Explaining My Solutions: An Integrated Model of Peer Tutoring for Facilitating Mathematical Communication Abilities

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Abstract: For elementary students, mathematical communication ability is a fundamental learning objective. Previous studies have showed that self-explanation could allow students to inspect what they have learned in the learning process, while peer tutoring could facilitate students to reorganize their learning knowledge as well as to express their idea. As a result, both could enhance students learning performance. Therefore, this study aims to design a system that may increase the mathematical communication abilities of elementary students. The study also conducts a learning activity which incorporates representation generation, self-explanation and peer-explanation in mathematical word problem solving. Preliminary evaluation shows that the integrated model may facilitate students' mathematical communication abilities.

Keywords: mathematical communication, self-explanation, peer-explanation, peer-tutoring

1. Mathematical Communication

Even if children have not learnt mathematics, they live in a world with numbers and shapes. Mathematics helps people understand the world by simplifying complex problems, solving them reasonably, and conveying the solution to other people persuasively. However, our primary education about mathematics focuses too much on problem solving and ignores the importance of mathematical communication. Mathematical communication involves adaptive reasoning (Kilpatrick, Swafford, & Findell, 2001, p. 170) and even argumentation (Andriessen, 2006).

In terms of adaptive reasoning, students have to acquire the ability to think logically, to explain a mathematical concept or procedure, and to justify their own or others' assertions. Adaptive reasoning also relates to the usage of representation (English, 1997). The ability to use appropriate representation can facilitate conceptual understanding, and problem solving. In terms of argumentation, students have to elaborate what they think, and to debate with sufficient evidences (Toulmin, 1958). When students attempt to build arguments, they aim to produce their mathematical ideas. For doing so, they may direct themselves to learn new concepts and procedures.

Self-explanation (or think aloud) is a domain-general learning strategy (Chi, de Leeuw, Chiu, & Lavancher, 1994), which emphasizes the linkage between prior knowledge and new one (Chi & van Lehn, 1991). Previous research has shown that successful problem solvers can generate more explanation (Chi, Bassok, Lewis, Reimann, & Glaser, 1989). Compared with self-explanation, peer-explanation is an interactive explanation strategy, which can be applied in a natural and social learning environment. Among various peer-explanation pedagogies, peer instruction is a widely adopted and effective pedagogy, which allows students to explain their own ideas for reducing misconceptions (Mazur, 1997).

Furthermore, students may benefit from tutoring others (Cohen, Kulik, & Kulik, 1982; Rohrbeck, Ginsburg-Block, Fantuzzo, & Miller, 2003) as well as preparing teaching materials (Ching, Chen, Chou, Deng, & Chan, 2005). Additionally, peer teaching facilitates spontaneous and appropriate use of diagrams in order to solve mathematics word problems (Uesaka & Manalo, 2007; 2011). Therefore, this study aims to design a system to support a peer tutoring model, which integrates generating representations, self-explanation and peer-explanation for facilitating students' mathematical communication ability.

2. Activity Design

As shown in FIGURE 1, the model consists of three main phases: material preparation, peer teaching and public teaching.

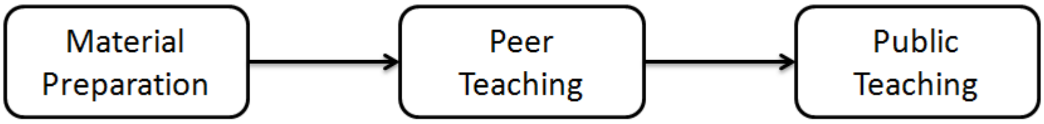


FIGURE 1. Activity Model

In the first phase, every two students are paired as a group and receive two similar but different mathematical word problems. They are told that they have to teach each other one of the two problems, and thus have to prepare their teaching materials. For doing so, they should solve their own word problems in a way to draw representations, to formulate expressions, and to calculate their answers on their own tablet PCs. They are also asked to practice their teaching by self explaining.

