of medium and low achieving students.

But not everyone liked computer-guided discovery learning. For example, the high-achieving students had different opinions. S5 responded that he liked the teacher instruction. He said "*computer instruction was boring, there were only math problems. It always asked me to finish the math questions. Unlike teacher would give us practice after teaching*." High achieving students could follow the teacher's teaching pace and relatively had no learning difficulties in the learning process. However, the variation based discovery learning only guided through questions, which was unable to meet the students' emotional needs and wanted a real teacher to promote their learning motivation and enthusiasm.

Some students were frustrated because some questions they tried many times yet not found the critical features. Some students needed additional guidance in variation based discovery learning to successfully find the critical feature between the questions presented. The teacher said: "*1/3 children were very excited and want to surpass themselves, but 2/3 children encountered bottlenecks. In variation based discovery learning, students had no way when they couldn't pass in one stage. So they would be very frustrated and afraid of this course.*" Even if the variation based discovery learning had cognitive scaffolding and prompted students to solve problems, it still unlike real teacher who could find out students' individual problems and provide suitable instruction. The current system couldn't do so precise detections of the problem difficulties. Students couldn't pass some stage by using variation based discovery learning cause them to stay put and feel depression.

Overall, most students expected teachers' oral encourage. Some students remarked that they were used to the traditional instruction which teachers and textbooks directly told the concepts and answers, even if they could answer by observing examples from the computer and quite had achievability, but still hoped to acquire knowledge from the direct instruction.

## 4. Conclusion

This study focused on the design method, learning effect, and learners' interesting in variation based discovery learning in 1:1 mathematics. This study proposed a guide strategy based on variation theory to control the similarities and differences between examples to facilitate students to be aware of the critical features of concepts for further analysis, reasoning and inductive to learn knowledge. According to this strategy, this study design computer-aided function to help students follow a certain learning process and discovery activities to ensure in correct exploring direction, and provide scaffolding to facilitate students' discovery process and describe results in a summarized form. This guide strategy gives students effective guidance and is able to retain the opportunity of self-discovery for students. Therefore, this study may bring new thinking direction for the research community of discovery learning.

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