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Exploring Patterns in Calculus Learning: An Achievement-Based Evaluation of the Funk-SVD System

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Abstract: This study applies a Funk-SVD collaborative filtering model to analyze calculus learning data across student groups with varying achievement levels. By examining recommended items, recommendation intensity, and conceptual focus, the study aims to explore students' knowledge structures and identify group-specific learning difficulties. The results indicate that high-achieving students were frequently recommended advanced theorems and integrative tasks; medium-achieving students were presented with items involving conditional reasoning and representational shifts; while low-achieving students primarily received reinforcement on basic definitions and common conceptual misconceptions. Although all three groups demonstrated certain shared difficulties in core topics such as integration and differentiability, they exhibited distinct cognitive demands overall. Based on these findings, it is recommended that instructors utilize the results to inform differentiated instructional design and make pedagogical adjustments tailored to students' learning profiles. In addition, future research may further address the recommendation gap caused by data sparsity and confidence asymmetry, aiming to optimize the system for enhancing access to key conceptual content across achievement levels.

Keywords: Funk-SVD, recommendation system, calculus learning, differentiated instruction, learning analytics

1. Introduction

Calculus is often viewed as an academic threshold in science and engineering programs, requiring students to integrate abstract reasoning, procedural skills, and spatial visualization. As student backgrounds become increasingly diverse, learning gaps have widened. Haciomeroglu et al. (2013) found that students favoring visual representations outperform those with analytical preferences, underscoring the impact of learning styles on calculus performance. Similarly, Eun-ho Moon et al. (2023) emphasized the need for phased, discipline-sensitive course design to address varying levels of prerequisite knowledge.

In Taiwan, the disconnection between high school mathematics and university calculus is especially pronounced among students admitted via non-traditional pathways. While many instructors offer remedial support, time and resource constraints often limit the depth of instruction. As a result, precalculus bridging courses have become a key strategy in higher education.

Beyond content knowledge, calculus success also depends on students' metacognitive strategies and engagement. Lust et al. (2013) and Duran et al. (2024) highlighted the importance of adapting learning tools, fostering collaboration, and promoting reflective practices. Yet, many students still struggle with self-regulated learning.

This study introduces a novel approach by applying the Funk-SVD recommendation system to analyze calculus learning. Unlike traditional studies using tests and surveys, this model identifies topic-specific learning needs across different performance levels. Given the limited educational research using recommendation systems, this study also evaluates the conceptual alignment and recommendation patterns generated by Funk-SVD, offering insights for differentiated instruction and personalized learning pathways.

2. Research Aims and Key Questions

This study applies the Funk-SVD recommendation model to examine calculus learning patterns across student groups with different achievement levels. It aims to explore students' conceptual understanding and knowledge structures, and to identify commonalities and differences in mastery of core concepts and skills. The guiding research questions are:

1. What learning trends and characteristics emerge from the recommended items for high-, medium-, and low-achieving students?
2. Do achievement groups differ in their difficulties with core calculus concepts based on the recommended items?

3. Related Work

In mathematics learning, knowledge is structured as a network of interconnected concepts rather than isolated units (Morton & Bekerian, 1986). Effective learning requires not only acquiring correct procedures but also forming coherent conceptual systems. Siyepu (2013) noted students often make conceptual and interpretive errors when learning derivatives. APOS-based instruction supports transitions from procedural to conceptual understanding, especially in trigonometric differentiation.

Students' representational preferences also affect their conceptual development. Tarmizi (2010) argued that visual representations of abstract ideas—like limits and rates of change—can reduce symbolic dependency and foster holistic understanding. This aligns with a concept-centered model where visual support strengthens knowledge structures.

Individual learning styles further influence calculus outcomes. Sternberg et al. (2008) advocated for integrating analytical, creative, and practical activities to support diverse learners. Mayer and Massa (2003) highlighted that learning preferences (e.g., visual vs. verbal) affect academic performance independently of cognitive ability.