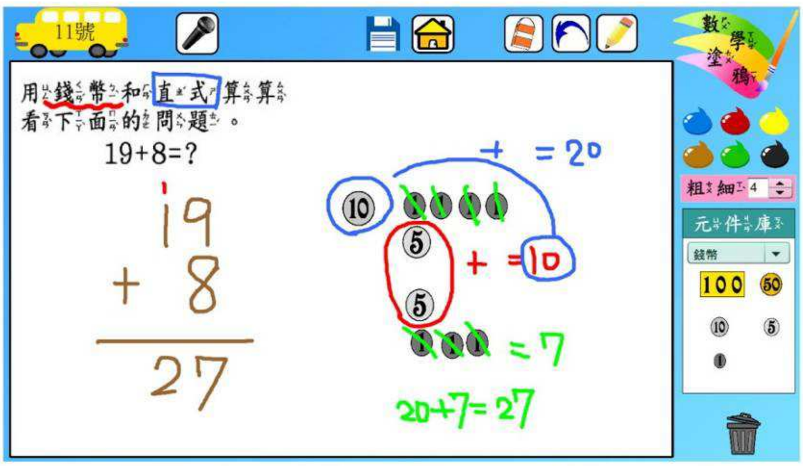


FIGURE 2. Interface

In the second phase, they reciprocally play as a tutor and a tutee. For this reason, the tutor has to teach his/her tutee how he/she solve the word problem. The tutor, more specifically, have to elaborate their representations and expressions to his/her tutee. After the elaboration, the tutee has to ask his/her tutor questions, and the tutor should answer the tutee.

In the third phase, the teacher conducts a session of public tutoring. The teacher may allow several tutor students to teach their own word problems. As the tutoring practices in the second phase, the tutor students have to elaborate their representations and expressions to the whole class. The other tutee students have to ask the tutor students questions related to the tutoring. The tutor students have to response these questions.

3. Preliminary Evaluation

3.1 Research questions

This study focuses on a main research question whether the peer tutoring model can facilitate students' mathematical communication ability. More specifically this research question can be divided into three sub-questions:

1. Can the peer tutoring model facilitate students' ability to explain self mathematical procedure?
2. Can the peer tutoring model facilitate students' ability to explain others' mathematical procedure?
3. Can the peer tutoring model facilitate students' ability to explain others' mathematical statements?

3.2 Settings

The participants were two second-year classes (N_1=25, N_2=26), in which students had similar mathematical communication abilities (see 3.3 for more details). One of the two classes was assigned as the experimental group, in which the integrated model of peer teaching was conducted for eight weeks. Another class was assigned as the control group, in which students received traditional courses of word problem solving.

The teacher in the experimental group could conduct the activity one or two sessions in a week and each session took 80 minutes. In this experiment, students were participated in the activity thirteen times in total. The materials were mathematical word problems, which involved addition, subtraction and multiplication. More specifically, in the first seven sessions, the word problems were about addition and/or subtraction, while they further involved multiplication in latter six sessions.

3.3 Measures

In this study, the dependent variable was the mathematical communication ability. For this purpose, a test on mathematical communication was conducted. This test, developed on the basis of a (Lin, & Lee, 2004), consisted of three sub-abilities: the ability to explain self procedures, the ability to explain others' procedure, and the ability to explain others' statements.