Self-regulated learning (SRL) also plays a key role. DiFrancesca et al. (2015) found that high-achieving students use metacognitive strategies effectively, while low-achievers rely on rote methods and show overconfidence. Structured interventions are thus essential. Duran et al. (2024) reported that active learning benefits students with negative math attitudes, reinforcing the need for adaptive instruction.

4. Methodology

4.1 Participants and Data Source

This study used a de-identified dataset from a Taiwanese university, comprising responses from over 850 first-year students enrolled in compulsory calculus courses during 2022–2023. Across two semesters, students completed six 150-minute quizzes, each with 10 true/false and 8 fill-in-the-blank items. A total of 108 distinct problems assessed core calculus topics, including differentiation, integration, and multivariable applications. Responses were recorded as binary values.

4.2 System Experimental Procedure

Based on the Funk-SVD model(Hsu & Hsu, 2025), the top 36 recommended items for each student—approximately 30% of the total item pool—were extracted as their personalized recommendation list. Students were then categorized into three achievement groups: the top 30% were classified as high-achieving, the middle 40% (31st–70th percentile) as medium-achieving, and the bottom 30% as low-achieving. For each group, the recommended items were aggregated and the top 10 most frequently recommended items were identified. To further investigate performance trends and knowledge gaps, the conceptual composition of the recommended items was cross-referenced with the knowledge topics of calculus, allowing for an analysis of the specific conceptual deficiency's characteristic of each achievement group.

4.3 Data Analysis Framework

To infer students' learning needs across achievement groups, this study analyzes the content attributes and recommendation intensity of items generated by the Funk-SVD system. Higher recommendation frequencies suggest lower proficiency or unstable understanding, marking these items as key reinforcement targets. By mapping each item's conceptual attributes to its cognitive level, the model helps infer students' knowledge construction stages and learning challenges.

The framework includes two levels:
(1) An inverse relationship between recommendation intensity and proficiency—frequent recommendations indicate predicted low performance or confidence; infrequent recommendations suggest mastery or undetected difficulties.
(2) The categorization of task characteristics and their corresponding learning demands in Table 1 is grounded in the Depth of Knowledge (DoK) framework proposed by Webb (1997). This framework classifies mathematical tasks according to the cognitive complexity they require, progressing from basic recall and procedural skills, to strategic reasoning and extended conceptual integration. By aligning each task type with a specific DoK level, the study captures not only the thematic focus of recommended items, but also the depth of reasoning students must engage in to complete them (see Table 1).

Table 1. *Cognitive Demands and Thematic Characteristics of Mathematics Tasks*

DoK Level	Associated Learning Topics	Description
DoK 1 – Recall	- Derivatives of Trigonometric Functions - Basic Integration - Elementary Differential Operations - Fundamental Differential (basic use)	Requires recalling definitions or formulas and performing routine procedures; does not involve conditional reasoning or multi-step logic.
DoK 2 – Skills and Concepts	- Definition of Differential - Partial Derivative - Critical Points - Differentiability Implies Continuity - Fundamental Differential (with condition recognition)	Involves identifying and applying core concepts and procedures with basic understanding; requires accurate execution but not advanced strategies.
DoK 3 – Strategic Thinking	- Chain Rule (in multivariable contexts) - Directional Derivative - The Convergence Theorem for Power Series (standard form) - The Mixed Derivative Theorem (conditional application) - Curvature (basic geometric interpretation)	Involves reasoning under conditions, multi-step processing, choosing appropriate strategies, and coordinating visual-symbolic representations.
DoK 4 – Extended Thinking	- Multivariable Chain Rule with Coordinate Transformations - Curvature in Parametric or 3D Contexts - Mixed Derivative Theorem with Variable Dependencies - Power Series Convergence across Representations - Multi-concept Integrative Examples	Involves integration of multiple concepts, abstract model construction, application in novel contexts, and extended reasoning over time.