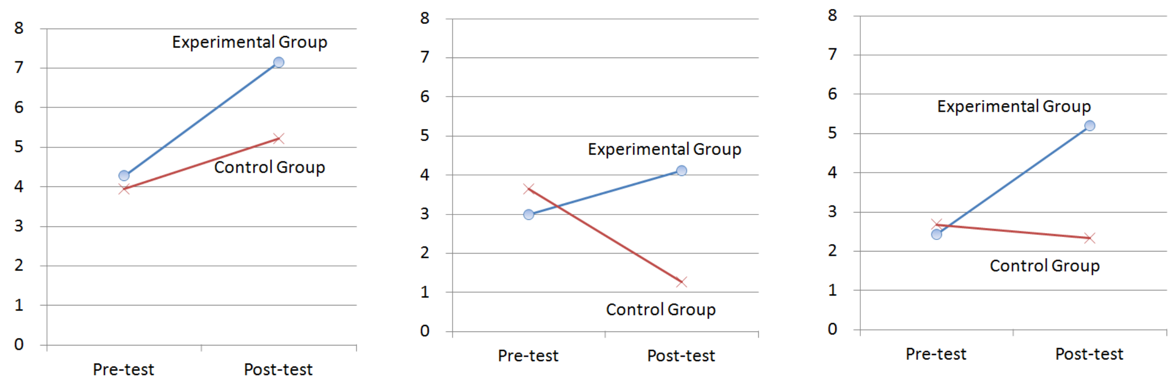
Independent t tests show that there are no significant differences between experimental and control groups in terms of explanation for self procedure ($t(49)=0.879$, $SE=0.362$, $p>0.05$), explanation for others' procedure ($t(49)=0.861$, $SE=0.759$, $p>0.05$), and explanation for others' statements ($t(49)=0.541$, $SE=0.466$, $p>0.05$). Therefore, the mathematical communication abilities of experimental and control groups are similar.

3.4 Results

FIGURE 3 illustrates the results of students' mathematical communication ability. First, in terms of the ability to explain self procedure, a two-way ANOVA reveals that there is a significant interaction between groups and time ($F(1, 49)=7.441$, $MSE=2.222$, $p<0.05$). As shown in FIGURE 3(a), both of the experimental and control group performed significantly better in the post-test than in the pre-test (the experimental group: $t(24)=7.955$, $SE=0.362$, $p<0.05$; the control group: $t(25)=2.744$, $SE=0.463$, $p<0.05$). An independent t test on the post-test further indicates that there is a significant difference between the experimental and control group ($t(49)=3.788$, $SE=0.509$, $p<0.05$). This result shows that the integrated model of peer tutoring can facilitate the ability to explain their own procedures. Although the ability of the control group increase as well, the students who have the experience of peer tutoring can improve more.

Second, in terms of the ability to explain others' procedure, a two-way ANOVA shows that there is a significant interaction between groups and time ($F(1, 49)=19.831$, $MSE=3.947$, $p<0.05$). Interestingly, while the experimental group improved significantly ($t(24)=2.206$, $SE=0.508$, $p<0.05$), the control group performed significantly worse in the post-test than in the pre-test ($t(25)=-3.990$, $SE=0.598$, $p<0.05$). The reason is probably that the pre-test asked students to distinguish and explain a wrong procedure, and the post-test asked student to distinguish and explain a correct procedure. The results may raise a further question 'can students explain more about a wrong procedure than about a correct one', which need further investigation.

Third, in terms of the ability to explain others' statements, a two-way ANOVA shows that there is a significant interaction between groups and time ($F(1, 49)=27.583$, $MSE=2.229$, $p<0.05$). Further analysis indicates that while the experimental group significantly improved their performance in the post-test ($t(24)=6.157$, $SE=0.448$, $p<0.05$), the performance of the control group did not change significantly ($t(25)=-0.892$, $SE=0.388$, $p>0.05$). The results suggest that the experience of peer tutoring can facilitate students to understand and explain others' statements.



(a) explanation for self procedure (b) explanation for others' procedures (c) explanation for others' statements

FIGURE 3. The effect on mathematical communication abilities

4. Concluding Remarks

This study aims to incorporate representation generation, self-explanation and peer-explanation into an integrated peer tutoring model in order to facilitate students' mathematical communication abilities. The results showed that the integrated model may significantly improve students' ability to explain self procedures, others' procedures, and others' statements. Furthermore, this study also revealed that students could explain procedures in a more complete and more contextualized way. Besides, students could generate more and more abstract representations, when they prepared teaching materials for teaching their classmates. Students were also found that they became more enthusiastic and confident about teaching in public. These findings suggested that the integrated model could help not only students' cognition, but also their affects.

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