5. Results

The findings of this study reveal that Riemann integration and the chain rule emerged as the most frequently recommended topics across high-, medium-, and low-achieving student groups. However, the uniformity in recommendation frequency does not imply that students experienced identical difficulties with these concepts. In fact, both topics are structurally central and broadly applicable in calculus learning, involving multi-layered cognitive processes ranging from basic recall to advanced reasoning. Further analysis indicates that students' difficulties with these core concepts varied in alignment with different Depth of Knowledge (DoK) levels. Specifically, low-achieving students primarily exhibited difficulties at DoK Levels 1–2, characterized by inadequate understanding of definitions and insufficient procedural fluency. Medium-achieving students faced challenges at DoK Levels 2–3, particularly in conditional reasoning and the correct application of conceptual rules. High-achieving students demonstrated challenges at DoK Levels 3–4, involving conceptual integration, cross-topic transfer, and abstract reasoning—such as applying the chain rule in multivariable contexts or extending the idea of Riemann integration to nonstandard structures like parametric curves or variable upper limits. These findings underscore the need for differentiated interpretation and instructional design, even when students encounter the same mathematical concepts.

A closer examination of each achievement group's recommendation profile and corresponding pedagogical implications is provided below:

High-achieving students were frequently recommended tasks involving advanced calculus concepts and conceptual transformation. Common topics included Riemann integrals (297 times), the chain rule (284), power series convergence (248), and the mixed partial derivative theorem (228), highlighting persistent challenges in deep conceptualization and strategic application (see Table 2). These tasks require high-level reasoning about continuity, differentiability, and structural transformation, suggesting that students have entered a phase of theoretical abstraction. In addition, the frequent appearance of geometrically-oriented items such as directional derivatives (219), partial derivatives (227), and curvature (224) further indicates difficulties in coordinating spatial intuition with symbolic representation. This pattern reflects students' cognitive demands at DoK Levels 3–4. Instruction should therefore focus on structured reasoning, error analysis, and conceptual reconstruction through tasks that emphasize multiple representations and higher-order thinking.

Table 2. *High-achieving group: Top Recommended Questions*

	Rank	Questions	Recommended Times	Topics
High-achieving group	1	Quiz_2, Question_13	297	Riemann Integral, Basic Integration
	2	Quiz_1, Question_8	284	Fundamental Differential, Chain Rule
	3	Quiz_4, Question_4	248	The Convergence Theorem for Power Series
	4	Quiz_2, Question_10	230	Critical Points, Example
	5	Quiz_1, Question_16	228	Derivatives of Trigonometric Functions, Fundamental Differential
	6	Quiz_5, Question_7	228	Elementary Differential Operations, The Mixed Derivative Theorem
	7	Quiz_6, Question_7	227	Definition of Differential, Partial Derivative, Example
	8	Quiz_5, Question_8	224	Curvature
	9	Quiz_6, Question_8	219	The Directional Derivative, Definition of Differential,

				Partial Derivative, Example
	10	Quiz_4, Question_5	216	Example

Medium-achieving students exhibited recommendation patterns reflecting the need to consolidate foundational concepts and strengthen conceptual linkage. Although topics like Riemann integrals (299) and the chain rule (287) appeared frequently, the source of difficulty was primarily associated with unclear conditional logic and procedural misapplication. This group also showed instability in reasoning with concepts such as "differentiability implies continuity" and in applying partial and directional derivatives (see Table 3). These challenges suggest that students are transitioning between DoK Levels 2 and 3, requiring support in linking concepts, interpreting geometric representations, and organizing mathematical ideas strategically. Instructional approaches should thus emphasize the refinement of conceptual connections, guided use of representations, and the development of structured mathematical thinking.

Table 3. *Medium-achieving Group: Top Recommended Questions*

	Rank	Questions	Recommended Times	Topics
Medium-achieving Group	1	Quiz_2, Question_13	393	Riemann Integral, Basic Integration
	2	Quiz_1, Question_8	374	Fundamental Differential, Chain Rule
	3	Quiz_6, Question_8	307	The Directional Derivative, Definition of Differential, Partial Derivative, Example
	4	Quiz_5, Question_8	303	Curvature
	5	Quiz_2, Question_10	293	Critical Points, Example
	6	Quiz_4, Question_4	289	The Convergence Theorem for Power Series
	7	Quiz_4, Question_5	278	Example
	8	Quiz_1, Question_16	278	Derivatives of Trigonometric Functions, Fundamental Differential
	9	Quiz_5, Question_7	272	Fundamental Differential, The Mixed Derivative Theorem
	10	Quiz_6, Question_7	270	Definition of Differential, Partial Derivative, Example

Low-achieving students were primarily recommended tasks focused on core concept reconstruction and the correction of misconceptions. Frequent items included Riemann integrals (301), the chain rule (281), and foundational topics such as "differentiability implies continuity" (211) and the power series convergence theorem (242) (see Table 4). Difficulties in this group often stemmed from fragile understanding of definitions, logical reasoning, and procedural clarity. Although they encountered multivariable topics such as curvature and critical points, symbolic errors and weak representational interpretation revealed a fragmented conceptual structure. This group's cognitive challenges align with DoK Levels 1–2. Instruction should therefore prioritize rebuilding foundational knowledge, clarifying procedural logic, and addressing conceptual biases. Recommended strategies include comparing similar concepts (e.g., partial vs. directional derivatives), constructing concept maps, diagnosing common errors, and incorporating semantic cues to shift students from rote memorization toward meaningful understanding, while reinforcing links between mathematical language, symbolism, and visualization.

Table 4. *Low-Achieving Group: Top Recommended Questions*

	Rank	Questions	Recommended Times	Topics
Low-Achieving Group	1	Quiz_2, Question_13	301	Riemann Integral, Basic Integration
	2	Quiz_1, Question_8	281	Fundamental Differential, Chain Rule
	3	Quiz_4, Question_4	242	The Convergence Theorem for Power Series
	4	Quiz_6, Question_8	227	The Directional Derivative, Definition of Differential, Partial Derivative, Example
	5	Quiz_5, Question_8	226	Curvature
	6	Quiz_5, Question_7	220	Fundamental Differential, The Mixed Derivative Theorem
	7	Quiz_6, Question_7	220	Definition of Differential, Partial Derivative, Example
	8	Quiz_1, Question_16	218	Derivatives of Trigonometric Functions, Fundamental Differential
	9	Quiz_1, Question_9	211	Differentiability Implies Continuity
	10	Quiz_4, Question_5	209	Example

In summary, the integration of recommendation system outputs with a DoK-based interpretive framework provides a more nuanced understanding of student learning challenges and informs targeted instructional interventions. Future research may explore the alignment between recommended tasks and students' actual performance trajectories, and how dynamic recommendation mechanisms can better support cross-level conceptual transfer in individualized learning environments.

6. Conclusion

This study applied the Funk-SVD collaborative filtering model to analyze calculus learning data across students with varying achievement levels, addressing two key questions: (1) What learning trends emerge from the recommended items for high-, medium-, and low-achieving students? and (2) Do these groups differ in their difficulties with core calculus concepts?

For the first question, the analysis revealed distinct learning patterns. High-achieving students were recommended advanced theorems and integrative tasks (e.g., multivariable calculus, curvature), reflecting a focus on conceptual deepening. Medium achievers received items involving conditional reasoning and procedural application, suggesting a transition toward relational understanding. Low achievers were recommended foundational topics related to definitions, symbolic errors, and misconceptions, indicating continued struggles with basic concepts and accuracy.

Regarding the second question, while core topics like Riemann integration and the chain rule appeared across all groups, the associated cognitive demands differed. High achievers struggled with abstract applications, whereas low achievers had difficulty with basic recall and execution. These differences highlight how similar recommendations can reflect qualitatively different learning challenges, which can be interpreted through the Depth of Knowledge (DoK) framework.

Overall, the study demonstrates the diagnostic potential of recommendation systems not only for content delivery, but also for informing differentiated instruction and identifying learning gaps.

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